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## Developing Mathematical Models for Global Solar Radiation Intensity Estimation at Shakardara, Kabul

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### Abstract

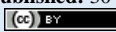
Solar energy properties such as Global Solar Radiation (GSR) intensity could be determined in either methods, experimentally or theoretically. Unfortunately, in most countries including Afghanistan, the first method which is more acceptable, but due to the high cost, maintenance and calibration requirements is not available. Therefore, an alternative widely used way is the second one which is model developments based on the meteorological (atmospheric) data; specially the sunny hours. The aim of this study at Shakardara area is to estimate atmospheric transparency percentage on 2017, determining the angstrom model coefficients and to introduce a suitable model for global solar radiation prediction. The hourly observed solar radiation intensity  $H$  ( $\text{WHm}^{-2}$ ) and sunshine hours  $S$  ( $\text{Hr}$ ) data at Shakardara Snow Survey Station for a full year 2017 were used to determine the Angstrom model coefficients for a linear and a nonlinear correlation. Then, by the application of solver function in Excel, the residuals for all correlations were minimized to obtain our linear and nonlinear models coefficients. Finally, to show the accuracy of models, Root Mean Square Error (RMSE) and Mean Bias Error (MBE) indices for all models are calculated and the estimated  $H$  data of three models are compared with the observed one to provide a graphical picture of models' accuracy. The results show an average value of 54% atmospheric transparency of sunny hours which was 2441 hours per year and 33% atmospheric transparency of monthly mean daily GSR which is  $123.61 \text{ WHm}^{-2}$ . Moreover, the angstrom model's coefficients found to be  $a=-0.1$  and  $b=0.9$  while for our linear model the same coefficients determined to be  $a=0$  and  $b=0.6$ . Calculation of RMSE and MBE for each model showed that the suggested models for GSR predicting in Shakardara area could be ordered from most accurate to less one as our nonlinear model with 99.16%, angstrom modified model with 96.67% and our linear model with 95.56%.

**Keywords:** Solar Radiation, Angstrom Model, Transparency, Solar Constant, Shakardara.

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**Ethical:** This study follows all ethical practices during writing.

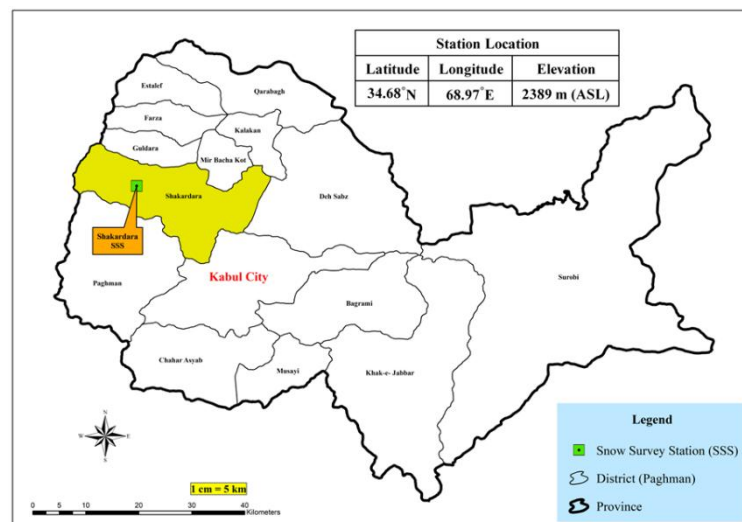
### 1. Introduction

Energy is the unique power reason for all kind of developments in the present world. Hence, knowing some adequate, measured and trustable information about the solar power state of a region is a must essential. Solar energy properties

(Global Solar Radiation (GSR) or Solar Radiation Intensity (SRI), sunshine duration hours, sunbeam incidence angle, sunrise/sunset hour angle and day lengths) could be determined in many ways: experimentally and theoretically. Unfortunately, in most developed countries the first method which is more acceptable, but due to the high cost, maintenance and calibration requirements is not available [1]. Moreover, because of different reasons; there is always some gaps in the observed GSR data series while for most of the solar based projects, a long period continuous GSR data set is required. Therefore, an alternative widely used way is the second one which is based on the meteorological (atmospheric) data; specially the sunny hours. Sunshine hours could be easily measured and is much more effective factor than other meteorological data on solar energy. Because of this, many empirical and mathematical models are developed, calibrated then adjusted and confirmed for specific areas and somehow worldwide. The inputs of most of these models is meteorological data sets such as air temperature, relative humidity, wind speed, sunshine hours and solar radiation which is easily available at many meteorological stations [2]. Among many different models, the general form of Angstrom model is mostly famous to be used around the world. The operation of this model is based on sunshine hours data as input to find the GSR. This model correlates the ratio of solar energy and sunny hours on the top of atmosphere to the same two parameters on the ground level [3]. It means that the Angstrom model is showing the clearness index of the atmosphere in a specific area. The main aim of this study is to determine the Angstrom' model coefficients values firstly, and then to elaborate mathematical models for estimation of GSR from sunshine hours at the snow survey station (SSS) of Shakardara in Kabul province based on the general form of Angstrom model. Although, some similar studies were conducted in other countries, but there is no any empirical developed model to estimate the solar energy intensity for Afghanistan context yet. GSR has been determined either experimentally measurements or mathematically estimates in many countries. For instance, Aljawi, et al. [4] have measured the solar ultraviolet radiation intensity at ground level in Bangi, Malaysia on 2014. Also, the total and spectral solar radiation irradiance (300–1100 nm) have been measured by Duay [5] at the same region in Malaysia. He used in his experiment a fiber optic spectrometer (AvaSpec ULS 2048x64-USB- spectrometer) and has done the measurements for 18 days within Jan – Mar 2014 in the direction of zenith sky. These two studies were conducted to apply a direct measurement of the GSR intensity using AvaSpec spectrometer. The focused point for both studies were selected at the National University of Malaysia (UKM) with coordinates of 2.92°N, 101.7° E at 50 meters above the sea level. Moreover, Harees, et al. [6]; at Iran, Togrul [7] at Kyrgyzstan, Muzathik, et al. [8], Muzathik, et al. [9] at Terengganu, Malaysia, Taha at Egypt [10] and Gana and Akpootu on 2013 at Nigeria [11] have estimated the solar radiation intensity based on meteorological data using different mathematical models.

## 2. Materials and Method

In this study, the hourly observed solar radiation intensity  $H$  ( $WHm^{-2}$ ) and sunshine hours  $S$  ( $Hr$ ) data at Shakardara Snow Survey Station was collected from the National Water Affairs Regulation Authority (NWARA) [12]. The used data was recorded by the Sutron Solar Radiation (SolRad) sensor installed in the mentioned station. The sensor was set up to measure GSR ( $H$ ) in ( $WHm^{-2}$ ) and sunshine duration in (*hours*) for each hour from the 1<sup>st</sup> Jan 2017 up to the end of Dec 2017. This hourly data set then was checked, controlled, tested and processed by the meteorology section of NWARA to turned to the daily values. In this study, the obtained daily data was turned to monthly average series. Shakardara SSS is installed at Shakardara district in the west part of Kabul province as illustrated in Figure 1



**Figure-1.**  
Kabul province administrative map and Shakardara snow survey location.

In the first step, the components of Angstrom model were calculated to correlate the two different transparency indices ( $H/H_0$  and  $S/S_0$ ) based on the Angstrom (1924) model [9]. The Angstrom model is given as:

$$\frac{H}{H_0} = a + b \frac{S}{S_0} \quad (1)$$

Where  $H$  is the monthly average daily GSR ( $Wh\ m^{-2}$ ),  $H_0$  is the monthly average daily extraterrestrial solar radiation,  $S$  is the monthly average of clear sky sunshine duration (hour),  $S_0$  is the monthly average of day length (hour) and  $a$  and  $b$  are the coefficients of the model to be determined in some way. As mentioned before, the values of  $H$  (in  $Wh\ m^{-2}$ ) and  $S$  (in hour) are recorded for each hour at Shakardara SSS during 2017 and the values of  $H_0$  (in  $Mj\ m^{-2}day^{-1}$ ) and  $S_0$  (in hour) could be obtained as below [3]:

$$H_0 = \frac{86400}{\pi} I_{sc} \left[ 1 + 0.033 \cos\left(\frac{360n}{365}\right) \right] [\cos\varphi \cos\delta \sin\omega + \omega \sin\varphi \sin\delta] \quad (2)$$

$$S_0 = \frac{2}{15} \omega \quad (3)$$

Where  $I_{sc}$  is the solar constant and is equal to  $1367 \pm 45 W \times m^{-2}$ ,  $\varphi$  is the latitude of the location (degree),  $\delta$  is the sun declination angle (degree),  $\omega$  is the sunset – hour angle (degree) and  $n$  is the sequential counted number of the day starting at 1<sup>st</sup> January.  $a$  and  $b$  are the coefficients of the correlation to be determined for Shakardara SSS in this study. In the next step, Angstrom model was applied to establish a simple linear correlation between  $S/S_0$  and  $H/H_0$  and the Angstrom model coefficients are found. To ensure that the inaccuracy of predicted data is minimized, there are two ways to be followed in our case. One way is to compare the observed and estimated data and hence, must reduce the residual value to find the new values of the coefficients. In this case, the Angstrom modified linear model will be reduced to a simpler linear model which is our linear model. To do this, by the help of solver function process in Excel, the squared residual of observed and estimated data was minimized. The second way is to establish a nonlinear correlation between  $S/S_0$  and  $H/H_0$  and then reduce the residual value by comparing the nonlinear predicting data with that of observed one. In this case by application of the solver function of Excel the squared residuals were tried to minimize and our nonlinear model was obtained.

### 2.1. Comparison Techniques

To show the accuracy of models' operation and compare the estimated data of different models, some statistical quantities and indices for all models are calculated. Also, the estimated  $H$  values of three models are graphically compared with the observed one to provide a pictorial insight of the models' accuracy. In our case, the coefficient of correlation ( $r$ ), the Mean Bias Error ( $MBE$ ) and the Root Mean Square Error ( $RMSE$ ) are calculated to test the models' operation and to compare the models' predicted values with those of measured.

#### The coefficient of correlation

The coefficient of correlation for both linear and nonlinear could be obtained as Yazdanpanah, et al. [3]:

$$r = \frac{\sum(H_{est} - \bar{H}_{est})(H_{obs} - \bar{H}_{obs})}{\sqrt{((H_{est} - \bar{H}_{est})^2)((H_{obs} - \bar{H}_{obs})^2)}} \quad (4)$$

For a better developed model and high correlated data, the value of  $r$  should approach to 1 as nearly as possible.

#### The mean bias error

The mean bias error is calculated as Yazdanpanah, et al. [3]:

$$MBE (\%) = \frac{100}{\bar{H}_{obs}} \sum_{i=1}^n \left( \frac{H_{est} - H_{obs}}{n} \right) \quad (5)$$

The value of  $MBE$  could be positive or negative, but a value close to zero shows the best performance of the model. Positive values of  $MBE$  show overestimates and negative values of  $MBE$  show underestimated performance of the model.

#### The Root Mean Square Error

The root mean square error is computed as Muzathik, et al. [9]:

$$RMSE = \left[ \frac{100}{n} \sum_{i=1}^n (H_{est} - H_{obs})^2 \right]^{\frac{1}{2}} \quad (6)$$

As we can see,  $RMSE$  could have only positive values and a model with a best performance will have a value of  $RMSE$  very close to zero. The more greater the value of  $RMSE$ , the poor accuracy of predicted value of  $H$  by the model.

## 3. Results and Discussions

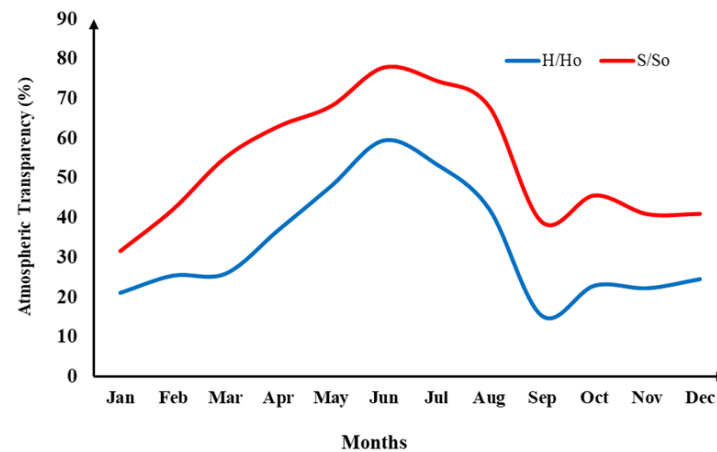
Some solar properties for Shakardara SSS are calculated to establish the correlation between two transparency indices ( $S/S_0$  and  $H/H_0$ ) and hence, to determine the angstrom model empirical constants  $a$  and  $b$ . Also, before creating the correlation, the monthly variation of the above indices is plotted to show the trend of model variables. The calculated solar properties and the trend of the transparency indices are presented in Table 1 and Figure 2, respectively.

As we can see in Table 1, the average value of the solar radiation that could penetrate to atmosphere and reaches to the ground surface at Shakardara area is around 33% of the solar extraterrestrial radiation. This is equivalent to  $10.94 \text{ MJ m}^{-2} \text{ day}^{-1}$  which is around third parts of the average value of the solar constant. Moreover, in an average day length of 11.99 hrs, the mean value of sunny hours is 6.67 hrs. This means that the average transparency rate of atmosphere based on sunshine hours in Shakardar is 53.79%.

**Table-1.**

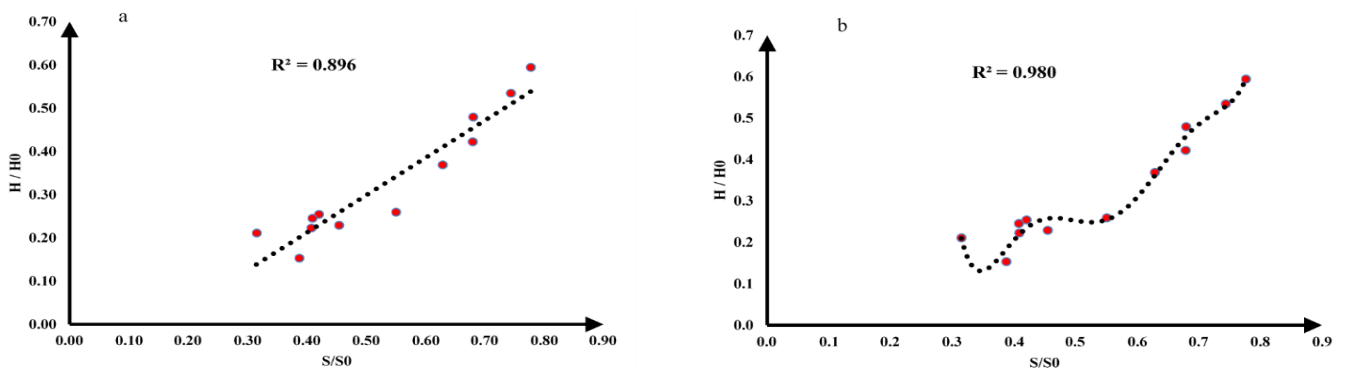
Some Solar Properties at Shakardara SSS in 2017.

Solar Property	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Mean
$H_0 (\text{MJ m}^{-2} \text{ day}^{-1})$	18.50	23.26	29.73	35.91	39.97	41.51	40.64	37.30	31.76	25.12	19.53	17.02	30.02
$H (\text{MJ m}^{-2} \text{ day}^{-1})$	3.91	5.92	7.71	13.25	19.16	24.68	21.73	15.75	4.87	5.76	4.35	4.18	10.94
$S_0 (\text{MJ m}^{-2} \text{ day}^{-1})$	10.0	10.7	11.8	12.9	13.8	14.3	14.1	13.3	12.2	11.1	10.2	9.7	11.99
$S (\text{MJ m}^{-2} \text{ day}^{-1})$	3.1	4.5	6.5	8.1	9.4	11.1	10.5	9.0	4.7	5.0	4.1	4.0	6.67
$H/H_0 (\%)$	21.11	25.44	25.95	36.91	47.94	59.45	53.48	42.23	15.33	22.92	22.27	24.55	33.13
$S/S_0 (\%)$	31.49	42.00	55.12	62.89	67.99	77.72	74.38	67.85	38.82	45.51	40.81	40.86	53.79

**Figure-2.**

Monthly Variation of Atmospheric Transparency at Shakardara SSS in 2017.

The monthly variation of atmospheric transparency indices in Figure 2 are closely parallel and hence, a pretty correlation could be established between them. Establishing a linear correlation between  $S/S_0$  and  $H/H_0$  results in Angstrom model coefficients determination. The linear correlation between the aforementioned atmospheric transparency indices is plotted in Figure 3 (a). with  $R^2 = 0.896$  which is in the acceptable range of correlation coefficient. Also, a nonlinear correlation between  $S/S_0$  and  $H/H_0$  is created as in Figure 3 (b). to obtain a more accurate relation for monthly mean daily GSR estimation at Shakardar area.

**Figure-3.**(a) Linear Correlation between  $H/H_0$  and  $S/S_0$  and (b) nonlinear Correlation between  $H/H_0$  and  $S/S_0$ .

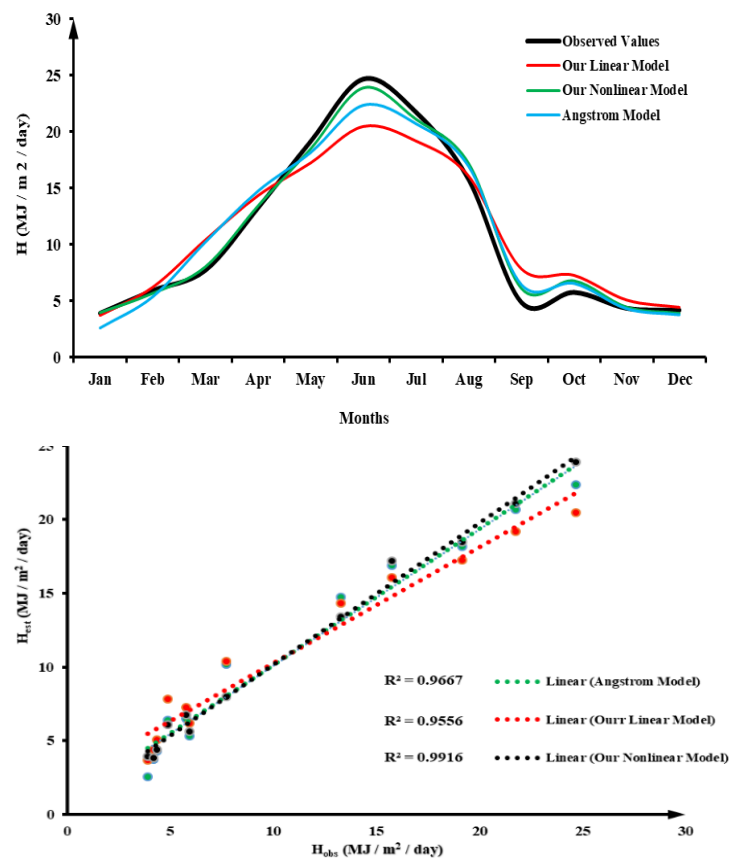
Based on the linear correlation between  $S/S_0$  and  $H/H_0$  the angstrom model coefficients  $a$  and  $b$  are found to be  $-0.134$  and  $0.865$  respectively. Moreover, this angstrom's modified model was used to predicted the monthly mean value of daily GSR and the results were compared with those of measured one at Shakardara SSS. By applying the so called "residual minimization method" and using Solver Function of Excel, we have compared the measured and predicted values of angstrom modified model and the nonlinear model to obtain our linear and nonlinear models respectively. Some properties of the angstrom modified model and our linear and nonlinear models are presented in Table 2.

**Table-2.**

Suggested parameters for different models to estimate the monthly mean daily GSR on horizontal surface at Shakardara SSS.

No	Model Name	Model Form	RMSE	MBE (%)	Reliability (%)
1	Angstrom	$H = a^* \left( 0.865 \frac{S}{b} - 0.134 \right)$	1.9	0.4	96.67
2	Our Curved Model	$H = a \left[ 5556 \left( \frac{S}{b} \right)^6 + 25782 \left( \frac{S}{b} \right)^4 + 7376 \left( \frac{S}{b} \right)^2 + 129.15 \right]$	0.5	0.1	99.16
3	Our Linear Model	$H = 0.634 \frac{a \times S}{b}$	4.0	0.6	95.56

As it is obvious from Table 2, the RSME values range between 0.5 to 4.0 with the smallest value for our nonlinear polynomial model ( $=0.5$ ) and the largest value is for our linear model. Hence, the most accurate monthly mean daily GSR data could be predicted using our curved model. Moreover, the values of MBE show that all models are predicting an overestimate of the observed data. However, our curved model predicts the monthly mean daily GSR with the smallest overestimates error. The statistical analysis and comparison of all models' predicted data with that of observed one gives the reliability of model's operation. As we can see in Table 2, the predicted data using our curved model is corresponding to the measured data with 99.16% accuracy.



**Figure-5.**

Correlation between measured monthly mean daily GSR data and predicted radiation.

As we can see in Figure 4, the trend of all model's predicted data is mostly parallel to that of observed data. The models work very well in the months with low values of GSR except in March and the most deviation of predicted data is occurred in the hot season especially in Jun. Moreover, Figure 5 illustrates that the most accurate GSR data is produced by our nonlinear model which is about 99.16% of the measured data and the most less accurate GSR data (about 96.67% of the measured data) is predicted by our linear model. However, both Figure 4 and 5 extremely recommend our nonlinear model to estimate the GSR based on sunshine duration hours in Shakardara areas.

#### 4. Conclusion

This research was aimed to calculate the transparency rate and to elaborate some mathematical models for estimation of global solar radiation at horizontal ground surface in Shakardara area and anywhere with similar climate conditions. Hence, we have tried to determine the Angstrom model coefficients and elaborate two other mathematical models for solar radiation estimation based on sunny hour available data at Shakardara district. The results show that Shakardara area is 54% sunny over the year and receiving around 33% of all incoming solar radiation energy on the top of atmosphere. Moreover, among three examined models, our polynomial model is highly recommended to be used for global solar radiation intensity based on sunshine hours data. This is helpful since most of the time the hydrometeorological stations are not equipped with actinometric instruments.

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