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Data-driven decision-making for enhancing aerobic and anaerobic performance in runners

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Abstract

The ability to undertake aerobic and anaerobic activities is critical for runners since these activities have an impact on endurance, speed, and overall performance. The capacity for aerobic exercise allows for prolonged effort throughout long runs, whereas the power of anaerobic exercise allows for rapid bursts of speed and efficient recovery. This article presents a data-driven decision-making technique that incorporates a cubic set with a q-rung Orthopair fuzzy set called cubic q-rung Orthopair fuzzy set (Cq-ROFS) to investigate the aerobic and anaerobic performance of runners. An introduction to Cq-ROFS is provided first, followed by its definition. Secondly, we suggest the Cq-ROF weighted average operator as a means of efficiently aggregating Cq-ROF information. Third, in order to determine the weights of attributes, a Cq-ROF-criterion impact loss method is built. The Cq-ROF-weighted aggregated sum product assessment method is then developed for the purpose of measuring runners' aerobic and anaerobic performance. In the end, we use comparative analysis to show how our proposed strategy is superior to others.

Keywords: Aerobic and anaerobic performance of runners, Cq-ROFS, Criterion impact loss method, Weighted aggregated sum product assessment method, and Weighted average operator.

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1. Introduction

Decision-making is an effective mechanism for identifying the primary options among the available choices. Numerous researchers have been provided with a combination of concepts to obtain accurate results. Historically, judgments were made based on precise numerical data. This strategy is less effective for making appropriate selections. Over time, the challenges associated with methodology have made it harder for decision-makers to manage the ambiguity of information using conventional methods. Fuzzy sets (FS) are the result of scholars presenting the data. By introducing the idea of membership to express subjective judgments, Zadeh [1] introduced fuzzy set (FS) theory in 1965. Following this, other more sophisticated varieties of fuzzy sets have been suggested, such as interval-valued fuzzy sets (IVFS) [2], intuitionistic fuzzy sets (IFS) [3], Pythagorean fuzzy sets (PFS) [4], and q-rung orthopair fuzzy sets (q-ROFS) [5]. To overcome the restrictions on information expression in the PFS and IFS, the q-ROFS generalizes these systems by ensuring that the sum of the q-power of the membership degree and the q-power of the non-membership degree cannot exceed the interval [0, 1]. For varied q, IFS and PFS both stand for certain cases of the q-ROFS. It is arguable that q-ROFS is broader. As a result, q-ROFS gives decision-makers a wider selection of options to convey their uncertainty [6, 7]. The provision of both interval values and related fixed-value evaluation information may be required of decision-makers in practice. This merged fuzzy data cannot be included in the previously specified singular form of fuzzy set and its extended collection. Jun, et al. [8] introduced the idea of the cubic set (CS) as a novel extension of fuzzy sets (FS). An interval-valued fuzzy set (IVFS) and a fuzzy set (FS) are two examples of deterministic and interval information that are combined in a CS. As an alternative to FS, CS can accommodate decision-makers in real-world scenarios who need to articulate many evaluation formats for an assessment item by simultaneously covering many types of fuzzy evaluation information. A number of procedures and properties of the cubic Iterated Function System (IFS) that are relevant to decision-making with multiple criteria were proposed by Faizi, et al. [9]. Using geometric aggregation operations, Riaz and Tehrim [10] studied cubic bipolar fuzzy sets within the framework of group decision-making involving several attributes. The similarity and Pythagorean reliability metrics of multivalued neutrosophic cognitive systems were investigated by Wang and Zhao [11]. For decision-making with multiple attributes, Saeed, et al. [12] used cubic Pythagorean fuzzy soft sets. In their work, Farhadinia and Farhadinia [13] created cubic hesitant fuzzy sets and presented a variant of these sets called triangular cubic hesitant fuzzy sets. The cubic q-rung orthopair fuzzy was first proposed by Zhang, et al. [14]. They analyzed the use of Heronian mean operators in group decision-making with multiple attributes. When dealing with large amounts of uncertain data accompanied by phase transitions, cubic fuzzy information falls short in capturing the partial ignorance of data and its fluctuations during a single execution phase. In light of this, we presented a novel hybrid model, 2 Cq-ROFS, that combines q-ROFS with CS. Complex multi-attribute group decision-making problems are better addressed by it since it incorporates more information than elaborate FSs and CSs. Naz, et al. [15] studied the CILOS-TOPSIS method for evaluating hydrological geographical areas relevant to watershed management in Pakistan using a 2-tuple linguistic cubic q-rung orthopair. Using the current weighted aggregated sum product assessment (WASPAS) approach and Pythagorean fuzzy sets, Ou and Chen [16] analyzed e-commerce websites, including those from Chnsenti Corp, the IMDB dataset, and the Amazon review dataset. Zou, et al. [17] examined the operating hazards of batteries in energy storage power stations by modifying the WASPAS method for multi-attribute decision-making using triangular fuzzy neutrosophic numbers. Using a multi-objective genetic algorithm and WASPAS, Panda, et al. [18] optimized process parameters for milling Ti-6Al-4V material. The most dangerous part of the Kerman water conveyance tunnel was located by Ghorbani, et al. [19] using multi-criteria decision-making methods, such as the COCOSo, WASPAS, and PROMETHEE II models. Fuzziness is better characterized by combining CS with Cq-ROFS, which is called Cq-ROFS [14]. It allows decision-makers to take into account a broader spectrum of uncertainty, including issues like information gaps and disagreements across expert groups, which leads to a more accurate assessment of the decision-making process. Modern decision-makers frequently use the weighted average (WA) operator for data aggregation since it can be enhanced by merging it with the Cq-ROFS. When using subjective techniques, the decision-makers' subjective opinions are used to calculate the virtue weights. As a result, the CILOS technique [20] is a useful ranking decision-making strategy and an efficient way to determine the weights of attributes. Under the Cq-ROFS framework, the optimal decision is chosen by the WASPAS method [21]. In order to synthesize informational data, Cq-ROFS is presented, and the WA aggregation operator is developed. Finally, in order to construct the attribute weights and priorities of the alternatives, the Cq-ROFS-based CILOS-WASPAS method is put into place. For more information about decision-making, see [22]. Given that we do not believe the above discussions have ever taken place before, our endeavor is very novel. Below is a detailed outline of the study's framework:

The primary ideas of the study are laid out in Section 2. The CILOS-WASPAS model is established for decision-making difficulties in Section 3. Part 3, Section 4 offers a case study and compares it to existing methods. Here, in Section 5, we provide a concise and clear summary of the paper's results and conclusions.

2. Basic Knowledge

In this section, the basic knowledge of q-ROFS and Cq-ROFS are provided.

Definition 2.1. [5] Let $R=\{\sigma_1,\sigma_2,...,\sigma_\tau\}$ be a standard set. A q-ROFS $\tilde{\beth}$ on $\Re$ is characterized as:

$$\hat{u}=\{(\sigma_\dagger,(\phi_{\hat{u}}(\sigma_\dagger),\psi_{\hat{u}}(\sigma_\dagger)))\,|\,\sigma_\dagger\in R\},$$

where $\phi_{\hat{u}}(\sigma_\dagger),\psi_{\hat{u}}(\sigma_\dagger)\ :\Re\to[0,\ 1]$ denotes the membership and non-membership degrees,

correspondingly, proving:

$$0\le(\phi_{\hat{u}}(\sigma_\dagger))^q+(\psi_{\hat{u}}(\sigma_\dagger))^q\le 1\ \ ,(q\ge 1)$$

The indeterminacy degree is:

$$\tilde{n}_{\hat{u}}(\sigma_\dagger)=(1-(\phi_{\hat{u}}(\sigma_\dagger))^q-(\psi_{\hat{u}}(\sigma_\dagger))^q)^{1/q}$$

A q-ROFN is represented as $” =(f,\psi)$.

Definition 2.2. [20] Let $\Re = \{\sigma_1, \sigma_2, \ldots, \sigma_{‡}\}$ A Cq-ROFS $\tilde{C}$ on $\Re$ is:

$$\tilde{C} = \{(\sigma, (\tilde{\zeta}(\sigma), \upsilon(\sigma)) \mid \sigma \in \Re\} \qquad , q \geq 1,$$

where:

$\tilde{\zeta}(\sigma) = ([\phi_{\in}^{-}(\sigma), \phi_{\in}^{+}(\sigma)], [\psi_{\in}^{-}(\sigma), \psi_{\in}^{+}(\sigma)])$ is an interval-valued q-ROFS with:

$$0 \leq \phi_{\in}^{+}(\sigma)^q + \psi_{\in}^{+}(\sigma)^q \leq 1,$$

$$0 \leq \phi_{\in}^{-}(\sigma)^q + \psi_{\in}^{-}(\sigma)^q \leq 1,$$

$\upsilon(\sigma) = (\phi_{\in}^{-}(\sigma), \psi_{\in}^{-}(\sigma))$ is a q-ROFS.

The indeterminacy degree of $\sigma \in \tilde{C}$ is:

$$\tilde{n}(\sigma) = ([\tilde{n}_{\in}^{-}(\sigma), \tilde{n}_{\in}^{+}(\sigma)], \tilde{n}_{\in}^{-}(\sigma)),$$

where:

$$\tilde{n}_{\in}^{-}(\sigma) = (1 - \phi_{\in}^{+}(\sigma)^q - \psi_{\in}^{+}(\sigma)^q)^{1/q}$$

$$\tilde{n}_{\in}^{+}(\sigma) = (1 - \phi_{\in}^{-}(\sigma)^q - \psi_{\in}^{-}(\sigma)^q)^{1/q}$$

A simplified form of Cq-ROFN is:

$$\tilde{C} = (([\phi_{\in}^{-}, \phi_{\in}^{+}], [\psi_{\in}^{-}, \psi_{\in}^{+}]), [\phi_{\in}^{-}, \psi_{\in}^{+}]).$$

For more simplification, a Cq-ROFS can be written as $[\phi_{\in}^{-}, \phi_{\in}^{+}] = u, [\psi_{\in}^{-}, \psi_{\in}^{+}] = \dot{u}$ and $(\phi_{\in}^{-}, \psi_{\in}^{+}) = \ddot{u}$

3. Methodology

This section presents a modification of the CILOS and WASPAS methods to a multi-attribute decision-making problem in a Cq-ROF framework. The process involves applying the Cq- ROFWA operator to combine multiple inputs into a single decision. This operator makes it possible to calculate the total values of the alternatives depending on the specified attributes. In a Cq-ROF environment, the modified CILOS-WASPAS approach offers an organized approach to rank alternatives. To address the decision-making problem, we consider a set of b alternatives $\mathbf{A} = \{\mathbf{A}_1, \mathbf{A}_2, ..., \mathbf{A}_b\}$, d attributes $\mathbf{N} = \{\mathbf{N}_1, \mathbf{N}_2, ..., \mathbf{N}_d\}$. Let $\pi = (\pi_1, \pi_2, ..., \pi_d)^T$ be the weighting vector for the attributes, where

$$\pi_{‡} \in [0,1], \sum_{‡=1}^{d} \pi_{‡} = 1.$$

The process for applying the Cq-ROF-CILOS-WASPAS method is outlined in the following steps.

Step 1. Establish the Cq-ROF evaluation matrix

$$B_{†‡} = [\tilde{C}†‡]_{b \times d} = ([\phi_{†‡}^{-}(\sigma), \phi_{†‡}^{+}(\sigma)], [\psi_{†‡}^{-}(\sigma), \psi_{†‡}^{+}(\sigma)]), (\phi_{†‡}^{-}(\sigma), \psi_{†‡}^{-}(\sigma).$$

Step 2. Form an aggregated Cq-ROF decision matrix (represented by r) by using the Cq-ROFWA operator from Equation (3.1).

$$\text{Cq-ROFWA}\ (\tilde{C}_1, \tilde{C}_2, \ldots, \tilde{C}_d) = \bigoplus_{‡=1}^{d} \pi_{‡} \tilde{C}_{‡}$$

$$= \begin{bmatrix} ([(1 - \prod_{‡=1}^{d} (1 - (\phi^{-})_{‡}^{q})^{\pi ‡})^{1/q}, (1 - \prod_{‡=1}^{d} (1 - (\phi^{+})_{‡}^{q})^{\pi ‡})^{1/q}], [\prod_{‡=1}^{d} (\psi^{-})_{‡}^{\pi ‡}, \prod_{‡=1}^{d} (\psi^{+})_{‡}^{\pi ‡}]) \\ [(1 - \prod_{‡=1}^{d} (1 - (\phi^{-})_{‡}^{q})^{\pi ‡})^{1/q}, \prod_{‡=1}^{d} (\psi^{-})_{‡}^{\pi ‡}] \end{bmatrix}$$

Step 3. Equations (3.2) and (3.3) are used to normalize positive and negative attributes, respectively.

$$r^{*}_{\dagger\ddagger} = \frac{r_{\dagger\ddagger}}{\max\limits_{\dagger} r_{\dagger\ddagger}}, \qquad \dagger = 1,\ldots,b, \qquad \ddagger = 1,\ldots,d \qquad (3.2)$$

$$r^{*}_{\dagger\ddagger} = \frac{\max\limits_{\dagger} r_{\dagger\ddagger}}{r_{\dagger\ddagger}}, \qquad \dagger = 1,\ldots,b, \qquad \ddagger = 1,\ldots,d \qquad (3.3)$$

where $r^{*}_{\dagger\ddagger}$ represents the normalized value of the aggregated decision matrix for the $\dagger$ -th alternative in the $\ddagger$ -th attribute.

Step 4. Employ the CILOS weighting method to calculate the weights of attributes:

Step 4.1. To construct a square matrix B, we select the values $x_{k\dagger\ddagger}$ from the matrix X that correspond to the maximum values of the $\dagger$-th attribute. This selection process is performed according to Equation (3.4), where $k^{\dagger}$ represents the number of rows associated with the $\dagger$-th attribute. In the resulting square matrix B, the highest values for each attribute are positioned along the main diagonal. Subsequently, we formulate the matrix p, which represents the relative loss of attribute significance. This is achieved by applying Equation (3.5).

$$\text{B} = \left\| b_{\dagger\ddagger} \right\|, \qquad b_{\dagger\dagger} = x_{\dagger}, \qquad b_{\dagger\ddagger} = x_{k\dagger\ddagger}, \qquad (3.4)$$

$$\text{p} = \left\| P_{\dagger\ddagger} \right\| \qquad (3.5)$$

Step 4.2. The process of determining the relative impact loss matrix is conducted in the following manner:

$$P_{\dagger\ddagger} = \frac{\chi_{\ddagger} - b_{\dagger\ddagger}}{\chi_{\ddagger}} = \frac{b_{\dagger\dagger} - b_{\dagger\ddagger}}{b_{\dagger\dagger}}, \qquad (p_{\dagger\ddagger} = 0; \dagger, \ddagger = 1,2,3,\ldots,d) \qquad (3.6)$$

where, $p_{\dagger\ddagger}$ represents the relative loss linked to the $\ddagger$ -th attribute when the $\dagger$ -th attribute is selected as the best.

Step 4.3. The weight system matrix F is constructed based on the matrix (3.7) provided below. Subsequently, the weights $q'_{\ddagger} = (q'_1, q'_2, \ldots, q'_d)^T$ for each attribute are obtained by solving a system of linear homogeneous equations described in Equation (3.8).

$$F = \begin{bmatrix} -\sum\limits_{\dagger=1}^{d} P_{\dagger 1} & P_{12} & \cdots & P_{1d} \\ P21 & -\sum\limits_{\dagger=1}^{d} P_{\dagger 2} & \cdots & P_{2d} \\ \cdots & \cdots & \cdots & \cdots \\ P_{d1} & P_{d2} & \cdots & -\sum\limits_{\dagger=1}^{d} P_{\dagger d} \end{bmatrix}_{d \times d} \qquad (3.7)$$

$$F \times q'_{\ddagger} = 0 \qquad (3.8)$$

Step 4.4. The attribute weights $q'_{\ddagger}$ are obtained by solving the homogeneous linear system of equations using the procedure proposed by Atanassov [3].

$$q'_{\ddagger} = F^{-1} A \qquad (3.9)$$

where A be a vector close to 0. In order to determine the value of A, we make the assumption that the first element of A is approximately 0, while the remaining elements are all zeros. This allows us to represent the vector A in the following form:

$$\text{A} = [0.49\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]^{T} \qquad (3.10)$$

consequently, the weight vector $q'_{\ddagger}$ normalized to ensure that the sum of its elements satisfies the condition $\sum\limits_{\ddagger=1}^{d} q'_{\ddagger} = 1$

$$ý_{†}^{(1)} = \sum_{‡=1}^{d} r_{‡†}^{*} \ .\pi_{‡} \ , \qquad † = 1,...,d \qquad (3.11)$$

where $\pi_{‡}$ denotes the weight of the ‡ -th attribute $(\pi_1, \ \pi_2, \ ..., \ \pi_d)$, and $ý_{†}^{(1)}$ represents the additive relative importance of the † -th alternative.

Step 6. The multiplicative relative importance is determined by:

$$ý_{†}^{(2)} = \prod_{‡=1}^{d} (r_{†‡}^{*})^{\pi ‡}, \qquad † = 1,...,d \qquad (3.12)$$

where $ý_{†}^{(2)}$ denotes the multiplicative relative importance of the † -th alternative.

Step 7. The integrated criterion combining additive and multiplicative approaches is defined as:

$$ý_{†} = \frac{1}{2}(ý_{†}^{(1)} + ý_{†}^{(2)}) = (\sum_{‡=1}^{d} r_{‡†}^{*} \ .\pi_{‡} + \prod_{‡=1}^{d} (r_{†‡}^{*})^{\pi ‡}) \ , † = 1,...,d \qquad (3.12)$$

An enhanced version with tunable parameter λ is given by:

$$ý_{†} = \lambda \sum_{‡=1}^{d} r_{‡†}^{*} \ .\pi_{‡} + (1-\lambda) \prod_{‡=1}^{d} (r_{†‡}^{*})^{\pi ‡}, \qquad \lambda \in [0,1] \qquad (3.14)$$

Step 8. Alternatives are ranked in descending order of Q-values. The highest Q-value corresponds to the best alternative.

4. Case Study and Results

Aerobic and anaerobic metabolism are two distinct energy systems that fuel human movement. Aerobic metabolism, also known as oxidative phosphorylation, utilizes oxygen to break down carbohydrates, fats, and proteins into energy. This process is efficient and sustainable, allowing for prolonged exercise. Anaerobic metabolism, on the other hand, produces energy without the use of oxygen. It primarily relies on the breakdown of glucose through glycolysis, a process that generates lactic acid as a byproduct. In the context of running, both aerobic and anaerobic systems play crucial roles depending on the distance and intensity of the race. For longer distances such as marathons, aerobic metabolism is the primary energy source. Endurance runners must train to increase their aerobic capacity, which is the body's ability to utilize oxygen efficiently. This involves consistent training at a moderate intensity, such as long, slow runs and tempo runs. Shorter distances, such as sprints and middle-distance races, heavily rely on the anaerobic system. These explosive events require high-intensity bursts of energy that exceed the body's ability to supply oxygen to the muscles. Anaerobic training focuses on developing the body's capacity to produce energy quickly without oxygen. This includes exercises like interval training, sprint repeats, and strength training. The interplay between aerobic and anaerobic systems is complex and varies depending on the individual runner, the specific race distance, and the pace. While aerobic training forms the foundation for endurance running, incorporating anaerobic training can enhance speed, power, and overall performance. A well-rounded training program should include a combination of both aerobic and anaerobic exercises to optimize performance across different running distances. In conclusion, understanding the interplay between aerobic and anaerobic metabolism is crucial for runners of all levels. By incorporating both types of training into their programs, runners can improve their endurance, speed, and overall performance. Whether it's a marathon or a sprint, optimizing both aerobic and anaerobic systems is key to achieving peak running performance. For this, there are five alternatives $A = \{A_1, A_2, A_3, A_4, A_5\}$, and the best one must be found. The attributes of each alternative are $N = \{N_1, N_2, N_3\}$. Five exercises to enhance aerobic and anaerobic performance in runners are given as: cycling A_1; walking A_2; rowing A_3; squats A_4; pushups A_5. Step 1. Establish the Cq-ROF evaluation matrix as given in Table 1.

Step 1. Establish the C*q*-ROF evaluation matrix as given in Table 1.

Table 1.
Decision-making matrix.

	N₁			**N₂**			**N₃**		
	u	$\dot{u}$	$\ddot{u}$	u	$\dot{u}$	$\ddot{u}$	u	$\dot{u}$	$\ddot{u}$
A_1	[0.3,0.4]	[0.2,0.5]	(0.2,0.4)	[0.1,0.3]	[0.2,0.4]	(0.1,0.3)	[0.3,0.5]	[0.2,0.3]	(0.3,0.4)
A_2	[0.2,0.3]	[0.3,0.4]	(0.2,0.3)	[0.3,0.4]	[0.1,0.3]	(0.2,0.3)	[0.2,0.4]	[0.2,0.4]	(0.1,0.3)
A_3	[0.4,0.5]	[0.3,0.4]	(0.3,0.4)	[0.2,0.4]	[0.1,0.2]	(0.2,0.3)	[0.3,0.5]	[0.2,0.3]	(0.2,0.4)
A_4	[0.3,0.4]	[0.2,0.5]	(0.2,0.4)	[0.2,0.4]	[0.1,0.3]	(0.1,0.3)	[0.4,0.5]	[0.3,0.4]	(0.2,0.4)
A_5	[0.2,0.3]	[0.3,0.4]	(0.1,0.3)	[0.4,0.5]	[0.2,0.4]	(0.3,0.4)	[0.3,0.4]	[0.2,0.5]	(0.2,0.4)

Step 2. Form an aggregated C*q*-ROF decision matrix by using the C*q*-ROFWA operator from Equation (3.1). The computed results are given in Table 2.

Table 2.
Aggregated matrix.

	A	$\dot{A}$	$\ddot{A}$
A_1	[0.3000,0.5000]	[0.2000,0.3000]	(0.3000,0.4000)
A_2	[0.2000,0.4000]	[0.2000,0.4000]	(0.1000,0.3000)
A_3	[0.3000,0.5000]	[0.2000,0.3000]	(0.2000,0.4000)
A_4	[0.4000,0.5000]	[0.3000,0.4000]	(0.2000,0.4000)
A_5	[0.3000,0.4000]	[0.2000,0.5000]	(0.2000,0.4000)

Step 3. The aggregated matrix is normalized by using Equation (3.2) because all the at- tributes are of benefit type. The computed results are given in Table 3.

Table 3.
Normalized matrix.

	A	$\dot{A}$	$\ddot{A}$
$\mathbf{A}_1$	0.8889	0.7143	1.0000
$\mathbf{A}_2$	0.6667	0.8571	0.5714
$\mathbf{A}_3$	0.8889	0.7143	0.8571
$\mathbf{A}_4$	1.0000	1.0000	0.8571
$\mathbf{A}_5$	0.7778	1.0000	0.8571

Step 4. Employ the CILOS weighting method to calculate the weights of attributes. The computed weights are given as: $\pi = (0.1933, 0.4706, 0.3361)^T$.

Steps 5 & 6. The computed results for additive and multiplicative relative matrices are given in Table 4.

Table 4.
Additive and multiplicative relative importance matrix.

Alternatives	$\mathbf{A}_1$	$\mathbf{A}_2$	$\mathbf{A}_3$	$\mathbf{A}_4$	$\mathbf{A}_5$
Results	0.8441	0.7243	0.7961	0.9520	0.9090
Results	0.8343	0.7125	0.7922	0.9495	0.9045

Steps 7 & 8. The computed results for joint generalized criterion matrix and ranking are given in Table 5.

Table 5.
Joint generalized criterion matrix and ranking.

Alternatives	$\mathbf{A}_1$	$\mathbf{A}_2$	$\mathbf{A}_3$	$\mathbf{A}_4$	$\mathbf{A}_5$
Results	0.8392	0.7184	0.7941	0.9507	0.9068
Ranking	**III**	**V**	**IV**	**I**	**II**

Table 6.
Comparison analysis.

Cubic PFS [23]					
Results	0.8420	0.7220	0.7980	0.9490	0.9080
Ranking	$\mathbf{A}_4 \succ \mathbf{A}_5 \succ \mathbf{A}_3 \succ \mathbf{A}_1 \succ \mathbf{A}_2$				
Cubic PF-EDAS method [24]					
Results	0.5877	0.5000	0.3571	0.5000	0.4109
Ranking					
Cubic IFS [25]					
Results	0.8392	0.7184	0.7941	0.9507	0.9068
Ranking	$\mathbf{A}_4 \succ \mathbf{A}_5 \succ \mathbf{A}_1 \succ \mathbf{A}_3 \succ \mathbf{A}_2$				
Cubic IF-TOPSIS method [26]					
Results	0.5247	0.2816	0.4215	0.7869	0.7212
Ranking	$\mathbf{A}_4 \succ \mathbf{A}_5 \succ \mathbf{A}_1 \succ \mathbf{A}_3 \succ \mathbf{A}_2$				
Cubic IF-WASPAS method [27]					
Results	0.8450	0.7250	0.8000	0.9480	0.9100
Ranking	$\mathbf{A}_4 \succ \mathbf{A}_5 \succ \mathbf{A}_3 \succ \mathbf{A}_1 \succ \mathbf{A}_2$				

We have established techniques to address the suggested problem and apply our framework to the analysis of the results in order to determine the ability and efficacy of the proposed approach. Using these methods, we attentively compute the evaluation results for the selection of the most effective exercise for enhancing aerobic and anaerobic performance in runners. Therefore, we solved the problem at hand using various methods and sorted the results in Table 6. Table 6 indicates that the ranking outcomes of the existing five methods vary somewhat; however, $Ą_4$ emerges as the optimal selection. From the comparison results it is clear that, the suggested methodology effectively garnered significant interest from academics. Our proposed method is especially suitable for combining C*q*-ROFS data in a decision-making environment.

5. Conclusions

This research suggested an innovative Cq-ROF approach and used computational calculations to analyze its capabilities over several current methodologies. The Cq-ROF-CILOS approach was used to compute the weights while taking into account the correlations between the attributes. Then, using the Cq-ROF-CILOS approach, the alternatives under consideration were assessed and rated. The decision-making process was clarified in depth, and an illustration was given to show how the suggested approach was workable. The key contribution comes to the conclusion that the suggested decision approach ranks the alternatives by taking into account the correlations between the attributes, which helps to mitigate any preconceptions and prejudices. This study makes three major contributions. (1) To convey complex and unpredictable choice facts in the decision-making process, an innovative technique entitled Cq-ROFS was put forth. It is effective, robust, and adaptable. The Cq-ROFS offers more flexibility in fully expressing assessment facts by employing the capabilities of both q-ROFS and IV q-ROFS. The knowledge domain that Cq-ROFS can portray grows as q does. (2) Relying on the Cq-ROFS, an operator was created to include attribute contents into the decision-making phase. In addition to reducing the negative impact of excessive or irrational assessment contents on the final decision findings, this aggregation operator entirely absorbs the capabilities of the WA operator and reflects the interrelationships between any variety of attributes. Because of these characteristics, the suggested operator is suitable and competent to handle actual decision-making issues. (3) Relying on the Cq-ROFWA operator, a novel Cq-ROF-CILOS-WASPAS decision-making technique was put forth. The suggested approach's superiority and accuracy were thoroughly examined. Our suggested approach is more potent and adaptable than some current approaches, as shown by computational measurements.

Our suggested strategy's primary drawback is that it requires higher computation than certain other approaches to decision-making. Furthermore, fuzzy appraisals may be used in situations when it is challenging to assess the items using quantitative measurements. Our suggested approach is nevertheless more appropriate and adequate to handle real-world decision-making techniques than certain others, despite its benefits and advantageous aspects. The subsequent areas of investigation will be the main concentration of our upcoming studies.

Despite our presentation of the Cq-ROFS in the decision-making environment, there are still certain restrictions. The Schweizer-Sklar t-norm and t-conorm can be used to considerably enhance them. Heronian mean power-aggregation operators and Muirhead mean operators for Cq-ROFS are two other progressive aggregation operators that we would want to examine. Future studies may enhance additional decision-making techniques for Cq-ROFS with better scientific accomplishments to broaden the range of instances, such as the CO-PRAS methodology [28], the EDAS methodology [29], the MABAC methodology [30], and so forth. The decision-maker is regarded as completely rational in the framework that is being presented. However, in reality, decision-makers do not always operate in a totally logical manner. Therefore, further investigation will be conducted on the irrational traits of decision-makers [31]. Future studies could focus on defining the Cq-ROFS function and its associated derivative, differential, definite, and indefinite integral.

The suggested approach can be used in various domains under uncertain conditions, including risk assessment, hotel placement, venture selection, and other areas for future research.

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