International Journal of Innovative Research and Scientific Studies, 8(3) 2025, pages: 3467-3477

IJIRSS
International Journal of Innovative Research & Scientific Studies

ISSN: 2617-6548

URL: www.ijirss.com

Characteristic Properties of Plane-Parallel Accelerating Unsteady Turbulent Pressure Flaw

Arestak Sarukhanyan[1*], Garnik Vermishyan[2], Hovhannes Kelejyan[3]

[1,3]Department of Water Systems, Hydraulic Engineering and Hydropower National University of Architecture and Construction of Armenia 105 Teryan str., Yerevan, Armenia, 0009.
[2]Department of Mathematics, National University of Architecture and Construction of Armenia Teryan str., 105, Yerevan, Armenia, 0009.

Corresponding author: Arestak Sarukhanyan (*Email: arestak.sarukhanyan@gmail.com*)

Abstract

One of the urgent problems in the study of applied hydrodynamics is the study of structural changes in turbulent accelerating plane-parallel pressure flow of viscous fluid. The investigation of structural alterations in turbulent accelerating plane-parallel pressure flow of viscous fluid is one of the most pressing issues in applied hydrodynamics research. It aims to identify regularities of hydrodynamic parameter changes in fluid channels of hydro-automation equipment in the case of unsteady flow of viscous fluid. When a viscous fluid flows unsteadily, the problem here is to find regularities in the hydrodynamic characteristics that vary in the fluid channels of hydro-automation equipment. This will allow for the correct design of individual components of hydraulic equipment, ensuring the precise and uninterrupted operation of the equipment. This will enable the accurate design of each hydraulic equipment component, guaranteeing the equipment's exact and continuous performance. The study of the plane-parallel unsteady accelerating flow of viscous fluid was carried out based on calculating the kinematic coefficient of turbulent viscosity. The kinematic coefficient of turbulent viscosity was calculated to examine the plane-parallel unstable accelerating flow of viscous fluid. Under conditions of axisymmetric change of the flow, a boundary problem was formulated, of which the boundary conditions were chosen in accordance with the pressure gradient and arbitrary distribution regularities of velocities in the section. The regularities of the dynamic viscosity coefficient change in turbulent flow were taken as a special condition. A boundary value problem is formulated under general initial and boundary conditions. A method for integrating the boundary value problem is developed, and regularities for changing instantaneous velocities over a cross-section are obtained. Analytical solutions were obtained, which allow for obtaining regularities of velocity change in the flow direction at any time. To get the regularities of velocity change in the flow direction at any moment, analytical solutions were produced. Based on the general solutions of the problem, solutions were obtained for accelerating flow under the influence of a constant pressure gradient on the fluid at rest. Solutions for accelerating flow when a continuous pressure gradient is applied to the fluid at rest were derived from the problem's general solutions. Computer-aided analyses have been used to plot graphs of the distribution of velocities in the section, average velocities, and the change in the coefficient of flow for different values of time. A graph of the change in energy loss over time under unsteady flow conditions has also been plotted.

Keywords: Graph of changes, hydrodynamic parameters, plane-parallel, unstable flow, viscous fluid.

DOI: 10.53894/ijirss.
Funding: This study received no specific financial support.
History: Received: 28 March 2025 / **Revised:** 1 May 2025 / **Accepted:** 5 May 2025 / **Published:** 22 May 2025

Competing Interests: The authors declare that they have no competing interests.
Authors' Contributions: All authors contributed equally to the conception and design of the study. All authors have read and agreed to the published version of the manuscript.
Transparency: The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.
Publisher: Innovative Research Publishing

1. Introduction

Hydromechanical phenomena occurring in pressure systems are often accompanied by unsteady flow of the working medium, which causes changes in velocity, pressure, and density of the medium.

Unsteady flow of the working medium frequently coexists with hydromechanical processes in pressure systems, resulting in variations in the medium's velocity, pressure, and density. When studying engineering problems, the one-dimensional non-stationary flow model is considered a computational model, in which the hydrodynamic parameters of the flow are regarded as averaged values over the section. The one-dimensional non-stationary flow model is regarded as a computer model for engineering issues, where the hydrodynamic characteristics of the flow are taken into account as averaged values across the cross-sections. Deviations of the averaged values from the hydrodynamic parameters at a point are taken into account using the coefficients of kinetic energy and momentum. Therefore, one of the important issues of non-stationary flow is the question of how the flow of the particles of air is affected by the flow of the particles of air. Therefore, one of the important issues of non-stationary flow is the study of the patterns of change of the mentioned coefficients, for which it is necessary to perform structural studies of non-stationary flow.

Often, to perform these studies, it is necessary to use complex mathematical models, and the results obtained are not suitable for practical applications. Therefore, in order to introduce certain simplifications into these calculations, we proceed from the quasi-stationary model of flow proposed by Popov [1]. In this case, the non-stationary flow is viewed as a sequence of stationary flows. At each instant of time, the average velocity of the current is equal to the average velocity of the apparently stationary flow, thus the non-stationary flow is viewed as a set of successive stationary flows. However, experimental and theoretical studies Luoqin et al. [2]; Sun [3] and Luchini [4] show that in reality the instantaneous velocity distribution curves can differ significantly from the quadratic parabola law, due to which the energy loss cannot be determined only by the friction stress occurring at the stationary wall. Depending on the law of change of instantaneous velocity in the section, the friction stress occurring at the wall will also change. In order to obtain regularity, the change of friction stress occurring at the stationary wall in the case of non-stationary flow, it is necessary to study the pattern of velocity change in the effective cross-section.

The pattern of velocity change in the effective cross-section must be examined in order to determine the change in friction stress that occurs at the stationary wall in the event of unsteady flow.

2. Literature Review and Problem Statement

The flow of viscous fluids in pressure systems is usually accompanied by an unsteady flow regime, in which, in the effective cross-section, the hydrodynamic parameters of the flow change with time. Depending on the flow regime, these dependencies are different, since the main characteristic factors of the flow are the tangential stresses arising between the fluid layers. Schlichting and Gersten [5], based on the viscosity law of Newton and taking into account tangential stresses, came to the idea that when the flow is laminar, the tangential stresses arise between the layers. Popov [1] and Emtsev [6] carried out studies under laminar non-stationary flow conditions, as a result of which regularities of changes in the hydrodynamic parameters of the flow and calculation formulas for energy losses have been obtained. However, Emtsev [6] arrived at the conclusion that plane-parallel laminar flow is one of the least common flows of this nature. Plane-parallel unsteady flows are mainly in the turbulent regime. Based on this circumstance, the study of plane-parallel non-stationary turbulent flow has important practical applications and theoretical interest. The turbulent regime is mostly where plane-parallel unstable flows occur. This situation makes the study of plane-parallel non-stationary turbulent flow both theoretically and practically significant. Gromeka [7] studied the regularities of changes in the hydrodynamic parameters of the flow under conditions of unsteady viscous fluid in a narrow cylindrical tube, and as a result, the patterns of changes in pressure and velocity were obtained.

The friction stresses, according to Prandtl [8] and Boussinesq [9], arising between flow layers during turbulent flow depend on the distance from the stationary wall. Sarukhanyan et al. [10] prove that solutions obtained using a function introduced to account for turbulent stresses depending on the distance from a stationary wall in a turbulent flow provide insights into structural changes in the flow. However, the accepted calculation formula for turbulent stresses provides accurate results in relatively narrow flow paths. Therefore, the obtained solutions are applicable only for narrow fluid-carrying paths. Fundamental research on the distribution of turbulent stresses in wide fluid-carrying paths was carried out by Prandtl [8], who introduced the concept of a mixing path.

The patterns of changes in the hydrodynamic parameters of under turbulent flow of a viscous fluid in circular cross-section pipes and in the case of plane-parallel flow have been studied by She et al. [11]. For this purpose, a variable function is introduced for the mixing path in the turbulent stress calculation formula, which ensures the symmetry of the flow. As a result, a universal formula was obtained, which includes both logarithmic and power distribution formulas for the turbulent stress distribution. Experimental studies conducted by Chen et al. [12] prove the reliability of the research. Thanks to carefully conducted, high-precision x-probe experiments performed by Baidya et al. [13] was assessed the effect of the velocity component perpendicular to the wall on the magnitude of the turbulent viscosity coefficient was assessed. Luoqin et al. [2] present the main achievements in the research of the transition period and turbulence for the last thirty years. Despite the achieved results, further deepening of studies is required to identify the emerging turbulent tension. The state of the problem is analyzed by identifying the general analytical formula of the non-uniform distribution of the average velocity over the entire region of the turbulent flow for all Reynolds numbers within the framework of Prandtl's mixing length theory. Considering the Prandtl mixing length model, a closed-form solution for the average velocity profile of a plane turbulent flow is obtained by Sun [3]. In order to confirm the universal logarithmic law of the average velocity profile, which is the basis of turbulent fluid mechanics, experiments and numerical simulations were conducted by Luchini [4]. It is proven that the discrepancy may arise due to the geometric dimensions of the channel and the effect of the pressure gradient. When this effect is taken into account, the logarithmic law of the average speed profile is satisfactorily acceptable. Numerical modeling of turbulent flow after a sharp increase in flow rate was carried out. It has been proven that a turbulent flow with a low Reynolds number can undergo a transition process that resembles a laminar-turbulent transition. He and Seddighi [14] found that. The flow is initially laminar, but later in the transitional phase, localized turbulent spots are formed, which grow spatially, merge with each other, and eventually, the flow becomes completely turbulent.

When moving a viscous liquid, as the speed of the flow increases, the process of transfer and turbulence occurs. Even for the well-studied case of flow in a pipe, it was not possible to determine at what Reynolds number the flow would be either permanently turbulent or eventually laminar. Thanks to carefully designed experiments and computer modeling, Avila et al. [15] managed to identify the conditions for maintaining turbulence. In order to reduce the hydraulic resistance, Jozsa [16] developed a strategy for controlling the turbulent flow. Incompressible liquid flow in a rectangular channel and round pipes caused by uniform and traveling wave oscillations of the wall is investigated. Obtained conditions for the reduction of turbulent friction. Numerical modeling of the accelerated flow was carried out by Kharghani and Pasandidehfard [17]. Two accelerations that affect the turbulent flow are considered. The obtained results illustrate that the applied accelerations can seriously dampen the oscillations. In the work of Chovniuk et al. [18], a general method of numerical solution of the problem of unsteady flow of viscous incompressible liquid in plane channels of arbitrary shape was developed. An effective solution to the task is achieved by using adaptive networks. The mathematical model of the flow is based on two-dimensional Navier-Stokes equations and Poisson's equation for pressure, which are solved on the basis of the finite difference method. Numerical modeling of liquid flow in a plane curved elbow was carried out at Reynolds number Re = 1000.

3. The Aim and Objectives of the Study

The purpose of the research is to develop a method for calculating hydraulic losses in a plane channel in non-stationary developing turbulent plane-parallel pressure movements.

To achieve this goal, the following tasks are solved:

- Formulate a boundary value problem to identify patterns of change in the hydrodynamic parameters of a developing turbulent plane-parallel pressure flow.
- To develop a method for solving the boundary value problem by identifying regularities in the changes of hydrodynamic parameters during the development of turbulent plane-parallel pressure flow.
- Build graphs of changes in axial velocities, coefficients of the amount of flow and kinetic energy, energy losses and shear stresses, depending on time and pressure gradient.

4. Materials and Methods

4. 1. *Choosing a Calculation Scheme*

The calculation of energy losses in turbulent non-stationary flow has important practical applications and theoretical interest. Research on the calculation of energy losses in plane-parallel non-stationary flow is based on the conclusion that energy losses are caused by friction stresses arising on the stationary wall of the channel. However, this conclusion, which corresponds to the quasi-stationary model of non-stationary flow, does not correspond to reality. In reality, during non-stationary flow, energy losses occur not only from the change in the fluid velocity but also from the deformation of the velocity profile:

$$h_{\mathrm{w}} = \frac{l}{\rho g}\frac{\partial p}{\partial x} + \frac{l}{g}\left(\beta\frac{dV}{dt} + \frac{V}{2}\frac{d\beta}{dt}\right) \qquad (1)$$

The second term of the above equation is called inertial pressure of which the first term $\frac{l}{g}\beta\frac{dV}{dt}$ is conditioned by the velocity change and the second one $\frac{l}{g}\cdot\frac{V}{2}\cdot\frac{d\beta}{dt}$ is conditioned by the velocity diagram deformation where $V = \int_0^1 u(x,t) \cdot dx$ is the average velocity of the flow, $\beta = \frac{\int_0^1 u^2(x,t) \cdot dx}{V^2}$ is the Boussinesq's momentum factor.

Therefore, to determine the energy losses during non-stationary flow, it is necessary, depending on the initial and boundary conditions of the problem, to identify the patterns of change in the hydrodynamic parameters in the effective cross-section, which determine the energy losses.

As is known, in turbulent flow, friction stresses arise between the fluid layers, the magnitude of which depends on the position of the point in the flow. The farther the point is from the stationary wall, the greater the friction stresses. The tangential stresses arising between the fluid layers in turbulent flow depend on the distance from the stationary wall and the velocity gradient in the vertical direction of flow.

As is known, in turbulent flow, friction stresses arise between the fluid layers, the magnitude of which depends on the position of the point in the flow. The farther the point is from the stationary wall, the greater the friction stresses. The tangential stresses arising between the fluid layers in turbulent flow depend on the distance from the stationary wall and the velocity gradient in the vertical direction of flow. According to Prandtl's conjecture, the tangential stresses arising in turbulent flow are determined by Schlichting and Gersten [5] and Prandtl [8].

$$\tau = \rho \ell^2 \left(\frac{du}{dy} \right)^2 \tag{2}$$

where ℓ is the mixing path length

$$\ell = \varepsilon . y \tag{3}$$

where ε is a relativity coefficient which is accepted as equal to 0.4. Prandtl [8] y is the distance of the layer from the fixed wall.19

The calculation formula (2) for the tangential stresses arising between fluid layers in a turbulent flow was obtained as a result of analyses of studies of stationary turbulent flows. The high-accuracy coincidences between the results obtained with this formula and experimental studies prove the correspondence of the proposed formula to the ongoing phenomena.

Due to the lack of a calculation formula for the tangential stresses arising between the flow layers under turbulent unsteady flow, let us consider the same principle. We will see that the tangential stresses arising between the fluid layers during turbulent unsteady flow can be calculated using the same Equation 2.

4.2. Statement of the Problem and Formulation of the System of Differential

If we separate the elementary mass of the fluid from the non-stationary turbulent flow and formulate its differential equation of flow, we will have (Figure 1)

$$\rho \frac{\partial \bar{u}}{\partial t} = -\frac{\partial p}{\partial z} + \frac{\partial \tau}{\partial y^*}. \tag{4}$$

Considering that $y^* = h - y$ the last equation will take the following form

$$\rho \frac{\partial \bar{u}}{\partial t} = -\frac{\partial p}{\partial z} - \frac{\partial \tau}{\partial y}. \tag{5}$$

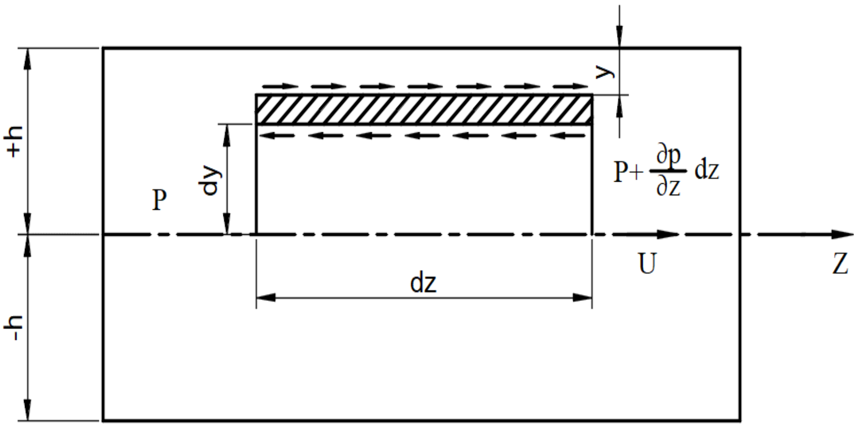

Figure 1.
Computational diagram for plane-parallel turbulent flow.

In the case of axisymmetric and isothermal plane-parallel flow, the pressures at each point of the effective cross-section are the same and depend only on time, i.e.

$$\frac{1}{\rho}\cdot\frac{\partial p}{\partial z} = f(t): \tag{6}$$

Thus, the study of non-stationary accelerating turbulent plane-parallel flow is reduced to the integration of the equation

$$\frac{\partial \bar{u}}{\partial t} = -f(t) - \frac{\partial \tau}{\partial y} \tag{7}$$

under the following boundary conditions:

$$\bar{u}(y,t) = \phi(y), \text{ when } t=0, \quad (0<y<2h) \tag{8}$$

Let's introduce some dimensionless variables

$$U(x,t) = \frac{\bar{u}}{U_0}, \quad \frac{y}{h} = x, \quad \frac{P}{P_0} = p, \quad \sigma = \frac{z}{h}, \quad -\frac{p_0}{\rho h U_0}\frac{\partial p}{\partial \sigma} = F(t) \tag{9}$$

Where is the average velocity of the effective cross-section

$$U_o = \frac{1}{h}\int_0^{+h}\varphi(y)dy: \tag{10}$$

Eq.(7) with dimensionless variables will have the following form

$$\frac{\partial U}{\partial t} = -F(t) - \frac{1}{\rho h U_0}\frac{\partial \tau}{\partial x}, \tag{11}$$

As for boundary conditions, (9) will have the following view

$$U(x,t) = 0, \quad \text{when} \quad x=0, \quad t>0, \tag{12}$$

$$U(x,t) = \phi(x), \text{ when } t=0, \quad (0<x\le 1), \tag{13}$$

$$\frac{\partial U(x,t)}{\partial x}\Big|_{x=1} = 0. \tag{14}$$

The solution of Eq.(11) according to Tikhonov and Samarskii [19] can be found in the form of a sum

$$U(x,t) = U_1(x,t) + U_2(x,t), \tag{15}$$

where $U_1(x,t)$ function is the solution of the homogeneous equation

$$\begin{cases} \dfrac{\partial U_1(x,t)}{\partial t} = -\dfrac{1}{\rho h U_0}\dfrac{\partial \tau}{\partial x}, \\ \tau = \rho\varepsilon^2 U_0\left(x\dfrac{\partial U_1(x,t)}{\partial x}\right)^2 \end{cases} \tag{16}$$

In case of inhomogeneous (12,13,14) boundary conditions and $U_2(x,t)$ function is the solution of the inhomogeneous equation in case of homogeneous boundary conditions

$$\begin{cases} \dfrac{\partial U_2(x,t)}{\partial t} = F(t) - \dfrac{1}{\rho h U_0}\dfrac{\partial \tau}{\partial x}, \\ \tau = \rho\varepsilon^2 U_0\left(x\dfrac{\partial U_2(x,t)}{\partial x}\right)^2 \end{cases} \tag{17}$$

Let us look for the solution of the Equation 16 system in the following form` $U_1(x,t) = V(x)\exp(-\beta\cdot t)$

where $\beta = \dfrac{\varepsilon^2}{h}$, we get

$$V(x) = \frac{d}{dx}\left(x\frac{dV(x)}{dx}\right)^2\exp(-\beta\cdot t_0), \qquad 0\le t_0\le t \tag{18}$$

To solve Eq.(18), we introduce $W(x)$ auxiliary function

$$W(x) = x\frac{dV(x)}{dx}, \tag{19}$$

After that Eq.(18) will have the following form

$$\begin{cases} W(x) = x\dfrac{dV(x)}{dx}, \\ V(x) = 2aW(x)\dfrac{dW(x)}{dx} \end{cases} \qquad (20)$$

The general real solution of the system (20) will be

$$W(x) = \frac{1}{\sqrt{a}}\sqrt{2ac_1 + \log x}\text{, where } \quad a = \exp(-\beta \cdot t_0), \qquad (21)$$

Having $W(x)$ function from Eq.(19) $V(x)$ is determined

$$V(x) = \frac{2}{3\sqrt{a}}\left[2aC_1 + \log x\right]^{3/2} \qquad (22)$$

where C_1 is an arbitrary constant of which value is determine from boundary condition $V(0) = 0.$ Since the point $x = 0$ is a unique point in the problem, assuming that $V(0.01) = 0$, we get:

$2aC_1 + \log 0.01 = 0$, from which

$$2aC_1 = 4.61 \qquad (23)$$

Let us look for $U_2(x,t)$ function in the following form

$$U_2(x,t) = \varphi(t)V(x) \qquad (24)$$

Substituting the value of $U_2(x,t)$ function from Equation 24 in the Equation 17 system, we have

$$\frac{d\varphi(t)}{dt}V(x) = F(t) - \beta\varphi^2(t)\frac{d}{dx}\left[x\frac{dV(x)}{dx}\right]^2. \qquad (25)$$

Taking into account Eq.(18), Eq.(25) will take the following form:

$$\left[\frac{d\varphi(t)}{dt} + \frac{\beta}{a}\varphi^2(t)\right]V(x) = F(t) = B_0 = Const \qquad (25)$$

In order to determine the value of the $\varphi(t)$ function, we integrate Equation 25 with respect to x, taking into account that $x = 0$ is a unique point for $V(x)$, and performing the integration for the interval [0.01;1], we have

$$\left[\frac{d\varphi(t)}{dt} + \frac{\beta}{a}\varphi^2(t)\right]\int_{0.01}^{1} V(x)dx = F(t)\int_{0.01}^{1} dx = 0.99B_0\ . \qquad (26)$$

Taking into account Eqs. (21 and 22), we have

$$\int_{0.01}^{1} V(x)dx = \int_{0.01}^{1} \frac{2}{3\sqrt{a}}\left[4.61 + \log x\right]^{3/2} dx = 4.72509\,, \qquad (27)$$

By substituting the value of expression (27), Equation 26 will be reduced to the following form:

$$\frac{d\varphi(t)}{dt} + \frac{\beta}{a}\varphi^2(t) = 0.20952B_0\,. \qquad (27)$$

Taking into account the boundary conditions of the problem, we get: $b = -2{,}41962;\ \dfrac{0.99}{b} = -0{,}409155$ Under these values of constant coefficients, Equation 27 will take the following form

$$\left[\frac{d\varphi(t)}{dt} + \frac{\beta}{a}\varphi^2(t) - \gamma = 0\right]. \qquad (28)$$

where $\gamma = 0{,}409155B_0$.

The solution to Equation 28 will look like this $\varphi(t) = -\sqrt{\dfrac{a\gamma}{\beta}}Tanh\left[\sqrt{\dfrac{\beta}{a}}\gamma(t - C_0)\right]$ (29)

Having the value of the function $\varphi(t)$, we obtain the function $U_2(x,t) = \varphi(t)V(x)$:

$$U_2(x,t) = -\sqrt{\frac{a\gamma}{\beta}}Tanh\left[\sqrt{\frac{\beta}{a}}\gamma(t - C_0)\right]V(x). \qquad (30)$$

Substituting the values of $U_1(x,t)$; $U_2(x,t)$ functions into Eq.(15), we get

$$U(x,t)=\frac{2}{3}\left\{\exp(-\beta\cdot t)-\sqrt{\frac{a\gamma}{\beta}}Tanh\left[\sqrt{\frac{\beta}{a}}\gamma(t-C_0)\right]\right\}(4{,}61+\log x)^{3/2} \tag{31}$$

The obtained solutions allow to obtain the axial velocities at any point of the effective cross-section at any time.
The average velocity of the section is determined from the velocity distribution, Equation 31

$$\overline{U}(t)=\int_0^1 U(x,t)dx=\int_0^1\frac{2}{3}\left\{\exp(-\beta\cdot t)-\sqrt{\frac{a\gamma}{\beta}}Tanh\left[\sqrt{\frac{\beta}{a}}\gamma(t-C_0)\right]\right\}(4{,}61+\log x)^{3/2}dx \tag{32}$$

The value of the C_0 constant is determined from the (13) initial condition, when $\phi(x)=A_0=Const$, we have

The value of the C_0 constant is determined from the initial condition (13) when $\phi(x)=A_0=Const$, we have

$$\frac{2}{3}\left\{1+\sqrt{\frac{a\gamma}{\beta}}Tanh\left[C_0\sqrt{\frac{\beta}{a}}\gamma\right]\right\}(4{,}61+\log x)^{3/2}=A_0. \tag{33}$$

Integrating the last equation in the x $[0.01;1]$ interval, we get

$$\frac{2}{3}\left\{1+\sqrt{\frac{a\gamma}{\beta}}Tanh\left[C_0\sqrt{\frac{\beta}{a}}\gamma\right]\right\}7.08763=0.99A_0, \tag{34}$$

from which

$$\frac{2}{3}\left\{1+\sqrt{\frac{a\gamma}{\beta}}Tanh\left[C_0\sqrt{\frac{\beta}{a}}\gamma\right]\right\}7.08763=0.99A_0; \tag{35}$$

$$Tanh\left[C_0\sqrt{\frac{\beta}{a}}\gamma\right]=\frac{0.690856}{\sqrt{B_0}}(0.22A_0-1). \tag{36}$$

$Tanh[z]=A,\quad (|A|<1)$ equation in the interval $-\infty<z<\infty$ has only one real root regardless the sign of A constant.

Therefore, if we choose an arbitrary constant C_0 so that the condition $C_0\sqrt{\frac{\beta}{a}}\gamma=-1$ is satisfied, we will have $Tanh[-1]=-0..761594..$ Substituting this value into Equation 35, we will obtain the values of B_0 and $\frac{a\gamma}{\beta}$:

$$\sqrt{B_0}=0.907118(1-0.22A_0);\qquad \frac{a\gamma}{\beta}=\frac{1-022A_0}{0.761594} \tag{36}$$

Substituting these values in Eq.(32), we have the solution of the problem

$$\overline{U}(t)=\frac{2}{3}\left\{\exp(-\beta\cdot t)-\frac{1-022A_0}{0.761594}Tanh\left[\frac{1-022A_0}{0.761594}\beta\cdot t-1\right]\right\}(4{,}61+\log x)^{3/2} \tag{37}$$

Based on the general solutions to the problem, let us consider the flow of a fluid at rest in the case of a constant change in the pressure gradient. Under these conditions, we have $A_0=0$, $B_0=0.822863$

$$\overline{U}(t)=\frac{2}{3}\{\exp(-0.1\cdot t)-1.313Tanh[0.131\cdot t-1]\}(4{,}61+\log x)^{3/2}. \tag{38}$$

With this velocity distribution formula, velocity change graphs were plotted using a computer analysis for different values of the pressure gradient.
According to Equation 38, the average velocity of the effective cross-section will be

$$V=\{4.72509e^{-0.1t}-6.20423\text{Tanh}[1-0.131304t]\} \tag{39}$$

5. Research Results of Structural Changes in the Hydrodynamic Parameters of the Plane-Parallel Turbulent Accelerating Flow

5. 1. Results of Computation of Structural Changes of the Turbulent Accelerating Flow.

A regularity of variations in instantaneous axial velocities was derived from the results of integrating the boundary value problems (38). Both graphs of changes in velocities over time and graphs of changes in axial velocities over time at various positions of the effective cross-section were plotted using numerical computations.

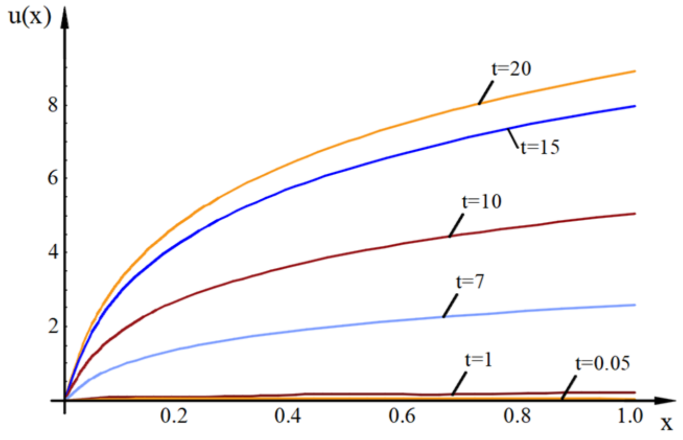

Figure 2.
Graph of axial velocity change at different points of the effective cross-section.

$t \to \{0.5,0,1,3,7,10,15,20\},, \{x, 0.01,1\}, A_0 = 0, B_0 = 822863$

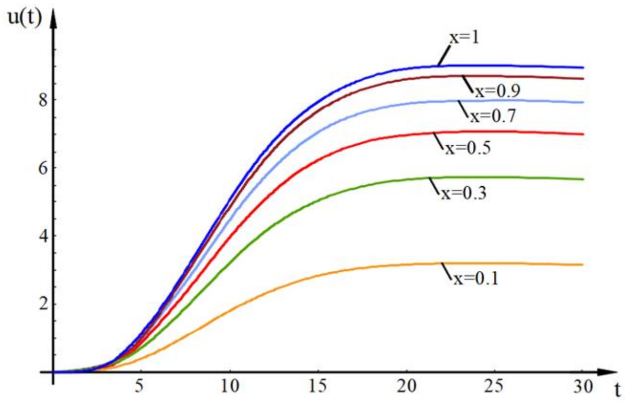

Figure 2.
Graph of axial velocity change over time.

$x \to \{0.01,0.1,0.3,0.5,0.7,0.9,1\}, \{t, 0,30\} \quad A_0 = 0, B_0 = 822863$

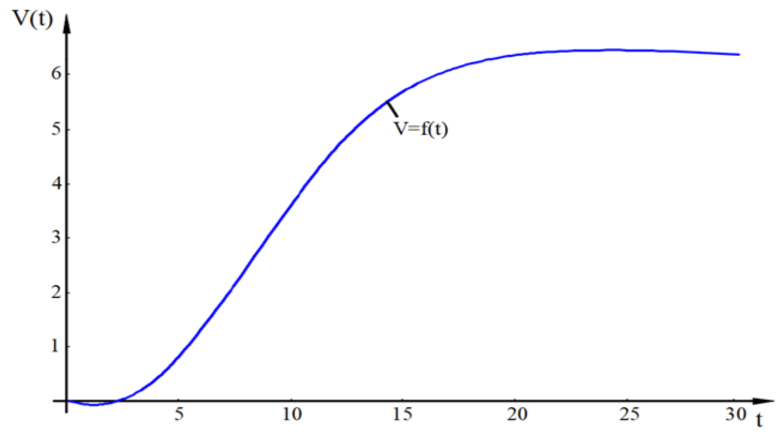

Figure 3.
Graph of average velocity change of effective cross-section over time.

The plotted graphs indicate that the stabilizing process takes place at $t \to 20$.

Graphs illustrating the change in shear stresses over the effective cross-section were also plotted based on the results collected, and they clearly show how the shear stresses changed over time.

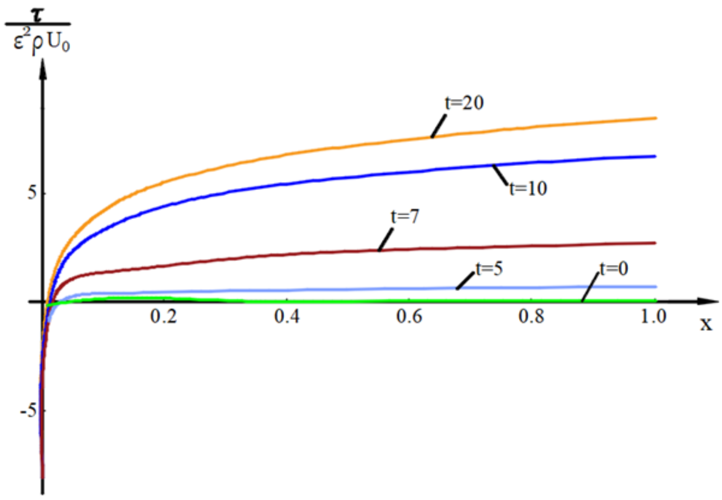

Figure 4.
Graph of change in shear stress over time $\{x, 0.01, 1\}, t \to \{0.5, 0, 1, 3, 7, 10, 15, 20\}$

Evaluating the formation of energy losses is the primary task of unsteady pressure flows. Given the significance of this problem, each c

A component of formula (1) was computed using the regularities found, and a graph showing variations in energy losses was plotted (Figure 5).

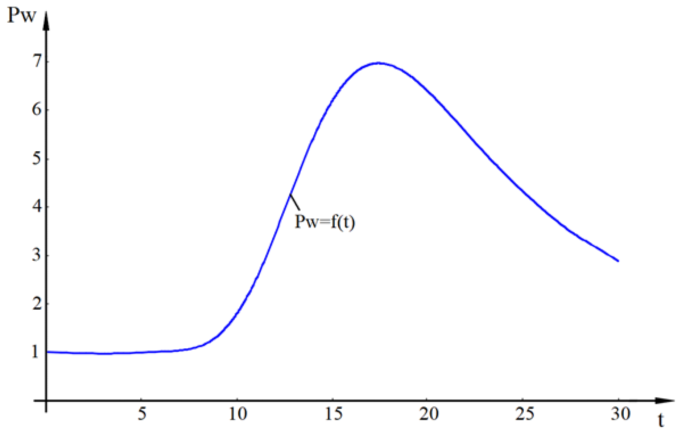

Figure 5.
Graph of energy losses over time.

The generated graph indicates that energy loss increases during the period of movement instability and then decreases after a certain amount of time. The graph then turns into a straight line, which represents flowless movement, once the process has stabilized.

6. Discussion of the Results of Studies on the Structural Change of Hydrodynamic Parameters of the Developing Turbulent Plane-Parallel Pressure Flow

Optimizing the operation of mechanical equipment is the primary goal of structural improvement; this should guarantee effective operation with low energy consumption. To achieve these goals, it is necessary to conduct studies of the hydromechanical phenomena occurring in them, the results of which will accurately describe the phenomena occurring in reality. Theoretical interest and practical applications motivate research into the phenomena of transient flow of viscous fluids. Studies on the laminar non-stationary flows of viscous fluids have been carried out in this direction. Because viscous forces in laminar flow are proportional to the first power of the velocity gradient, the differential equations of flow take on simpler forms, which explains this.

One characteristic property of a viscous fluid's turbulent non-stationary flow is that it is uncertain how the tangential stresses that form between fluid layers relate to the velocity change.

According to research in this area, there isn't a single, comprehensive strategy for dealing with this problem.

In order to account for the tangential stresses that arise between fluid layers, the findings of research carried out in the case of stationary turbulent flow were used as the foundation for analyzing non-stationary turbulent flow.

It is assumed that the results of the research conducted under these conditions of acceptance will characterize the phenomena occurring in reality with high accuracy. The calculation of tangential stresses was based on the formula proposed by Prandtl, on the basis of which a boundary value problem was formulated. As a result of integrating the proposed boundary problem, the patterns of change in axial velocities, tangential stresses, and average velocity depending on time were obtained.

The following regularities of change in axial velocities, tangential stresses, and average velocity with time were found by integrating the suggested boundary problem:

Further advancement of this study is associated with the refinement of the calculated dependence of turbulent shear stress on coordinates. However, when refining the calculated dependence, it is necessary to proceed from the assumption of the possibility of integrating the formulated

Refinement of the computed dependency of the turbulent tangential stress on coordinates is linked to further advancement of this study. However, it is required to start with the assumption that the defined boundary value issue can be integrated and that regularities of structural changes of hydrodynamic parameters can be obtained before describing the calculation dependency.

The analytical solutions obtained as a result of integrating the boundary value problem make it possible to perform computer-based experimental studies and obtain a graph of the change in energy loss.

Graphs plotted by computer-aided calculation according to Equations 38 and 39 demonstrate the development of processes of plane-parallel pressure turbulent flow. Analysis of numerical computation results and obtained graphs (Figures 2 to 5) showed that the degree of development of the process depends on time and the coordinate of the point of the effective cross-section. The shape of the speed distribution diagram is stable at each fixed section. It changes over time (Figures 2 to 5) due to the deformation of speed distribution charts. Outside the unsteady section, the speed distribution diagrams remain unchanged. The study of the development of the accelerating plane-parallel compressive flow of a viscous liquid was carried out using the Boussinesq and Prandtl hypothesis on the distribution of turbulent tangential stresses.

The integration findings will be approximations as the suggested solution is predicated on the theory of the mechanism of turbulent shear stresses in stationary turbulent flows. Nonetheless, engineering computations yield findings with a high enough degree of precision.

Further development pertains to the improvement of the turbulent tangential stress formula during non-stationary plane-parallel pressure fluctuations, depending on the problem's importance. The process stabilization time, which in this case is 20 seconds, was ascertained by the examination of the numerical computation results and the resulting graphs.

7. Conclusions

1. To investigate the patterns of change in the hydrodynamic parameters of a viscous fluid under conditions of non-stationary plane-parallel turbulent flow, a boundary value problem has been defined, which is based on the change in the dynamic coefficient of viscosity of a fluid proposed by Boussinesq [9]. It is assumed that this change depends on the square of the product of the distance from the stationary wall and the axial velocity gradient.

2. A method for solving the boundary problem has been developed, which resulted in analytical solutions for the regularities of velocity changes in unsteady flow. Graphs representing regularities of change in average cross-section velocities, tangential stresses, and energy losses have been plotted, which allow conclusions and generalizations to be drawn. The resulting graphs make it possible to track the process of changing the hydrodynamic parameters of the flow and assess the quantitative and qualitative effects of the geometric parameters of the problem on them.

References

[1] D. N. Popov, *Non-stationary hydromechanical processes*. Moscow: Machinery Engineering, 1982.

[2] L. Luoqin, S. N. Gadde, and R. J. Stevens, "Universal wind profile for conventionally neutral atmospheric boundary layers," *Physical review letters,* vol. 126, no. 10, p. 104502, 2021.

[3] B. Sun, "Closed form solution of plane-parallel turbulent flow along an unbounded plane surface," *Surface,* 2021. https://doi.org/10.20944/preprints202111.0008.v4

[4] P. Luchini, "Universality of the turbulent velocity profile," *Physical Review Letters,* vol. 118, no. 22, p. 224501, 2017. https://doi.org/10.1103/PhysRevLett.118.224501

[5] H. Schlichting and K. Gersten, *Boundary-layer theory*. Berlin: Springer-Verlag Berlin Heidelberg, 2017.

[6] B. T. Emtsev, "Technical hydrodynamics." Moscow: Machine Building Publisher, 1978, p. 463.

[7] I. S. Gromeka, "On the theory of fluid flow in narrow cylindrical pipes // M," *Publishing house of the USSR Academy of Sciences,* pp. 149 –171, 1952.

[8] L. Prandtl, "Bericht uber Untersuchungen zur ausgebildeten Turbulens," *Z. Angew. Math. Mech,* vol. 5, p. 136, 1925.

[9] J. Boussinesq, "Reports of the ac.de Sc. - T.113," pp. 49-51, 1891.

[10] A. Sarukhanyan, G. Vermishyan, and H. Kelejyan, "Research of unsteady turbulent flow in a plane-parallel pipe jaer of nuaca," *Erevan,* 2024. https://doi.org/10.54338/27382656-2024.6-006

[11] Z.-S. She, X. Chen, and F. Hussain, "Quantifying wall turbulence via a symmetry approach: A Lie group theory," *Journal of Fluid Mechanics,* vol. 827, pp. 322-356, 2017.

[12] X. Chen, F. Hussain, and Z.-S. She, "Quantifying wall turbulence via a symmetry approach. Part 2. Reynolds stresses," *Journal of Fluid Mechanics,* vol. 850, pp. 401-438, 2018.

[13] R. Baidya, J. Philip, N. Hutchins, J. Monty, and I. Marusic, "Distance-from-the-wall scaling of turbulent motions in wall-bounded flows," *Physics of Fluids,* vol. 29, no. 2, p. 020712, 2017. https://doi.org/10.1063/1.4974354

[14] S. He and M. Seddighi, "Turbulence in transient channel flow," *Journal of Fluid Mechanics,* vol. 715, pp. 60-102, 2013. https://doi.org/10.1017/jfm.2012.498

[15] K. Avila, D. Moxey, A. De Lozar, M. Avila, D. Barkley, and B. Hof, "The onset of turbulence in pipe flow," *Science,* vol. 333, no. 6039, pp. 192-196, 2011.

[16] T. I. Jozsa, "Analytical solutions of incompressible laminar channel and pipe flows driven by in-plane wall oscillations," *Physics of Fluids,* vol. 31, no. 8, 2019. https://doi.org/10.1063/1.5104356

[17] M. Kharghani and M. Pasandidehfard, "Turbulence structures in accelerated flow over a flat plate with non-zero pressure gradient," *Journal of Applied Fluid Mechanics,* vol. 15, no. 2, pp. 311-324, 2022. https://doi.org/10.47176/jafm.15.02.32337

[18] V. Chovniuk, A. Kravchuk, Moskvitina, and I. Pefteva, "Numerical simulation of unsteady flow of a viscous incompressible liquid in plane channels of arbitrary shape of heat exchangers," *Ventilation Illumination and Heat Gas Supply,* 2020. https://doi.org/10.32347/2409-2606.2020.34.41-55

[19] A. N. Tikhonov and A. G. Samarskii, "Equations of mathematical physics." Moscow: Nauka, 1999, p. 799.