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Simplified mathematical modeling of hydrodynamic forces in twin-propeller ships for enhanced maneuverability during asymmetric operations

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Abstract

This paper presents a novel modeling approach for representing the propulsion and steering system of twin-propeller, twin-rudder high-speed ships as an equivalent single-propeller, single-rudder system. The aim is to provide a practical method when hydrodynamic data for twin-propeller systems are limited, especially during maneuvering at high speeds. The method uses nominal size substitutions for the propellers and rudders, along with additional forces and moments, to replicate the behavior of the original system. It incorporates yaw moments derived from a measured experimental database and compares them with equivalent moments at various rudder angles to enhance the prediction of ship responses under asymmetric operations. The proposed method demonstrates good accuracy in testing the ship's maneuvering capabilities (turning and zigzag maneuvers) compared to both real data from a high-speed warship and traditional simulation methods. The inclusion of new moment and force components derived from experimental data improved the single P&R model when applied to twin P&R ships, reducing model error by 8.4% in the case of one propeller operating at 50% capacity, and by 12.4% in the case of one propeller failure. The approach simplifies the calculation process for twin-propeller, twin-rudder systems and remains effective even with limited hydrodynamic data resulting from asymmetric operations. This method enables efficient modeling of high-speed naval ships for training simulations and supports straightforward control calculations during emergency conditions.

Keywords: Dynamic ship simulation, hydrodynamic forces, maneuverability enhancement, predictive capability, single propeller failure, twin-propeller, twin-rudder.

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1. Introduction

1.1. Introduction

High-speed warships, which require high maneuverability (rapid changes in heading and agility), typically employ more than one propeller and rudder. In most cases, they are equipped with twin propellers and twin rudders.

Under high-speed operating conditions, hydrodynamic differences between the port and starboard sides lead to discrepancies and complexities in calculating thrust forces between the two propellers and steering forces between the two rudders, leading to precise steering difficulty. Due to hydrodynamic characteristics are not always available or fully shared, particularly in warships, for example, hydrodynamic data such as flow velocity differences across the two propellers when the ship changes course may be unavailable. This makes a challenge to determine the corrected yaw moment acting on the ship accurately.

Moreover, in emergencies requiring rapid course changes, warships often employ asymmetric thrust control by adjusting the power output of the two propellers. In cases where an engine or propeller malfunctions, reducing overall propulsion efficiency and thrust force, it requires an accurate model for computing thrust and yaw moments in order to maneuver precisely. While a twin-propeller, twin-engine model could address this issue, constructing such a model requires extensive data collection to determine the thrust force coefficients accurately.

A solution needs to be developed to simplify the calculation, simulation model, and maneuvering prediction in the control of high-speed naval ships equipped with a twin-propeller, twin-rudder system, particularly when limited hydrodynamic data is available. This applies to states requiring high maneuverability (such as rapid movement or sharp turns) or cases of propulsion system failure. This method allows the use of hydrodynamic force models of single-propeller, single-rudder ships without needing complex hydrodynamic data on flow through the propellers in twin-propeller, twin-rudder systems while still achieving accurate results.

1.2. Literature Review

Typically, studies on ship dynamic modeling and propulsion systems focus on single-propeller and single-rudder models, such as those employing regression models [1, 2] or the MMG (Manoeuvring Mathematical Modeling Group - Japanese research group) approach by Yoshimura [3]; Kose [4] and Yasukawa and Yoshimura [5]. Additionally, there have been investigations into mathematical models for twin-propeller, twin-rudder ships using complex methods or extending the MMG-based solutions developed for single-propeller, single-rudder configurations to twin-propeller, twin-rudder systems by Lee et al. [6] and Kobayashi and Ishibashi [7]. However, these approaches require detailed calculations of the hydrodynamic differences between port and starboard sides or rely on empirical data to accurately describe the thrust forces of individual propellers under conditions such as turning maneuvers or propulsion failures.

To address such scenarios, You and Kim [8] and You and Choi [9] while utilizing twin-propeller, twin-rudder models where one propeller experiences a failure, have proposed methods for determining and predicting maneuvering parameters, including propeller thrust (through propeller rotational speed) and the required rudder angle to ensure proper vessel response.

More difficultly, for twin-propeller, single-rudder ships, the interaction forces between the two propellers and the single rudder are highly complex. Using the single MMG model, Kondo et al. [10] proposed a method to simplify the mathematical representation by combining the two propellers into an equivalent single-propeller model using nominal dimensions.

In another study by Yasukawa et al. [11] to address the twin-propeller, twin-rudder configuration and analyze the complexity of calculating the forces acting on the rudders on both sides, the authors and their colleagues introduced an approach that combines the two rudders into an equivalent single-rudder model while maintaining the twin-propeller configuration. This ensures that when there is a thrust imbalance between the propellers such as during turning maneuvers or asymmetric operations a yaw moment is generated.

Further advancing this approach, Yoshimura et al. [12] proposed a single-propeller, single-rudder model by reanalyzing the hydrodynamic forces of a twin-propeller, twin-rudder ship. This simplification allows for easier maneuvering predictions while also enabling the use of the hydrodynamic database for single-propeller, single-rudder ships to be applied to twin-propeller vessels. However, this study concluded that when the ratio of ship length L to the propeller spacing is small, the yaw moment induced by propulsion force differences between the two propellers becomes negligible, even during course-changing maneuvers, where asymmetric flow conditions between the port and starboard sides lead to variations in thrust forces. Consequently, this moment was omitted from the study.

For high-speed military ships, which typically operate at high velocities and exhibit a high ship-length-to-propeller-spacing ratio, as well as requiring rapid maneuvering (where, in addition to rudder steering, the one-side engine is adjusted to reduce power for enhanced turning efficiency of the ship), it becomes essential to account for this yaw moment in the calculations.

2. Methodology: Simplification of the Mathematical Model for Thrust Forces and Moments in Twin-Propeller, Twin-Rudder Ships

To simplify the mathematical models of twin-propeller, twin-rudder systems for practical application in predicting the maneuvering performance of high-speed naval vessels in special scenarios, mathematical transformations are applied using the single-propeller, single-rudder ship dynamics model for the fundamental equations.

2.1. Fundamental Ship Dynamics Model

The 3DOF mathematical model of a single-propeller, single-rudder ship is based on a set of nonlinear differential equations describing the vessel's motion under the influence of longitudinal force (X), lateral force (Y), and yaw moment (M) in a ship-fixed coordinate system:

$$m(\dot{u} - vr - x_g r^2) = X_H + X_P + X_R \tag{1}$$

$$m(\dot{v} + ur + x_g \dot{r}) = Y_H + Y_R \tag{2}$$

$$I_{zz,eff}\dot{r} + m x_g(\dot{v} + ru) = M_H + M_R \tag{3}$$

where: m - the mass of the ship; $I_{zz,eff} = I_{zz} + J_{zz}$- the effective moment of inertia of the ship; J_{zz} - the additional moment of inertia due to hydrodynamic effects; I_{zz} - the moment of inertia of the ship about the yaw axis (z-axis); u, v - the longitudinal and lateral velocities, respectively; $\dot{u}$, $\dot{v}$ - the time derivatives of the longitudinal and lateral velocities; r - the angular velocity around the ship-fixed coordinate system; $\dot{r}$ - the time derivative of the angular velocity.

Equation 3 can be transformed as:

$$\begin{aligned} I_{zz,eff}\dot{r} &= M_H + M_R - m x_g(\dot{v} + ru) \\ &= M_H + M_R - x_g(Y_H + Y_R) \end{aligned} \tag{4}$$

where x_g represents the coordinate of the ship's center of gravity. When $x_g = 0$ (i.e., the center of gravity coincides with the origin of the coordinate system), there is no need to account for the additional moment caused by the inertial force due to the offset between the center of gravity and the origin.

The forces and yaw moments acting on the hull, rudder, and propeller in the single-propeller, single-rudder system are analyzed and calculated as follows:

$$X_H = C_{xH}(\beta, \bar{\omega})0.5\rho U^2 Ld \tag{5}$$

$$Y_H = C_{yH}(\beta, \bar{\omega})0.5\rho U^2 Ld \tag{6}$$

$$M_H = C_{mzH}(\beta, \bar{\omega})0.5\rho U^2 L^2 d \tag{7}$$

$$X_R = [C_{xR}(\beta, \bar{\omega}, \delta, C_T) + C_{xIN}(\beta, \bar{\omega}, \delta)]0.5\rho U^2 Ld\bar{A}_R \tag{8}$$

$$Y_R = [C_{yR}(\beta, \bar{\omega}, \delta, C_T) + C_{yIN}(\beta, \bar{\omega}, \delta)]0.5\rho U^2 Ld\bar{A}_R \tag{9}$$

$$M_R = [C_{yR}(\beta, \bar{\omega}, \delta)I_R + C_{mzIN}(\beta, \bar{\omega}, \delta)]0.5\rho U^2 L^2 d\bar{A}_R \tag{10}$$

$$X_P = \rho n^2 D_P^4(1 - t)K_T(J, P_P/D_P) \tag{11}$$

where: β - drift angle; $\overline{\omega}$ - relative path curvature; δ - rudder deflection angle; C_{xH}, C_{yH}, C_{mzH} - non-dimensional bare hull aerodynamic forces in the longitudinal and lateral directions, and the non-dimensional yaw moment on bare hull, respectively; C_T - non-dimensional propeller thrust; $\overline{A_R}$- relative rudder area, defined as $\bar{A}_R = A_R/LT$, where A_R is the rudder area, L is the length on the waterline, and d is the draught at midship; C_{xR}, C_{yR} - non-dimensional rudder force in the longitudinal and lateral direction, respectively; $C_{xIN}, C_{yIN}, C_{mzIN}$ - non-dimensional hull-propeller interaction forces in the longitudinal and lateral direction, and non-dimensional hull-propeller interaction moment, respectively; n - propeller rotational frequency; t - thrust deduction fraction (accounting for the effects of hull shape and appendages); P_P/D_P - propeller pitch ratio; J - advance coefficient of the propeller; K_T - thrust coefficient.

The coefficients $C_{xH}, C_{yH}, C_{mzH}, C_{xR}, C_{xIN}, C_{yR}, C_{yIN}, C_{mzIN}$ are obtained through statistical analysis and regression analysis, as presented in studies [1, 2].

This mathematical model is designed for a single-propeller, single-rudder system but can be extended to a twin-propeller, twin-rudder configuration by incorporating additional moments generated by the new force pairs, including the propeller force pair and the rudder force pair, as well as accounting for the differences in hydrodynamic coefficients (of ship-hull, rudders and propellers) between port and starboard (especially during the ship's turning process). However, determining the hydrodynamic parameters between the two sides becomes complex when asymmetric operations occur between the two sides, or even in the turning process. To address this problem, typically, regression methods can be used for the twin P&R model based on measured data, or the hydrodynamic characteristics can be calculated based on CFD (Computer Fluid Dynamics) models. Similar to the study by the CFD method based on smoothed particle hydrodynamics (SPH) is used to simulate the forces and moments acting on the scaled-down ship model, from which Fourier series analysis is conducted to extract hydrodynamic coefficients, helping to simulate maneuvering trajectories such as turning circle and zigzag. Separately computing two single-propeller models and then combining them may provide an approximate solution, but it introduces inaccuracies compared to real-world conditions. To minimize these errors, the model is simplified, and additional data from empirical databases are incorporated. At this stage, Equation 4 must be updated, assuming the center of gravity is located at the origin.

$$I_{zz,eff}\dot{r} = M_H + M_R + N_P \tag{12}$$

here, N_P represents the moment induced by the thrust force asymmetry between the propellers; and the new moment ΔY_R generated by the rudders is updated through Y_R the forces acting on the rudder will be presented in detail in the following sections.

2.2. Propeller Thrust in the Simplified Model

As is well known, the thrust force generated by a propeller can be expressed as:

$$T = \rho n^2 D^4 K_T \tag{13}$$

where, D is the propeller diameter; n is propeller rotation frequency; ρ fluid density $(kG\ /m^3)$, K_T is thrust coefficient, which can be obtained experimentally from a geometrically similar model propeller and is a function of the advance coefficient: $J = \frac{V_a}{nD} = \frac{U(1-W_p)}{nD}$, V_a - is the propeller advance velocity; U - axial velocity, W_p - wake scaling factor.

For twin-propeller ships, when both propellers are identical and operate under the same conditions, according to Yoshimura, et al. [12] we have:

$$\begin{aligned} T &= T_{(s)} + T_{(p)} \\ &= \rho n^2 D^4 K_{T(s)} + \rho n^2 D^4 K_{T(p)} \\ &= \rho\left(K_{T(s)} + K_{T(p)}\right) n^2 D^4 \\ &= \rho\left(\frac{K_{T(s)} + K_{T(p)}}{2}\right)\left(\frac{n}{\sqrt{2}}\right)^2\left(\sqrt{2}D\right)^4 \\ &= \rho \widetilde{K}_T n^{*2} D^{*4} \end{aligned} \tag{14}$$

where, nominal equivalent dimensions for equivalent single-propeller model to obtain the same results as a twin-propeller system, are defined:

$$n^* = \frac{n}{\sqrt{2}}, D^* = \sqrt{2}D \tag{15}$$

where $\widetilde{K}_T$ is the thrust coefficient of the equivalent single-propeller model, which can be determined from the thrust coefficient function (or graph) based on

$$J^* = \frac{V_p\left(1 - W_p\right)}{n^* D^*} = \frac{V_p\left(1 - W_p\right)}{\frac{n}{\sqrt{2}}\sqrt{2}D} = \frac{V_p\left(1 - W_p\right)}{nD} = J \tag{16}$$

Thus, the new advance coefficient for the equivalent single-propeller remains the same as the twin-propeller system. This allows the thrust coefficient $\widetilde{K}_T$ could be retrieved using the same J, and ensuring that $\widetilde{K}_T$ remains unchanged compared to the original $K_{T(s)}, K_{T(p)}$. However, this assumption holds only when both propellers are physically identical and operate under identical conditions, i.e., with the same rotational speed. In cases where the propellers operate at different power levels (different n) or suffer blade damage (different D), the actual thrust of the twin-propeller system is given by, where the port-side propeller is assumed to operate asymmetrically with the starboard propeller $\left(n_{(p)} < n_{(s)}\right)$, or has blade damage $\left(D_{(p)} < D_{(s)}\right)$:

$$\begin{aligned} T &= T_{(s)} + T_{(p)} \\ &= \rho n_{(s)}^2 D_{(s)}^4 K_{T(s)} + \rho n_{(p)}^2 D_{(p)}^4 K_{T(p)} \end{aligned} \tag{17}$$

In such cases, it becomes challenging to determine the exact value of $D_{(p)}$ after damage or to compute the thrust coefficients K_T , especially when hydrodynamic asymmetry exists between the port and starboard sides, particularly during turning maneuvers. To address this, we proceed with the following transformation:

$$\begin{aligned} T &= \rho n_{(s)}^2 D_{(s)}^4 K_{T(s)} + \rho n_{(s)}^2 D_{(s)}^4 K_{T(s)} - \left(-\rho n_{(p)}^2 D_{(p)}^4 K_{T(p)} + \rho n_{(s)}^2 D_{(s)}^4 K_{T(s)}\right) \\ &= \rho n^{*2}_{(s)} D^{*4}_{(s)} \widetilde{K}_{T(s)} - \Delta T \\ &= T^* - \Delta T \end{aligned} \tag{18}$$

where, ${n^*}_{(s)} = \frac{n_{(s)}}{\sqrt{2}} = \frac{n}{\sqrt{2}}, D^*_{(s)} = \sqrt{2}D_{(s)} = \sqrt{2}D, \widetilde{K}_{T(s)} = K_{T(s)} = K_T$ with n, D, K_T in the normal operation of power performance, physical dimensions, and thrust coefficient.

Here, the thrust difference between the two propellers is given by:

$$\begin{aligned} \Delta T &= \rho n^2_{(p)} D^4_{(p)} K_{T(p)} - \rho n^2_{(s)} D^4_{(s)} K_{T(s)} \\ &= T_{(s)} - T_{(p)} \end{aligned} \tag{19}$$

which can be determined through direct measurements or actively controlled thrust differentials during actual operation. Meanwhile, $T^* = \rho {n^*}^2_{(s)} {D^*}^4_{(s)} \widetilde{K}_{T(s)}$ represents the nominal thrust force for the equivalent single-propeller, single-rudder (Single P & R) model.

2.3. Forces Acting on the Rudders in the Simplified Model

The forces acting on each rudder, in the case where both propellers generate equal thrust, were modeled in Yoshimura et al. [12] as an equivalent single rudder with the following parameters: equivalent rudder area $A^*_R = 2A_R$, equivalent rudder height $h^* = \sqrt{2}h$. This transformation maintains the ratio between the equivalent propeller diameter and rudder height D^*/h^*, which is beneficial when considering the inflow conditions at the rudder downstream of the propeller.

However, when there is an asymmetry in the forces acting on each rudder due to asymmetric propeller operation, the total lateral force acting on the equivalent rudder becomes:

$$Y_R = Y^*_R - \Delta Y_R \tag{20}$$

where ΔY_R represents the difference in lateral force acting on the rudders due to thrust asymmetry. Assume that, the ratio between the nominal propeller thrust and rudder forces are the same with the ratio of the differences from asymmetric propeller thrust and rudder forces:

$$\frac{\Delta Y_R}{\Delta T} \approx \frac{Y^*_R}{T^*} \tag{21}$$

2.4. Yaw Moment Due to Asymmetric Propulsion Operation

The thrust asymmetry (unbalance ΔT) in the propulsion system of high-speed naval ships generates a yaw moment:

$$N_p = y_p(1 - t)\Delta T \tag{22}$$

where, t is the thrust deduction fraction, y_p is the lateral distance between the propeller and the longitudinal axis passing through the center of rotation.

Thus, the yaw moment induced by the thrust imbalance is a first-order function of the thrust difference. To facilitate lookup and analysis, a graph (Figure 1) can be constructed to illustrate the relationship between thrust difference and the non-dimensional yaw moment. Utilizing an experimental database from scaled ship models of geometrically similar vessels equipped with the same type of propellers, a correlation chart can be constructed to illustrate the relationship between the yaw moment induced by unbalanced propeller thrusts and the corresponding rudder deflection angles with this yaw moment value in normal operation (balanced thrust).

By normalizing these values based on the dimensions of the experimental ship models, the corresponding non-dimensional yaw moment is computed as follows:

$$N'_p = \frac{N_p}{\frac{\rho}{2} L^2 d U^2} \tag{23}$$

For the warship object under study, Table 1, N'_p is obtained from Figure 1 based on either measured thrust differences or actively controlled thrust variations in practical operations, then N_p is calculated using the ship's physical size. At low total operational power (50%), when the ship's speed is not high, even significant thrust imbalances result in a small yaw moment value which is the same as the moment value induced by a relatively small rudder deflection angle at normal operation (dashed-dot line -.-), typically between 1-4 degrees, which is consistent with findings in Yoshimura et al. [12]. However, at full operational power, when the ship is at maximum speed and suddenly experiences a loss or reduction of thrust in one propeller, the resulting yaw moment value becomes huge, this is the same order as a bigger rudder deflection angle at balanced thrust operation (dashed line ---), ranging from 2-12 degrees.

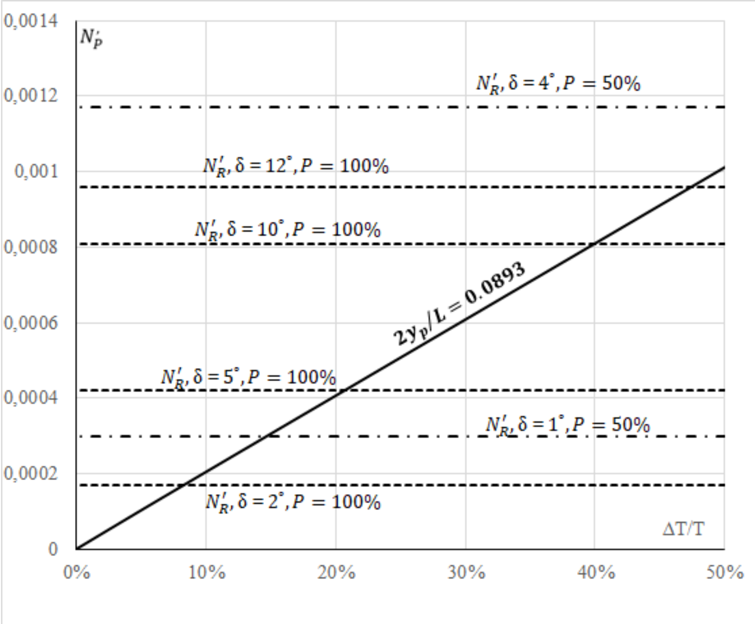

Figure 1.
Comparison of yaw moment induced by rudder and the difference of propeller thrusts.

Finally, the force imbalance on the rudder generates an additional yaw moment ΔN, which is already accounted for in the lateral force Y_R of the equivalent single-propeller model, as presented in the previous section.

3. Implementation, Results, and Discussion: Maneuvering Simulation for High Speed Warship by the Simplified Model

The subject of this study is a twin-propeller, twin-rudder warship. Table 1 presents the key parameters of the ship's hull, propulsion system, and rudder configuration.

Table 1.
Parameters for intercept firing problems.

Key Parameters	**DI**
L (length of ship Lpp m)	56.1
B (breadth of ship m)	10.2
d (mean draught m)	2.54
L/B	5.5
C_b (block coefficient)	0.367
$2y_p/L$ (distance between propeller/L)	0.0893
D_p (diameter of propeller mm)	1.8 (x2 pcs)
LCG from midship	20.41
A_R (rudder area m^2)	5.1(x2 pcs)
Aspect ratio	1.12

3.1. Validation of the Single-Propeller, Single-Rudder Model

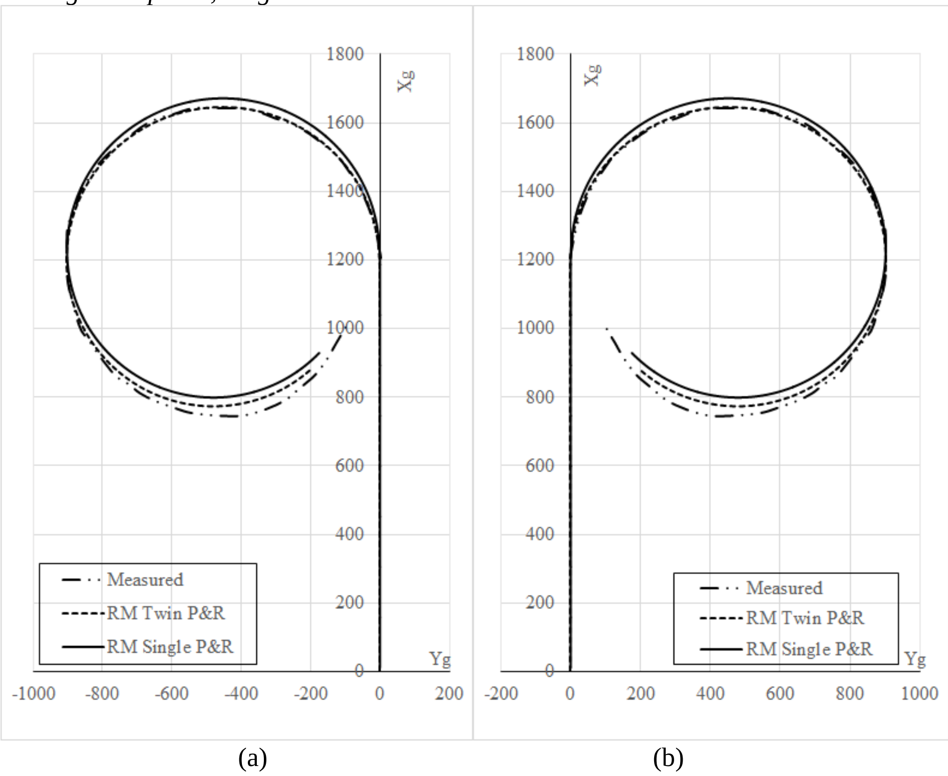

Figure 2.
Comparison of the ship's turning trajectory at 35 degrees and 43 knots (a) Turn left (b) Turn right.

First, the proposed equivalent single-propeller, single-rudder model (RM Single P&R) applied to the naval vessel simulator (developed by Viettel High Technology Industries Corporation) is validated by conducting maneuvering simulations and comparing the results with those obtained from the original twin-propeller, twin-rudder model (Regression analysis approach for hydrodynamic force and moment coefficients - RM Twin P&R) as well as the measured data extracted from the considered high-speed warship.

Figure 2 compares the 35-degree turning trajectory (at a maximum speed of 43 knots) of the naval vessel simulator obtained from the aforementioned models. The turning trajectory represents the vessel's heading control capability during fixed rudder angle tests (both rudder). The comparison results indicate that the equivalent single-propeller, single-rudder model agrees with the original twin-propeller, twin-rudder model, and closely matches the measured data extracted from the considered high-speed warship. However, in this experiment, the proposed simulation results are not as accurate as the complex twin P&R model, due to this case being in symmetric operation, the additional force and moment have not been applied yet (the adding only in asymmetric operation). Besides that, using an equivalent single P&R in symmetric operation is like a combination of two single models independently (no interaction), there is no consideration of the hydrodynamic parameters difference between the two sides (especially in the turning process) as the complex twin P&R model.

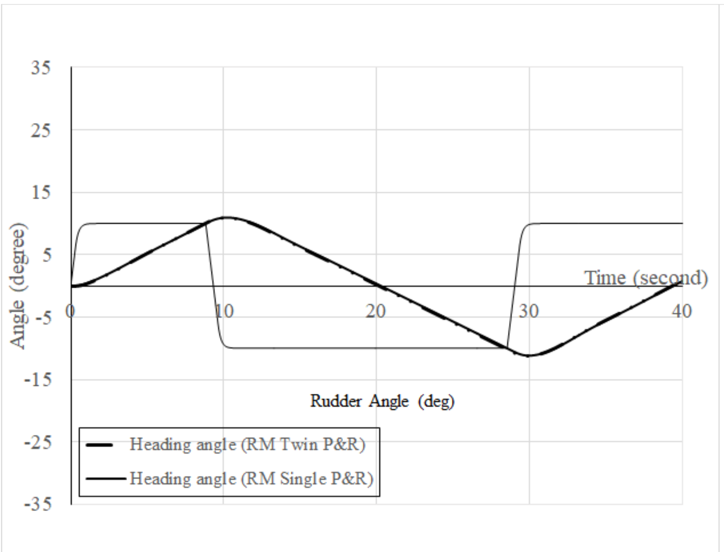

Figure 3.
Comparison of the ship’s trajectory in the 10°/10° zigzag test for the two computational models.

Figures 3 and 4 compare the simulation results obtained from the equivalent model and the original model through the 10°/10° and 20°/20° zigzag tests. The zigzag test is a standard maneuver used to evaluate the response and stability of a ship's control system when the rudder angle changes suddenly.

The comparison results demonstrate that maneuvering characteristics, such as rudder angle variations and ship heading response over time, are accurately simulated by the equivalent single-propeller, single-rudder model and are consistent with the original twin-propeller, twin-rudder model.

Based on the above comparisons, it can be concluded that the equivalent single-propeller, single-rudder model maintains the necessary accuracy in simulating the maneuvering behavior of naval ships while simultaneously reducing the complexity of the original model when calculating the forces and moments of both propellers and rudders.

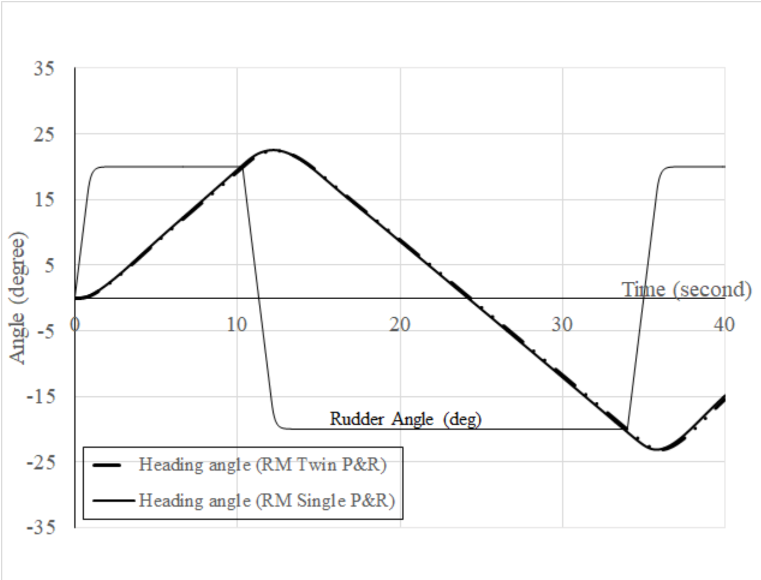

Figure 4.
Comparison of the ship's trajectory in the 20°/20° zigzag test for the two computational models.

For twin-propeller naval vessels, which are inherently faster and more maneuverable than other ship types, the thrust imbalance between the port and starboard propellers generates a yaw moment that significantly affects the vessel's maneuverability and course stability. This study employs the regression model-based equivalent single-propeller, single-rudder approach for naval vessels while evaluating and adding the yaw moment induced by asymmetric thrust between the two propellers under various operating conditions, including emergency turning maneuvers (partial thrust reduction on one propeller) and total failure of one propeller.

3.2. Emergency Turning Maneuver

An emergency turn is a rapid maneuvering technique used to avoid collisions or threats (such as missiles or torpedoes) or to respond to other emergencies at sea. One of the emergency turning methods for naval ships is performed as follows:

Step 1: Turn the rudder to its maximum deflection in one direction.

Step 2: Reduce the throttle on the same side as the rudder deflection.

This method allows the naval ships to turn quickly with minimal displacement distance.

Figure 5 illustrates the turning trajectories of the naval ship during emergency right and left turns using the computational models. When the vessel is traveling at a maximum speed of 43 knots, it executes a maximum rudder deflection of 35 degrees to the right (or -35 degrees to the left), followed by a 50% throttle reduction on the right side (or a 50% throttle reduction on the left side). In the figure: The solid line represents the results from the Regression Model (RM)-based equivalent single-propeller, single-rudder model, which accounts for the yaw moment induced by the thrust imbalance between the two propellers; the dashed line represents the results from the Regression Model-based single-propeller, single-rudder model, which neglects the yaw moment due to thrust asymmetry; the remaining line represents the results from the original Regression Model-based twin-propeller, twin-rudder model.

Table 2.
Turning Circle Diameters (m) for Emergency Left and Right Turns.

	Left	**Right**
RM Twin P&R	-788	787.7
RM Single P&R	-784.5	782.7
RM Single P&R without unbalance moment	-850.7	850.3

The results indicate that the right-turn (left-turn) trajectory of the equivalent single-propeller, single-rudder model closely matches the turning trajectory of the original twin-propeller, twin-rudder model. In contrast, the trajectory of the RM-based single-propeller, single-rudder model, which ignores the yaw moment (by unbalance), deviates from the other two models.

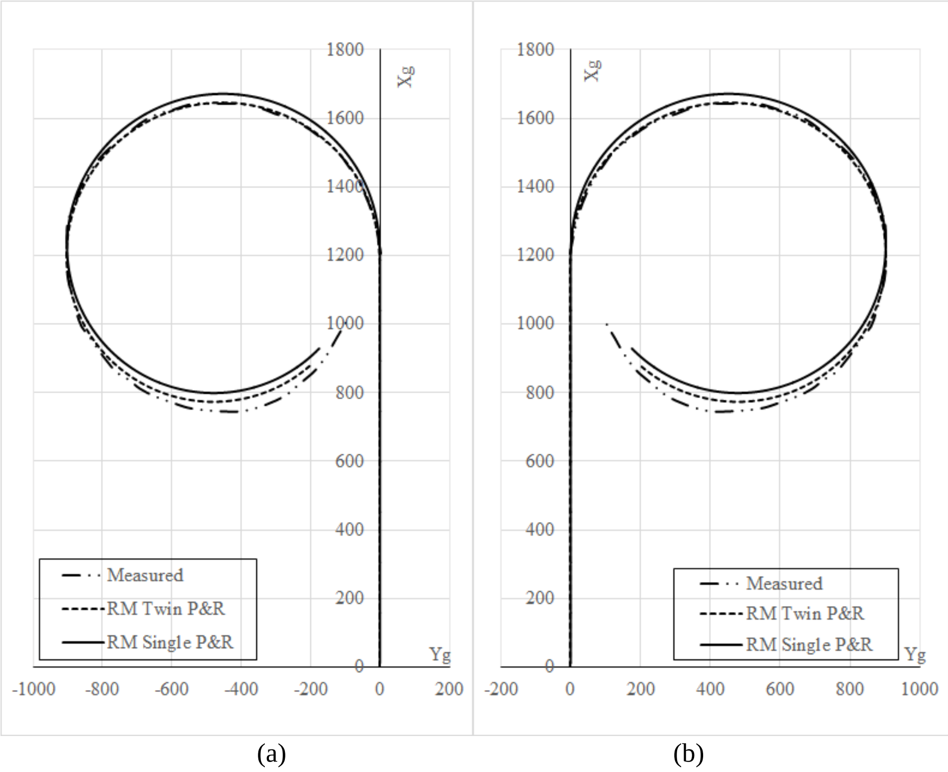

Figure 5.
Emergency left and right turning trajectories from computational models (a) Emergency left turning trajectory (b) Emergency right turning trajectory.

Table 2 summarizes the turning circle diameters for emergency right and left turns of the naval ship under the models. The turning diameter error of the RM-based single P&R model, when neglecting the yaw moment induced by thrust asymmetry, is approximately 8.4% compared to the other models.

3.3. Case of a Single Propeller Failure

When the right-side propeller fails, the left-turn trajectory becomes wider. Conversely, when the left-side propeller fails, the right-turn trajectory expands. This occurs because the asymmetric thrust from the operational propeller generates a yaw moment toward the failed propeller. As a result, the tactical diameter of the turning circle in the direction of the failed propeller is smaller compared to the opposite direction.

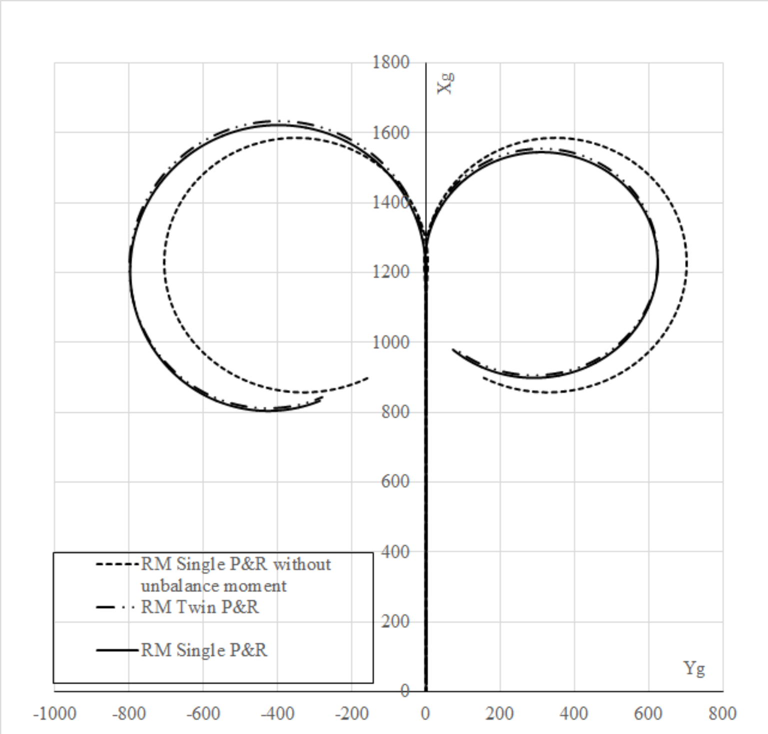

Figure 6.
Turning Trajectory of the ship with Right propeller failure based on computational models.

Figures 6 and 7 illustrate the turning trajectories of the ship in two scenarios: right propeller failure and left propeller failure (where the propeller fails while the ship is performing a turn at a maximum rudder deflection angle of ±35 degrees and a speed of 43 knots). Since the equivalent single-propeller, single-rudder model and the original twin-propeller, twin-rudder model both account for the yaw moment induced by the unbalanced thrust between the two propellers, the results indicate that the left-turn and right-turn trajectories from these two models align well with theoretical expectations and are nearly identical. In contrast, for the Regression Model (RM)-based equivalent single-propeller, single-rudder model, which neglects the yaw moment (due to thrust imbalance), the left-turn and right-turn trajectories are almost unaffected. Consequently, the turning trajectories in this case deviate significantly from those of the twin P&R model.

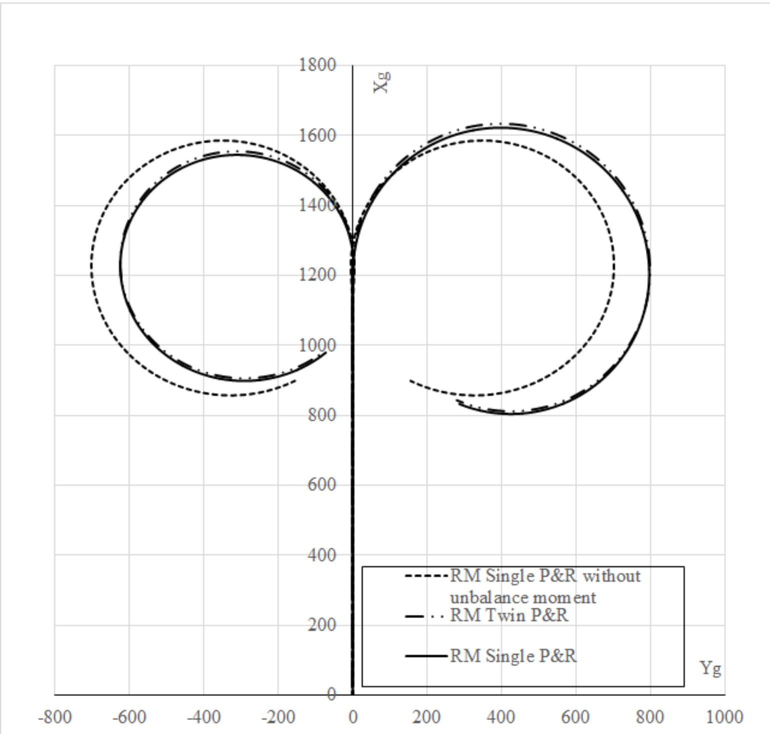

Figure 7.
Turning Trajectory of the ship with Left propeller failure based on computational models.

Tables 3 and 4 summarize the turning diameters for left and right turns in cases of right propeller failure and left propeller failure. The turning diameter obtained using the RM-based equivalent single-propeller, single-rudder model without unbalanced yaw moment deviates by approximately 12.4% compared to the other two models.

Table 3.
Turning Diameter (m) of the Ship with Right Propeller Failure.

	Left	Right
RM Twin P&R	-799	624
RM Single P&R	-798.8	625
RM Single P&R without unbalanced moment	-700.2	701.8

Table 4.
Turning Diameter (m) of the Ship with Left Propeller Failure.

	Left	Right
RM Twin P&R	-624.2	799.2
RM Single P&R	-625.5	798.9
RM Single P&R without unbalanced moment	-702.6	700.6

4. Conclusion

This study presents a simplified mathematical approach for modeling the dynamics of high-speed twin-propeller, twin-rudder ships by transforming them into an equivalent single-propeller, single-rudder mathematical model that utilizes the hydrodynamic database of single-propeller ships. This approach not only simplifies computational and simulation processes while preserving all essential force and moment components acting on the hull - such as total thrust, rudder-induced yaw moment, and yaw moment due to propeller thrust asymmetry - but also enhances the efficiency of maneuvering prediction, particularly in emergency operations and asymmetric propulsion conditions. The proposed method demonstrates accuracy in tests of the ship's maneuvering capabilities (turning and zigzag test) when compared to data collected from the real high-speed warship and traditional simulation techniques. This approach refines the single-propeller, single-rudder model when applied to twin-propeller, twin-rudder ships, reducing the 8.4% and 12.4% model error compared to the original single P&R model in scenarios where one propeller operates at 50% and 0% capacity respectively. However, a limitation of the proposed method is that when operating in a symmetric mode and turning, it often gives bigger different results than the complex traditional simulation method compared to measured data, but when simulating in the asymmetric operation, it brings the results closer to the accuracy of complex traditional simulation methods due to the incorporation of experimental data.

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