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An effective method and model: For optimizing production taking into account environmental

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Abstract

The proposed model includes in the optimization criterion, in addition to income, the possible payment in case of exceeding pollution norms, which stimulates production to introduce cleaner, environmentally friendly technology. The problems are often difficult to formalize and are characterized by vagueness of initial information, which requires the formulation of multi-criteria production optimization problems taking into account environmental standards in a fuzzy environment and the development of methods for their solution. The aim of the research is to create a model of production optimization taking into account environmental standards, to formulate and solve a multi-criteria optimization problem in a fuzzy environment, and to develop a heuristic method. In this paper, based on the modification of the principles of the main criterion, Pareto optimality, the formulation of a multi-criteria optimization problem considering environmental constraints is obtained, and a heuristic method of its solution is proposed for effective resolution in a fuzzy environment. The originality of the proposed method is that it allows obtaining an adequate solution in a fuzzy environment by maximizing the use of fuzzy information, knowledge, experience, and intuition of the decision maker and experts. A new model for optimizing production is proposed, which is applicable in the field of macroeconomics and microeconomics.

Keywords: Decision maker, Economic-mathematical model, Heuristic method, Payments for pollution, Production optimization, Waste management.

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1. Introduction

Nowadays, the issues of optimization of production with waste management and utilization are very urgent scientific and practical tasks of science and production. The acuteness and necessity of solving environmental problems are of a global nature and are realized practically in all countries of the world, including Kazakhstan. Balanced and sustainable development of society in today's world is possible only through a comprehensive solution to economic, environmental, and social issues. Existing studies addressing this topic lack sufficient investigation into waste management based on economic-mathematical models and taking into account fuzziness. The solution to these problems based on models and methods of optimization and computer technology allows for scientifically sound and efficient management of industrial waste, making a significant contribution to the body of knowledge aimed at solving the problems of sustainable production management.

The purpose of this work is to develop an ecological and economic model, in contrast to the known models, which allows effective management of production waste while taking into account environmental standards and the "cleanliness" of production in a fuzzy environment. The novelty and importance of the study lie in the development of a model of economic-environmental optimization of production, which, in addition to traditional resource constraints and pollution abatement costs, also considers environmental compliance conditions. The modern achievements of mathematical methods and computer technology, along with the need to solve economic-ecological optimization problems for sustainable production management, motivated this study. The developed ecological-economic model of production optimization, taking into account environmental standards, is universal and can be used for microeconomics at the level of individual production and for macroeconomics at the level of the production sector. Furthermore, the structure of the article includes a literature review, materials and methods, research methodology, research results discussions, and conclusions.

2. Literature Review

As the results of the literature review show, the principle of normative qualitative state of the environment, which is achieved by setting standards for waste pollution, was mainly put in the basis of solving environmental problems of the enterprise and waste management. These standards are supported by tax policy, which is both punitive and incentive in nature, including the use of subsidies, preferential lending, the introduction of payment systems, and fines for excessive levels of pollution. The main purpose of environmental payments is not to replenish the state budget, but to stimulate production enterprises to behave positively in terms of environmental protection and waste minimization [1]. The priority of international financial institutions and organizations is to encourage and support the sustainable development of the company from both an economic and environmental perspective [2, 3].

3. Hypothesis Development

3.1. Economic Mechanism of Nature Protection

The economic mechanism of nature protection in many countries represents ecologically oriented methods of nature use and waste management, which allow for regulating ecological and economic interests between society, nature users, and enterprises. In the works [4], these economic mechanisms include pollution charges, limits on waste volumes, fines for violations of environmental laws and regulations, low-interest loans, and favorable tariffs for companies for wastewater treatment and waste disposal. An incentive measure can be defined as any measure aimed at achieving the objectives of waste management, rational nature management, and environmental protection by companies, ensuring their sustainable development. The main and common type of economic incentive is payments for waste and pollution [5-7]. The system of these payments is common in Germany, France, the Netherlands, and other countries [8]. The above-mentioned works determine the degree of real influence of the tax policy of the European Union member states on environmental protection indicators. They also investigate how to ensure the sustainable development of society and how the fiscal measures of the European Union countries affect environmental protection and waste management performance.

$H_{1:}$ The economic mechanism of nature conservation is environmentally oriented methods of environmental and waste management, which allow regulating environmental and economic interests between society and natural resource users.

3.2. Payments And Incentives to Save Resources

Some researchers, Ukkonen and Sahimaa [9]; Botetzagias et al. [10] and Bozec [11] note that in France and in a number of other countries, various payments are practiced to save resources, limit waste, reduce pollutants, and pay for waste disposal services, etc. A number of countries have introduced a specific tax on payments for household waste collection services. One of the common types of economic incentives is fines for violations of waste limits and environmental legislation, which are generally accepted principles and are levied according to the degree and volume of pollution [12, 13]. In the article [14, 15], industrial companies were investigated and found to have a significant impact from economic incentives (environmental subsidies) on corporate investment in environmental protection and the waste management process according to Chinese data. In Japan and the UK, a tax method is used to reduce environmental risks, which incentivizes enterprises to develop. Industrial companies were studied and found to have a significant impact from economic incentives (environmental subsidies) on corporate investment in environmental protection and the waste management process according to Chinese data [16]. Many countries take measures to address waste collection, treatment, and management on a national, regional, or sectoral scale, and develop and implement waste management and recycling programs [17, 18]. Thus, the analyzed world experience shows that the authorized bodies of state management of natural resources and waste should be the holders of investment funds [19, 20]. Thus, the analyzed world experience shows that the authorized bodies of state management of natural resources and waste should be the holders of investment funds and be responsible for their distribution [21]. Some scientists note that the use of waste and natural resources requires the application of scientifically based economic and mathematical methods and methods based on computer technology [22].

$H_{2:}$ To save natural resources, other payments and incentives should be used, such as: restrictions and fees on polluting waste; payment for waste disposal services; fines for violating waste limits, etc.

3.3. Economic And Mathematical Methods and Models for Waste Management

Based on foreign and domestic experience of waste management [23-25] it can be noted that any waste management and utilisation project should be thoroughly justified using economic and mathematical methods, taking into account environmental criteria and constraints. Demand and supply should also be assessed, costs, profitability and environmental performance of the project should be calculated, and possible risks should be identified. To effectively address these, economic-ecological models can be used, which take into account environmental indicators and regulations Zheng and L.Y. [26] and Fuente [27]. Mokhov et al. [28] it can be noted that any waste management and utilisation project should be thoroughly justified using economic and mathematical methods, taking into account environmental criteria and constraints. It is also necessary to assess supply and demand, calculate costs, profitability and environmental performance of the project, and identify possible risks. To solve these tasks effectively, economic-ecological models that take into account environmental indicators and regulations can be used. Due to the complexity and complexity of waste management and utilisation tasks, the need to take into account economic6 environmental, social and other factors, the economic and mathematical models used for its solution are subject specific. Some types of problems and types of economic-mathematical models used in solving problems of waste management and utilisation are defined. Algorithmic bases of economic and mathematical modelling of network logistic processes in waste management and recycling are investigated in the work of Barykin et al. [29]. The authors of this work proposed an economic-mathematical model that allows to search for optimal solutions for economic-ecological criteria of waste management and utilisation in macroeconomics.

Authors of the article [30] proposed statistical models on the basis of which waste management and recycling problems are solved, taking into account risk factors affecting volumes, prices and costs of recyclers and consumers. In the works [31] a mathematical model for education forecasting and management was proposed development of stochastic models through scenarios for determining optimal waste management policies is proposed. This approach allows to find a reasonable compromise solution in the face of uncertainty. Zhumadillayeva et al. [32] in their research formulated balance models of functioning of the regional system, which takes into account material and financial components. Author of the work [33] investigated the problem of management and utilization of industrial waste and substantiated the necessity of using economic and mathematical modelling methods to solve these problems and develop the waste management industry. Consider examples of the use of economic and mathematical models and methods in the field of waste management and recycling. As part of the project «Development of the Model and Technologies of Logistics of the Communal Waste Transport» [34, 35] funded by the Ministry of Science of the Republic of Serbia, solved the problem of selecting an optimal waste management system in the city of Nish. Due to the complexity of the management system with its variable performance, a multi-criteria optimization method and a hierarchy analysis method were used for waste management in the city of Nish.

The aim of the project was to ensure maximum system efficiency and maximum user satisfaction in the city of Nish. Researchers at the University of Miami's Department of Industrial Engineering conducted a study to create a tool to model and optimize recycling and waste management infrastructure in the state of Florida [36, 37]. The proposed tool allows waste management and recycling to achieve a 75% recycling rate by 2024. The tool being developed includes the following components: a structured database that includes all waste-generating and processing facilities in the state of Florida, as well as infrastructure facilities. An evaluation module that allows the selection of counties of interest, after which the relevant conditions and agents are loaded from the database. The module automatically creates economic and operational links between agents. A resource allocation optimization module, including a hybrid agent-discrete model. The model consists of a module that checks that the attributes of each agent match the actual database content and the operational (tonnage), economic, and environmental (recycling rate) models. The analysis engine considers the feasibility and performance of all solution options.

A decision-making system based on continuous discrete modeling of waste management and recycling programs is proposed. The proposed decision-making and optimization system is designed to analyze and develop effective model-based waste management and recycling programs. The system includes a database and modules for resource allocation evaluation and optimization. The estimation module is designed to identify the sources of stochastic-type uncertainties in the system. These uncertainties are then parameterized and incorporated into the resource allocation optimization module, which includes a new discrete-continuous model of the system under consideration.

$H_{3:}$ Waste management and disposal projects must be carefully justified using economic and mathematical methods, taking into account environmental criteria and limitations.

$H_{4:}$ Based on economic and mathematical models, it is necessary to create a decision-making system for the analysis and development of effective waste management and disposal programs.

3.4. Dynamic Connections Between Agents of Waste Management Systems

The specificity of integrated waste management systems is that many specialized enterprises, vehicles, and infrastructure facilities are required. In addition, it is necessary to take into account the fact that the ownership of these facilities varies widely: from municipal and state ownership to numerous private firms and enterprises. In this context, the waste management system involves a variety of dynamic relationships between different agents [38]. Furthermore, many regions (oblasts, states) do not have a full cycle of waste recycling and must establish links with neighboring regions or even countries. Currently, scientists are implementing the project "Multi-objective agent-based modeling and optimization of 'one-stream' recycling programs" [39]. In this research, a tool consisting of a simulator module and an optimization module is being created. The different sources of uncertainty in the system will be parameterized and included in the simulation model. With the tool being developed, stakeholders will be able to simulate different scenarios until a compromise solution is reached. Thus, the literature review has shown that the problem of waste management and utilization is complex and multifaceted, and the economic and mathematical models used in this sphere are subject-specific. Economic and mathematical modeling in the sphere of waste management and utilization is best developed in solving the problems of disposal infrastructure facilities' location, determination of necessary capacities of these facilities, and description of material and financial flows arising between them.

Depending on the availability of input data, accepted hypotheses, complexity, and heterogeneity of the modeled waste management and disposal system, a wide range of models and methods are applied. There is no experience of economic-mathematical modeling of waste management and utilization systems in Kazakhstan yet, which is caused by the absence of a waste processing industry as such on the scale of the country and on the scale of separate regions. The economic-mathematical models developed in the analyzed works are oriented to the field of macroeconomics, at the level of the country or region. In addition, the proposed economic-mathematical models do not sufficiently address the problems of uncertainty arising from the vagueness of the initial information. Such a circumstance motivates this study, which is devoted to the development of models for the optimization of production by economic and environmental indicators, and suitable for microeconomics at the level of individual production, as well as taking into account the vagueness of some part of the initial information.

$H_{5:}$ Depending on the accepted hypotheses, the complexity and heterogeneity of the waste management and disposal system, it is necessary to apply a wide range of economic and mathematical models and methods, including those taking into account the dynamics of the system.

4. Materials and Methods

The main materials for the study were economic and environmental indicators of oil refineries on the example of the primary oil processing unit of the Atyrau oil refinery [40]. Desalinated oil and stable petrol are considered as the main products from this unit, i.e., 2 kinds of main products. Then the values of air and water pollution in monetary terms in the production of petroleum products, i.e., the cost of disposal of pollutants in excess of standards $\bar{z} = (z_1, z_2) = (58, 40)$ thousand tenge, determined on the basis of data from the object under study. When producing a unit of i-th petroleum product, the matrix of pollution intensities was identified as a result of statistical data processing by the formula $c_p = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}$. According to the facility data, it has been determined that 4 kg is required to produce 1 tonne of desalinated petrol and 5 kg of reagents are required for stable petrol. According to the norm, 100 kg of these reagents are allocated to the plant per week. In addition, according to the data of the plant for the production of 1 tone of desalted petrol 12 units of thermal energy, and for the production of 1 tone of stable petrol - 10. The standard amount of energy allocated to a facility per week is 250 units. The average price of 1 tone of desalted oil produced is 150 thousand tenge, and of stable petrol - 170 thousand tenge.

The methods of economic and mathematical models development were used in the research process [29, 41, 42] and multi-criteria optimization [21, 43]. Statistical methods are used to analyze the state of the enterprise [44, 45] and for assessment and description of non formalizable parameters, economic and environmental indicators of production - methods of system analysis and expert assessments [46-49].

Next, we formulate the problem of optimising production with waste management in the form of a fuzzy optimisation problem Let there be a vector of normalised m criteria $\mu_c(\bar{x}) = (\mu_c^1(\bar{x}), \dots, \mu_c^m(\bar{x}))$ и L constraints in the form of fuzzy instructions $f_{\mathbf{q}}(\bar{x}) \lessapprox b_q, q = \overline{1, L},$ assessing economic performance and taking into account environmental regulations. Suppose that with decision maker (DM),), experts constructed membership functions of fuzzy constraints $\mu_q(\bar{x}), q = \overline{1, L},$ a number of local criteria prioritised $I_c = \{1,2, \dots, m\}$ and the weight vector of constraints $\bar{\beta} = (\beta_1, \dots, \beta_L)$. There $\bar{x} = (x_1, \dots, x_n)$ – vector of input, mode parameters of the object, influencing local criteria and fuzzy constraints. Influence of the vector $\bar{x}$ on $\mu_c^1(\bar{x}), \dots, \mu_c^m(\bar{x})$ are determined on the basis of models of the primary oil refining process, and the fuzzy constraints are taken into account when constructing the accessory function $\mu_q(\bar{x}), q = \overline{1, L}$ [50, 51].

Then the problem of multi-criteria optimisation of production by economic and environmental indicators adapted to take into account the vagueness of the initial information in a general form can be written in the form:

$$\max_{\bar{x} \in X} \mu_c^i(\bar{x}), i = \overline{1, m},$$
$$X = \left\{\bar{x}: \arg \max_{\bar{x} \in X} \mu_q(\bar{x}), q = \overline{1, L}\right\}.$$

In the resulting formulation, the problem requires finding the maximum value of local m criteria $\mu_c^i(\bar{x}), i = \overline{1, m}$ at a single point, taking into account the requirements L. In production conditions, as a rule, in the area of effective solutions different criteria are in contradiction and it is impossible to find the optimal solution of fuzzy constraints of the given problem. In this regard, for correct formulation and obtaining the best solutions of the above problem, it is proposed to modify and use various principles of optimality (trade-off schemes). For example, based on the modification of the combination of the principles of optimality of the main criterion (for criteria) and Pareto optimality (for constraints), the general formulation of the reduced problem can be correctly written in the following form:

$$\max_{\bar{x} \in X} \mu_c^1(\bar{x}), \tag{1}$$

$$X = \left\{ \bar{x} \colon \arg() \geq \mu_R^i) \wedge \arg \max_{\bar{x} \in X} \sum_{q=1}^{L} \beta_q \mu_q(\bar{x}) \wedge \sum_{q=1}^{L} \beta_q \wedge \beta_q \geq 0, \; i = \overline{2,m}, q = \overline{1,L} \right\}. \quad (2)$$

In the production (1)-(2) $\wedge$ – logical signs «and», requiring the truth of all conditions that are related through them; μ_R^i– inquiries DM boundary values of local criteria $\mu_C^i(\bar{x})$, $i = \overline{2,m}$, apart from 1-го the main criterion taken into account as part of the constraints.

By changing the values $\mu_R^i, i = \overline{2,m}$ and vector of importance of constraints $\bar{\beta} = (\beta_1, \dots, \beta_L)$ we can obtain a set of solutions to the problem (1)-(2). Among the many solutions, the best solution is chosen by the DM on the basis of their preferences and taking into account the current situation in production and the demand for manufactured products on the market. The following heuristic method is proposed to solve the above formulation of the multicriteria optimization problem with fuzzy constraints. The proposed method is based on the modification of the optimality principles of the main criterion (MC) and Pareto optimality (PO) and the use of experience, knowledge and intuition DM in the process of improving and selecting the best solution and the capabilities of computer technology [52].

4.1. Heuristic Method MC+PO

1. For each q-го coordinates p_q, $q = \overline{1,L}$ DM the number of steps is set and a number of priorities are defined for local criteria $I_c = \{1,2, \dots, m\}$. In this case, the main criterion $\mu_c^1(\bar{x})$ should be prioritized 1.

2. DM the weight vector of importance of fuzzy constraints is defined $\bar{\beta} = (\beta_1, \dots, \beta_L)$.

3. For all local criteria6 except the main criterion, DM and experts the initial boundary values are determined $\mu_R^i, i = \overline{2,m}$.

4. To change the coordinates of the weight vector $\bar{\beta}$ step values are calculated by the formula $h_p = {}^1\!/_{p_q}, q = \overline{1,L}$.

5. Changing h_p in the interval [0,1] set of weight vectors is constructed $\bar{\beta}^1, \bar{\beta}^2, \dots, \bar{\beta}^N, N = (p_1 + 1) \cdot (p_2 + 1) \cdot \dots \cdot (p_L + 1)$.

6. The term set is defined, and the membership functions are constructed $\mu_q(\bar{x}), q = \overline{1,L}$, evaluating the degrees of fulfillment of the fuzzy constraints.

7. The maximization problem is solved $\mu_c^1(\bar{x})$ (1) taking into account the imposed restrictions, determined by the expression (2). In this case, the obtained single-criteria problem with constraints can be solved on the basis of suitable known methods of conditional optimization, for example, using the method of penalty functions. As a result of the solution, the current solutions are determined: values of the vector of input parameters $\bar{x}(\mu_R^i, \bar{\beta})$; values of the main $\mu_c^1\left(\bar{x}(\mu_R^i, \bar{\beta})\right)$ and other local criteria $\mu_c^i\left(\bar{x}(\mu_R^i, \bar{\beta})\right), i = \overline{2,m}$, and also degrees of fulfillment of fuzzy constraints$\mu_1\left(\bar{x}(\mu_R^i, \bar{\beta})\right), \dots, \mu_L\left(\bar{x}(\mu_R^i, \overline{\beta})\right)$.

8. The resulting current solutions are presented to the DM to analyze and select the best solution. If the current solutions do not satisfy the DM, the DM changes the values in order to improve the solution $\mu_R^i, i = \overline{2,m}$ и/или $\bar{\beta}$, And the search for the best solution is repeated, starting from point 4. When satisfying DM solutions are obtained, proceed to the next point.

9. Output of the final, DM-selected, best solutions: optimal values of the input parameter vector $\bar{\boldsymbol{x}}^*(\mu_R^i, \bar{\beta})$, Providing the maximum value of the main criterion $\mu_c^1\left(\bar{x}^*(\mu_R^i, \bar{\beta})\right)$, satisfactory values of local criteria $\mu_c^i\left(\bar{x}^*(\mu_R^i, \bar{\beta})\right), i = \overline{2,m}$, and maximum degrees of fulfillment of fuzzy constraints $\mu_1\left(\bar{x}^*(\mu_R^i, \bar{\beta})\right), \dots, \mu_L\left(\bar{x}^*(\mu_R^i, \bar{\beta})\right)$.

5. Methodology and Analysis

The task of optimizing production, taking into account environmental standards in general, can be written in the form of a mathematical programming problem. Let $F(x_1, x_2, \dots, x_R)$ – a criterion that evaluates economic indicators of the quality of the object (income) using the *R resources*. Let's assume that there are *k* types of pollution from this production, which are specified in the following matrix of pollution intensities:

$$c_p = \begin{pmatrix} c_{11} & c_{12} \dots & c_{1R} \\ c_{21} & c_{22} \dots & c_{2R} \\ \dots & \dots & \dots \\ c_{k1} & c_{k2} \dots & c_{kR} \end{pmatrix}. \quad (3)$$

Matrix elements (3) c_{ij} – means the amount *j*-го the contamination that results from the application of *i*- resource when using the selected technology. Then the vectors of pollution. Then the contamination vectors $\bar{w}$ can be determined by the formula:

$$\bar{w}^T = c_p \bar{x}^T \quad \text{or} \quad w_k = \sum_{j=1}^{n} c_{kj} x_j, j = \overline{1,m}, \quad (4)$$

where $\bar{x} = (x_1, x_2, \dots, x_m)$ – is the vector-string of produced output. Let A denote the matrix of coefficients of resource constraints, $\bar{b}$ – vector of constraints, which is determined by the capabilities of the production facility, and $\bar{w}^*$– vector of environmental standards, i.e., permissible wastes for each type of pollution. These standards are generally set according to the current norms of maximum permissible concentrations (MPC) of contaminants. Based on the above formalization, the problem of optimizing the production of products by economic and environmental indicators can be written as follows:

$$\max F(\bar{x}) \tag{5}$$

$$\begin{cases} \bar{x} \geq 0, \bar{z} \geq 0, \\ A\bar{x}^T \leq \bar{b}^T, \\ \bar{w} \lessapprox \bar{w}^*, \end{cases} \tag{6}$$

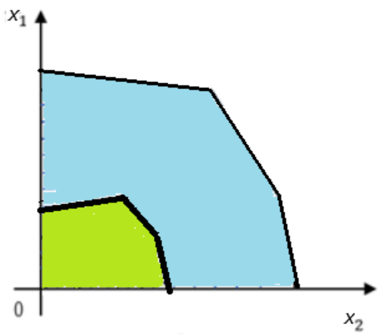
where $\lessapprox$ – fuzzy constraint imposed by a vector of environmental regulations, e.g, "less than", "the less the better", etc.

The obtained problem of optimization of production of products with observance of ecological norms of fuzzy character (5)–(6) allows to ensure sustainable development of production and can be solved by a heuristic method with modification of various principles of optimality for vagueness. For example, if the DM can identify the main criterion among local criteria, and can assign weight coefficients to the constraints, then the problem (5)–(6) is solved using the heuristic method proposed above, MC+PO.

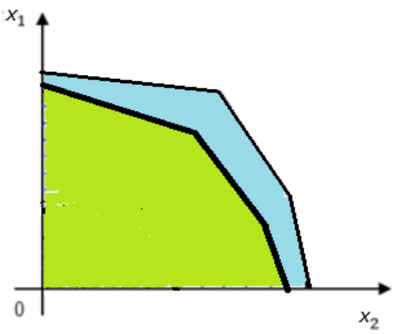
In the known optimization models of production, the admissible set is formed by the first two constraints given in (6). The novelty and difference from them in the optimization model given in this paper is that the optimization model still includes a third fuzzy constraint that takes into account environmental standards and requires "cleanliness of production". According to formula (4), the last condition in the constraint system (6) in the expanded form for each type of pollution can be rewritten as:

$$\sum_{j=1}^{n} c_{kj} x_j \lessapprox w_k^*, k = \overline{1, m}. \tag{7}$$

To ensure the requirements (7) when solving the optimization problem (5)–(6), production should choose more environmentally superior technologies or use "clean" resources instead of "dirty" resources. Otherwise, due to constraints (7), the outputs produced are $\bar{x} = (x_1, x_2, \ldots, x_m)$ will be so insignificant that it will not be economically viable. The above is illustrated more clearly in Figure 1. In this figure, the additionally introduced fuzzy constraint (7) is given by the green area and noticeably narrows the admissible set of solutions (blue area), on which the search for an optimal solution is carried out. The size and type of the admissible region depend on the resource processing technology used, i.e., the coefficients c_{ij} of the pollution intensity matrix. If "dirty" technologies are used, the region of admissible solutions is significantly narrower, the green region in Figure 1, b.



a



b

Figure 1.
Admissible set of solutions.

The model presented above (5)–(6) is applicable for macroeconomics, at the level of a country or a region, when production output can be identified with the GDP of the country or with the gross output of the region. In this case, condition (7) allows to the management of technological policy. For microeconomics at the level of individual production, this model is suitable, since the producer is interested primarily in achieving maximum production (6), and compliance with environmental standards (7) remains secondary, since this requirement is not reflected in the criteria.

To account for environmental norms in microeconomics, it is necessary to move to the value expression in the criteria and payment for violations, exceeding environmental norms.

Let *ap* – the aggregate price of the products produced by the facility, $\bar{z}$ – vector of pollution disposal costs when the relevant norms are exceeded, i.e., when the condition is violated $\bar{w} \lessapprox \bar{w}^*$ constraints (6):

$$\bar{z} = (z_1, z_2, \ldots, z_m). \tag{8}$$

In view of the above, the criterion assessing the income from the production of products $F(\bar{x})$, looks like:

$$F(\bar{x}) = apF(\bar{x}) - \bar{z}\,\bar{\delta}, \tag{9}$$

where $\bar{\delta} = (\delta_1, \delta_2, \dots, \delta_m)$ =– vector of "inclusions" of payments for pollution, the components of which are determined from the condition:

$$\delta_j = \begin{cases} 0, if \;\; w_j \le w_j^*, j = \overline{1,m} \\ 1, if \;\; w_j > w_j^*, j = \overline{1,m} \end{cases}, \tag{10}$$

where w_j – components of the pollution vector, determined by the formula (4), w_j^* – components of the vector of environmental standards, maximum allowable concentrations of pollutants:

$$\bar{w}^* = (w_1^*, w_2^*, \dots, w_m^*). \tag{11}$$

In order to simplify the task of ensuring the requirements, it is possible to include payments for environmental pollution in the payment for natural resources use, i.e., in the second term of profit (9) with the minus sign, meaning as payment for excess load on the natural environment. In this respect, the optimization criterion (9) can be regarded as a production function, the arguments of which are the volumes of produced products $\bar{x} = (x_1, x_2, \dots, x_m)$, and depending on the amount of pollution $\bar{w} = (w_1, w_2, \dots, w_m)$, maximum allowable standards $\bar{w}^* = (w_1^*, w_2^*, \dots, w_m^*)$ and payments for environmental pollution $\bar{z} = (z_1, z_2, \dots, z_m)$.

Product revenue optimization models using a vector of $\bar{x}$ at the technology characterized by a production function $\bar{F}(\bar{x})$, can be defined as follows: to find the maximum of the criterion (5), on the admissible set of solutions (6), taking into account the constraints (7). Then the proposed revenue optimization model with waste management (5)–(7) is universal and suitable for the purposes of microeconomics at the level of any enterprise, industrial complex, and sectoral outputs.

6. Research Results

Results of application of the proposed approach to the formulation and solution of the problem of revenue optimization with waste management at a particular facility – at the primary oil processing unit of the Atyrau Refinery. This facility processes crude oil and produces desalted oilx_1 and stable x_2 gasoline. The productions of these products are described by reagent and thermal energy limitations and elimination costs of contaminants $\bar{z}$. In case of breach of condition $\bar{w} \lesseqgtr \bar{w}^*$ in the constraint system (6) and pollution payments determined by formula (10). Numerical values of vector components $\bar{z}$,contamination intensity matrices c_p, Resource consumption rates and other numerical data obtained from the facility are given in Section 2, Materials and Methods.

Taking into account environmental standards and based on formula (10), pollution payments are determined as follows:

$$\delta_j = \begin{cases} 0, \; \left(c_{j1}x_1 + c_{j2}x_2\right) \le w_j^*, j = \overline{1,2} \\ 1, \; \left(c_{j1}x_1 + c_{j2}x_2\right) > w_j^*, j = \overline{1,2} \end{cases},$$

where c_{j1}, c_{j2} – respectively, the amount *j*-th product contamination x_1 и x_2.

Further, for the decision of the concrete problem of optimization of production on economic-ecological indicators, we use numerical data received from the primary oil processing unit of Atyrau oil refining plant and given in section 2. Then, inserting these data into optimization criteria (5), we obtain:

$$F(\,\bar{x}) = F(x_1, x_2) = 150x_1 + 170x_2.$$

In view of the fact that x_1 и x_2 express the quantity of products produced, they must satisfy the condition of positivity: $x_1 \ge 0$ and $x_2 \ge 0$. To determine the aggregate price of manufactured products, weighting coefficients of importance of products were determined on the basis of expert Information λ_1=0.3, λ_2=0.7. Then the aggregate price of products is defined as follows: $ap = 0.3 \cdot 150 + 0.7 \cdot 170 = 164\ 000$ tenge. Considering the aggregate price of the products produced, the criterion is defined in the form:

$$apF(x_1, x_2) = ap(x_1 + x_2) = apx_1 + apx_2 = 164x_1 + 164x_2.$$

Mathematical records of the limitation on reagents and thermal energy have the form of inequalities:

$x_1 + 5x_2 \le 100$ – for reagents;
$12x_1 + 10x_2 \le 250$ – for heat energy.

Including disposal costs in case of exceeding the relevant norms, for air pollution z_1 and wastewater contaminants z_2 in case of breach of condition $\overline{w} \lesseqqgtr \overline{w}^*$ and pollution charges δ_1, δ_2 the problem of production optimization taking into account environmental standards can be written as a mathematical programming problem:

$$\max(164F(x_1, x_2) - (z_1\delta_1 + z_2\delta_2)), \quad (12)$$
$$4x_1 + 5x_2 \leq 100. \quad (13)$$
$$12x_1 + 10x_2 \leq 250. \quad (14)$$
$$x_1 \geq 0, \text{и } x_2 \geq 0. \quad (15)$$

In the resulting optimization problem with waste management (12)–(14), payments for environmental pollution are already included for natural resource use. In this problem, the second term of profit with a minus sign means payment for production wastes that have an excessive load on the natural environment. At Atyrau oil refinery, for desalted oil and stable petrol produced at the primary oil refining unit the following values of permissible wastes are accepted:

$w_1^* = 75;\ w_2^* = 78.$

Based on these data and the matrix of contamination intensities $c_p = \begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}$, given in section 2, the optimization problem with waste management (12)-(14) of the primary oil processing unit of the Atyrau oil refinery is specified in the following form:

$$\max(164x_1 + 164\,x_2 - (58\delta_1 + 40\delta_2)), \quad (16)$$
$$4x_1 + 5x_2 \leq 100, \quad (17)$$
$$12x_1 + 10x_2 \leq 250. \quad (18)$$
$$x_1 \geq 0, \text{и } x_2 \geq 0. \quad (19)$$
$$\delta_1 = \begin{cases} 0, (2x_1 + 5x_2) \leq 75, \\ 1, (2x_1 + 5x_2) > 75, \end{cases};\ \delta_2 = \begin{cases} 0, (3x_1 + 4x_2) \leq 78, \\ 1, (3x_1 + 4x_2) > 78. \end{cases} \quad (20)$$

Since in the given optimization problem with waste control (16)–(20) the criterion and all constraint functions are linear for its solution, we can use known methods of solving linear programming problems. In this paper, for solving the problems (16)–(20), the software package "Manager" is used, which is the most convenient in operation and is suitable for solving the problems of optimization and decision making of various kinds. As a result of solving the problem of optimization of production with waste management (16)–(20) with the application of the software package "Manager" the following solutions were obtained. Maximum income 3690 thousand tenge $(164 \cdot 12.5 + 164 \cdot 10 = 3690)$, is ensured in the production of $x_1 = 12.5$ and $x_2 = 10$, compliance with the condition of the restrictions imposed for resources (17) и (18) $(4 \cdot 12.5 + 5 \cdot 10 = 100;\ 12 \cdot 12.5 + 10 \cdot 10 = 250)$. All resources are fully utilised and environmental standards (20) are fully complied with, so there are no pollution charges for the production of desalted oil and stable petrol.: $\delta_1 = 0$ and $\delta_2 = 0$ because $(2 \cdot 12.5 + 5 \cdot 10) = 75 \leq 75,\ (3x_1 + 4x_2) = 77.5 < 78.$

7. Discussion

The proposed model for optimizing profits from the production of products, taking into account environmental standards, is universal and applicable in both the field of macroeconomics and microeconomics. The obtained model will allow for the optimization of production according to economic criteria while considering environmental requirements, as well as managing waste and pollution in production. This model is solved using mathematical programming methods.

In well-known models of the production waste management problem, the admissible set is defined by two expressions: $\bar{x} \geq 0, \bar{z} \geq 0$ и $A\bar{x}^T \leq \bar{b}^T$.In these expressions. $\bar{x}$ – vector-string of manufactured products using required resources; $\bar{z}$ – payments for environmental pollution; A – matrix of resource constraint coefficients; $\bar{b}$ – vector of resource constraints. The novelty of the proposed model of optimization of production with waste management with observance of environmental standards is that in the admissible set, besides these constraints, are included additional constraints that take into account the requirements of environmental standards, i.e., on the "cleanliness" of production. Additional constraints on the "cleanliness" of production are represented by the expressions $\overline{w} \lesseqqgtr \overline{w}^*$, where $\overline{w}$ – vector of contaminants in the production of products, $\overline{w}^*$– vector of environmental standards, i.e., the permissible waste for each pollution. The introduced additional constraint, narrowing the area of acceptable solutions, requires that the amount of pollution does not exceed the permissible standards for each type of pollution.

To justify the advantage and efficiency of the results of the proposed model of optimization of production with waste management, taking into account environmental standards before the results of known models, a comparison table (Table 1) is made, which shows the results of optimization using the proposed model and known models.

Table 1.
Optimization results using known production optimization models and the proposed optimization model, taking into account environmental standards.

Optimization models used	Maximum value of optimization criterion	Ability to take into account environmental regulations ("cleanliness" of production)
Known models of production optimization	$F(\bar{x}) = F(x_1, x_2)$ =3690 thousand tenge	Environmental regulations are not taken into account
Proposed model of production optimization with waste management considering environmental regulations	$F(\bar{x}) = apF(x_1, x_2) - \bar{z}\,\bar{\delta}$ =3650 thousand tenge	Environmental standards, i.e., the "cleanliness" of production is taken into account on the basis of the introduced additional condition on permissible waste for each type of pollution $w_j \le w_j^*$

Source: Compiled by authors according to source data [4, 53] and obtained during calculations using the Manager software package.

From the results in Table 1, we can see that the values of the optimization criterion (income) when solving the specifically defined problem (16) and (20) with the use of the proposed and known models are equal (3690 thousand tenge per week). However, in the known models, since the conditions of "cleanliness" of production are not taken into account, the real income can be significantly lower due to environmental fines for violations of environmental standards. In the proposed model of production optimization with waste management, the need to take into account environmental standards stimulates production facilities to use cleaner, environmentally friendly technology, and income is guaranteed. In addition, such productions are supported by state authorities and have certain privileges for further development.

At the present stage of production development, the fulfillment of environmental standards, compliance with the cleanliness of production are particularly important requirements. In this regard, the proposed model should be applied to solve the problems of production optimization with waste management (16)–(20), taking into account the fulfillment of environmental standards. Thus, in contrast to the known models of production optimization, in the proposed model of production optimization with waste management to take into account the environmental factor in microeconomics, it is proposed to go to cost expressions in the criterion of $apF(\bar{x})$. When determining the value of the criterion, it is necessary to take into account payment for exceedances of pollution norms $\bar{z}\,\bar{\delta}$.

In the existing methods for solving fuzzy optimization problems, first, the original fuzzy problem is reduced to a set of crisp problems using the set of α levels. Then, the obtained set of crisp problems is solved using known methods. The solutions obtained at α levels are combined by the formula of fuzzy decision theory, which provides the solution to the original fuzzy problem. As it is known, in this case, a part of the initially collected fuzzy information representing the experience, knowledge, and intuition of the DM and experts is lost, which reduces the adequacy of the obtained solution in a fuzzy environment. In the formulated multi-criteria fuzzy optimization problem (1)-(2) and the heuristic method of its solution based on the combination and modification of the main criterion and Pareto optimality principles, the problem is both posed and solved in a fuzzy environment due to the belonging function and normalization. In this case, the original fuzzy problem is not transformed into a set of crisp problems, and the original fuzzy information is used as much as possible, which ensures the efficiency and adequacy of the obtained solutions in the fuzzy environment. In the proposed heuristic approach to solving the production optimization problem with fuzzy constraints, the DM iteratively improves the solutions by changing the values of the boundary values of local criteria and/or weight coefficients of fuzzy constraints until the best solution satisfying him is obtained. The proposed approach to solving a multi-criteria optimization problem in a fuzzy environment is based on the use of the experience, knowledge, and intuition of the DM and, to some extent, increases the amount of his work.

8. Conclusion and Further Research

An effective approach to the optimization of production with waste management based on economic and mathematical models is proposed. The general problem of multi-criteria optimization of production by economic and environmental indicators, adapted for the vagueness of some parts of the initial information, is formulated. The correct mathematical formulation of the formulated general problem is obtained by modifying and combining the principles of the main criterion and Pareto optimality, and a heuristic method of its solution in the presence of fuzzy constraints that take into account environmental standards is developed. The novelty of the proposed heuristic method lies in the maximum use of available fuzzy information, i.e., knowledge, experience, and intuition of DM and experts in the process of searching for and determining the effective, best solution under conditions of multi-criteria and vagueness. A new model of production optimization with waste management, taking into account economic norms, which is universal, i.e., applicable both in the field of macroeconomics and in the field of microeconomics, at the level of individual production, is proposed. In the proposed model of production optimization with waste management, the optimization criterion is represented in value terms based on the aggregate price of the produced products with the deduction of possible payments for exceedances of pollution norms. Thus, the proposed model of production optimization with waste management encourages industries to adopt cleaner, environmentally friendly technology. With the application of the proposed model of optimization of production with waste management, the problem of optimizing the production of desalted oil and stable gasoline at the Atyrau oil refinery, with consideration and observance of ecological norms, was solved. The optimization criterion for this optimization problem with waste management includes payments for exceeding environmental standards. The optimal solution to this problem, with observance of ecological norms, is determined with the application of the software package for solving optimization problems, Manager.

The developed heuristic method of multi-criteria optimization of production in a fuzzy environment will work effectively if there is necessary information, the possibility of selecting the main criterion, and assigning weights to fuzzy constraints. If this possibility is low, the effectiveness of the proposed method is significantly reduced. Therefore, in the future, it is planned to formulate mathematical formulations of production optimization problems by economic and environmental criteria in a fuzzy environment based on the combination and modification of other principles of optimality (maximin, equality, ideal point, etc.) and to develop methods for their solution. Then, depending on the current situation, availability, and accessibility of the initial information, the DM can choose a more appropriate formulation of the production optimization problem and method of its solution, which provides the possibility of choice and efficiency of the solution. In addition, the authors plan to apply the proposed model of production optimization with waste management, taking into account environmental standards, to solve the problems of production optimization by economic and environmental criteria of other objects and industries in the future. Based on the software implementation of the proposed model of production optimization, taking into account environmental standards, it is also planned to create a computer system to support the process of production optimization with waste management.

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