International Journal of Innovative Research and Scientific Studies, 8(4) 2025, pages: 110-122

IJIRSS
International Journal of Innovative Research & Scientific Studies

ISSN: 2617-6548

URL: www.ijirss.com

Enhancing public service agility and value co-creation through fuzzy logic integration algorithms

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Abstract

This study investigates the limitations of traditional evaluation methods, particularly Likert-type scales, in capturing complex, subjective constructs such as organizational agility and value co-creation within public service delivery. It proposes an alternative framework grounded in fuzzy logic theory to better reflect the nuanced perceptions of stakeholders. A qualitative research design was employed, using semi-structured interviews with public sector agents in Moroccan territorial administrations. Respondents evaluated service attributes using a self-anchored "free scale" and linguistic descriptors. These evaluations were transformed into triangular fuzzy numbers (TFNs) and analyzed using fuzzy similarity indices and scale invariance testing (D-test). The fuzzy logic-based method revealed significant perceptual differences not captured by the standard 5-point Likert scale. Semantic differential scales showed stronger alignment with the free scale. Findings replicated across samples confirmed the increased representational sensitivity and reduced distortion in perceptual data using fuzzy logic. Fuzzy logic offers a more flexible and theoretically robust alternative to traditional fixed-point scales in assessing subjective dimensions of public service, enhancing accuracy in measuring perceived service quality and co-creation. The study emphasizes the importance of measurement choice in public administration research. Applying fuzzy logic can yield richer, more actionable insights for policy design, performance evaluation, and citizen engagement strategies.

Keywords: Agility, Citizen behavior, Fuzzy logic, Public Service, Service managers, Value Co-Creation.

DOI: 10.53894/ijirss.v8i3.7743
Funding: This study received no specific financial support.
History: Received: 16 April 2025 / **Revised:** 20 May 2025 / **Accepted:** 22 May 2025 / **Published:** 11 June 2025

Competing Interests: The authors declare that they have no competing interests.
Authors' Contributions: All authors contributed equally to the conception and design of the study. All authors have read and agreed to the published version of the manuscript.
Transparency: The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.
Publisher: Innovative Research Publishing

1. Introduction

Effective management of local public services is essential for community well-being, but remains a complex challenge requiring advanced evaluation tools [1]. Municipalities are key environments for experimenting with practices to enhance service quality and public value [2]. In addition to strategic challenges in TQM implementation, public service evaluation faces specific methodological difficulties, notably in assessing perceived service quality, understanding value co-creation among diverse stakeholders [3] and fostering organizational agility to adapt to environmental shifts [4]. Likert scales [5] are commonly used to measure perceptions in the public sector, yet concerns persist about their ability to capture the nuanced and subjective nature of human judgment [6, 7]. Critics argue that such fixed-point tools inadequately reflect cognitive and affective complexity [8, 9], potentially introducing biases and limiting sensitivity to subtle perception gradients. This study addresses these limitations by introducing a fuzzy logic-based evaluation framework. Developed by Zadeh [10] fuzzy logic models ambiguity and subjective assessment by allowing partial membership in categories (e.g., 'moderately agile') via values between 0 and 1 [11, 12]. This approach facilitates approximate reasoning aligned with human judgment under uncertainty [13, 14]. Fuzzy logic has shown effectiveness in fields such as control systems and decision support [15, 16] and is increasingly relevant for evaluating complex constructs in public service management [17, 18]. This study operationalizes fuzzy membership functions using data from a "free scale" tool, which is less constrained than fixed-point formats, to better capture perceptions of agility and co-creation. We hypothesize that this approach reveals nuanced variations in perception that are typically masked by traditional methods. A mixed-methods design, combining fuzzy-scaled quantitative data with qualitative insights, ensures triangulation and enhances validity [19-21]. Ultimately, the study contributes to public administration scholarship by offering a more sensitive framework for evaluating stakeholder perceptions and service performance. It aims to inform evidence-based decision-making and foster innovation in local governance by demonstrating how fuzzy logic can enrich the understanding of service quality and co-creation dynamics [15].

2. Literature Review

The extant scholarly literature pertinent to the evaluation of local public services is synthesized herein, with a specific focus directed towards elucidating the identified limitations inherent within traditional measurement methodologies and exploring the theoretical potential afforded by fuzzy logic systems for the assessment of complex, subjective constructs such as organizational agility and value co-creation. This review endeavors to establish the necessary theoretical and conceptual context underpinning the proposed employment of fuzzy logic within the analytical domains of territorial governance and contemporary public service management.

2.1. Limitations of Traditional Evaluation Methods in Public Service Assessment

Assessing public service quality requires capturing nuanced subjective perceptions related to delivery, value, and organizational responsiveness. While Likert-type scales [5] and Thurstone-derived methods (Thurstone, 1928) are widely used; however, their effectiveness in public service contexts has been questioned. Scholars point to issues of reliability and validity, emphasizing their limited sensitivity to subtle perceptual differences and the full spectrum of human experience [6, 7, 22]. These tools often rely on psychometric assumptions, such as dominance models, that may not reflect the cognitive processes behind evaluative judgments in complex service scenarios [23]. Discrete categories can induce categorization bias and fail to represent gradual perceptual transitions [24]. Moreover, standardized scales may overlook contextual and individual factors shaping service perceptions [25, 26]. Cognitive biases [27, 28] and emotional responses [8, 9, 29] further complicate measurement, challenging the assumption of purely rational evaluations. This has led to interest in alternative methods, such as semantic differential scales [24, 30] and personalized, first-person evaluation tools that allow respondents to define their own scaling criteria [31-33].

2.2. Fuzzy Logic: Theoretical Foundations and Relevance

Fuzzy logic, as introduced in the seminal work of Zadeh [10], provides a formal mathematical framework explicitly developed to address the inherent vagueness, ambiguity, and imprecision that characterize many aspects of human reasoning, linguistic expression, and complex real-world systems [13, 34]. Representing a significant departure from the strict binary true/false distinctions mandated by classical Aristotelian logic and crisp set theory, fuzzy logic is fundamentally predicated upon fuzzy set theory. Within this paradigm, elements possess degrees of membership in a given set, typically represented by a continuous value within the interval [0, 1], rather than an absolute, dichotomous inclusion or exclusion [11, 12]. This foundational principle permits the formal representation and computational manipulation of linguistic variable terms drawn from natural language that represent imprecise concepts (e.g., 'high agility,' 'moderate co-creation,' 'somewhat satisfied') [14]. Furthermore, it enables the application of approximate reasoning methodologies, utilizing fuzzy rules (typically expressed as IF-THEN conditional statements) and associated inference mechanisms to derive conclusions from imprecise premises [35, 36]. Membership functions (which may assume various shapes, such as triangular, trapezoidal, or Gaussian) serve to mathematically define the specific degree of membership for any given input value within a fuzzy set, thereby enabling computations that reflect gradual transitions and overlapping categories rather than imposing sharp, artificial boundaries [11]. Its conceptualization provided indispensable tools for modeling complex systems wherein ambiguity and subjectivity are prevalent features rather than aberrations [37].

2.3. Applications of Fuzzy Logic in Management and Public Services

The established capacity of fuzzy logic systems to effectively manage uncertainty and informational imprecision has precipitated their application across a diverse array of scientific and engineering domains, including artificial intelligence, advanced control systems, pattern recognition, and sophisticated data analysis [36, 38-41]. Of particular relevance to the present investigation, fuzzy logic has demonstrated considerable utility within the development of decision support systems and the implementation of performance management frameworks, especially in contexts characterized by high complexity, dynamic conditions, and informational uncertainty [15, 16, 42]. Within the specific domains of public service management and territorial governance, fuzzy logic offers a pertinent methodological response to the persistent challenges posed by system complexity and the frequent necessity of integrating imprecise qualitative information alongside quantitative data. Illustrative applications documented in the literature include its utilization for modeling and measuring the performance of local government service quality delivery [17] facilitating public decision-making processes related to complex infrastructure planning initiatives [18] assessing service quality dimensions while explicitly accounting for vague human judgments [43] and managing environmental systems characterized by both cognitive and statistical uncertainties [44]. Extant studies provide empirical evidence supporting its utility in enhancing the robustness and nuance of decision-making processes within geographically defined territorial contexts, primarily through its capacity to formally accommodate the inherent complexities and uncertainties involved [45-48]. Consequently, its application appears well-aligned with the operational exigencies of effective territorial governance, which frequently involves the coordination of multiple administrative tiers and diverse local actors operating under conditions of significant ambiguity [49, 50].

2.4. Value Co-Creation and Organizational Agility in Public Services

This study focuses on two key constructs in modern public service management: value co-creation and organizational agility. Value co-creation shifts away from provider-centric models, emphasizing collaborative value generation through the active participation of stakeholders, especially citizens and service providers [1, 3, 51]. It involves complex behavioral, cognitive, and emotional processes aimed at enhancing service quality, democratic governance, and citizen satisfaction [4, 52]. However, such processes also risk "value co-destruction" if poorly managed [1, 53]. Organizational agility refers to the public sector's capacity to sense and respond quickly to changes in its environment [54]. This includes adapting services, processes, and strategies in response to citizen needs, policy changes, societal challenges, or technological disruptions. The link between these constructs lies in the way co-creation can generate real-time feedback, enhancing the organization's responsiveness and adaptability [53]. Given their complexity and subjective nature, assessing these constructs requires methods capable of handling ambiguity making fuzzy logic a particularly suitable approach.

2.5. Synthesis and Research Gap

In summary, the literature highlights the limitations of traditional psychometric scales in capturing the subjective complexity of public service evaluations, particularly for constructs like value co-creation and organizational agility. Fuzzy logic, with its strong theoretical foundation for handling vagueness and proven utility in complex governance contexts, offers a promising methodological alternative. Although fuzzy logic has been applied in public service research, its use as a primary evaluation tool based on respondent-defined scales for assessing both agility and co-creation within territorial communities remains underexplored. This study addresses this gap by developing and testing a fuzzy logic-based framework tailored to these constructs. Methodological rigor is ensured through triangulation, combining qualitative insights with fuzzy quantitative analysis, in line with best practices for validity in complex social research [19-21, 55]. The study also notes future prospects for integrating AI techniques to enhance analysis [56, 57]. Ultimately, this approach aims to provide a more nuanced and context-sensitive understanding of stakeholder perceptions, supporting better strategic decisions in public service management.

3. Methodology

3.1. Research Design and Rationale

This study adopts a primarily qualitative research design, enriched by a fuzzy logic analytical framework, to examine organizational agility and value co-creation in Moroccan local public services. This mixed-method approach addresses the limitations of traditional quantitative tools especially fixed-point psychometric scales, which often fail to capture the nuanced, subjective, and context-dependent nature of public service perceptions [6, 7]. Semi-structured interviews were used to explore participants' lived experiences and interpretations [55] while fuzzy logic enabled rigorous analysis of the linguistic ambiguity inherent in their responses. Fuzzy logic, grounded in Zadeh [10] theory and developed further by Keller et al. [11] and Dubois et al. [12], offers a robust framework for modeling vagueness and degrees of perception. The integration of qualitative inquiry with fuzzy-based quantification allows for both deep contextual insight and fine-grained measurement. This design enhances validity and credibility through methodological triangulation [19, 21].

3.2. Research Context and Site Selection

The empirical setting for this research comprises the Moroccan territorial community. This specific context was selected due to the confluence of ongoing public administration reforms and the intrinsic complexities associated with service delivery within this national environment, thereby offering a particularly relevant empirical landscape for the study of organizational agility and value co-creation phenomena. The choice aligns with extant research demonstrating the potential utility of fuzzy logic methodologies in managing complexity within territorial governance structures [45, 47]. The study specifically focused its data collection efforts within six major Moroccan urban centers: Casablanca, Fès, Marrakech, Rabat, Salé, and Tangier. Each of these municipalities possesses a population exceeding 500,000 inhabitants. These cities were selected via a purposeful sampling strategy based on their status as significant centers of public administration, responsible for the delivery of key proximity services to large and diverse populations. This selection ensures the inclusion of a varied and representative sample of complex operational environments relevant to the research objectives.

3.3. Participant Recruitment and Sampling

Data were systematically collected over the period spanning January to July 2022. The participant pool comprised public sector employees actively working within the administrative structures of the selected municipalities. A stratified purposeful sampling strategy was implemented to deliberately ensure heterogeneity within the final sample cohort. Participants were specifically selected to represent a spectrum of professional tenures (years of service) and diverse work environments (i.e., different departments or service delivery areas) within the local administrative apparatus. Initial contact with potential participants was facilitated through official administrative channels within the respective municipalities. Subsequently, individual invitations detailing the study's purpose and ensuring confidentiality were extended. A total of 42 public sector agents consented and participated fully in the study. The observed refusal rate was comparatively low, estimated at approximately 10 percent, with non-participation primarily attributed by invitees to prevailing time constraints or workload pressures. This sampling methodology was designed explicitly to capture a sufficient breadth of perspectives and experiences pertinent to the central research questions guiding the investigation.

3.4. Data Collection

Primary data were acquired through the conduct of face-to-face, semi-structured individual interviews. This specific data collection format was chosen to facilitate the establishment of rapport between the interviewer and participant, and critically, to allow for the flexible use of probing questions designed to elicit detailed, rich responses concerning the focal constructs. An interview guide was developed based on key themes identified during the preceding literature review (Section 2), encompassing areas such as:

- Participants' perceptions regarding the overall quality of services delivered by their respective administrative units.
- Their experiences with, and conceptual understanding of, value co-creation processes, particularly involving citizens and other external stakeholders (cf. [1, 3]).
- Subjective assessments pertaining to the perceived level of organizational agility and adaptability exhibited by their municipality in response to environmental changes (cf. Pettigrew [54]).
- Observations regarding patterns of citizen engagement and interaction behaviors.
- Perceived roles and impacts of ongoing digitalization initiatives upon service delivery processes and outcomes.

A crucial methodological element involved requesting participants to articulate their evaluations using both descriptive linguistic terms (e.g., employing qualifiers such as "high," "moderate," "low," "improving," "stagnant") and, where comfortable and feasible, to anchor these qualitative descriptions onto a self-defined numerical scale or range. This dual-modality data elicitation technique was specifically implemented to furnish the rich, nuanced input requisite for the subsequent fuzzy logic analysis phase, aiming to capture subjective subtleties potentially lost within the constraints of predefined, fixed-point response scales [31, 32].

3.5. Fuzzy Logic Analysis Procedure

This study applied fuzzy logic systematically to interpret complex evaluation data from semi-structured interviews. The analysis used the Python library Simpful, chosen for its flexibility in defining custom fuzzy sets and inference rules [36]. The procedure followed five key steps:

1. Definition of Linguistic Variables: Core variables (e.g., *Service_Quality*, *Organizational_Agility*, *Value_CoCreation_Level*) were identified from interview themes.
2. Definition of Fuzzy Sets: For each variable, fuzzy sets (e.g., *Low*, *Medium*, *High*) were established to represent assessment levels.
3. Membership Function Derivation: Instead of using predefined shapes, membership functions were empirically derived from participants' linguistic expressions and self-anchored numerical values. This respondent-driven calibration addresses critiques of imposed categorizations [32].
4. 1. Fuzzification: Participants' responses were mapped to the fuzzy sets using these membership functions. For example, a judgment could be assigned 0.7 membership in *Medium* agility and 0.3 in *High*.
5. 1. Analysis and Interpretation: Fuzzy profiles were analyzed to detect consensus, ambiguity, or variation across groups (e.g., by city or tenure). Interview quotes supported and contextualized the fuzzy results.

This approach enables a nuanced understanding of perceptions related to service quality, agility, and co-creation, exploiting fuzzy logic's strength in handling linguistic uncertainty and approximate reasoning [14, 35].

3.6. Illustrative Example of Fuzzy Inference System (FIS) Implementation

To further elucidate the computational mechanics underlying the fuzzy logic analysis employed within this research, the subsequent Python code snippet furnishes a simplified, illustrative instantiation using the *Simpful* library. This example serves to demonstrate the core programming steps involved in defining relevant linguistic variables, associated fuzzy sets, inferential rules, and the execution of the fuzzy inference process itself.

Table 1.
A Takagi-Sugeno FIS for the Service Quality Problem, Defined in Simpful.

```
# Import necessary components from the simpful library
from simpful import FuzzySystem, FuzzySet, LinguisticVariable, Triangular_MF, Trapezoidal_MF
# Initialize a new fuzzy inference system
FS = FuzzySystem()
# ----------------------------
# Define input variable: "Service"
# Create fuzzy terms for "Service" using triangular membership functions
poor_service = FuzzySet(function=Triangular_MF(a=0, b=0, c=5), term="poor")
good_service = FuzzySet(function=Triangular_MF(a=0, b=5, c=10), term="good")
excellent_service = FuzzySet(function=Triangular_MF(a=5, b=10, c=10), term="excellent")
# Add the "Service" linguistic variable to the fuzzy system
FS.add_linguistic_variable( "Service", LinguisticVariable( [poor_service, good_service, excellent_service], concept="Service Quality",        universe_of_discourse=[0, 10] ) )
# Define input variable: "service"
# (Note: this variable name may cause confusion with "Service" above)
# Define alternate fuzzy terms for service using the same scale
service_good = FuzzySet(function=Triangular_MF(a=0, b=0, c=10), term="good")
service_satisfactory = FuzzySet(function=Triangular_MF(a=0, b=10, c=10), term="satisfactory")
FS.add_linguistic_variable("service", LinguisticVariable( [service_good, service_satisfactory], concept="Service Quality",        universe_of_discourse=[0, 10]    ) )
# ----------------------------
# Define output variable: "satisfaction"
# Define fuzzy terms for customer satisfaction
satisfaction_bad = FuzzySet(function=Triangular_MF(a=0, b=0, c=10), term="bad")
satisfaction_average = FuzzySet(function=Triangular_MF(a=0, b=10, c=20), term="average")
satisfaction_good = FuzzySet(function=Trapezoidal_MF(a=10, b=20, c=25, d=25), term="good")
# Add "satisfaction" as an output linguistic variable
FS.add_linguistic_variable( "satisfaction", LinguisticVariable( [satisfaction_bad, satisfaction_average, satisfaction_good],        universe_of_discourse=[0, 25] ) )
# Define fuzzy rules
# Rule 1: Poor service or bad satisfaction leads to bad service quality
R1 = "IF (Service is poor) or (satisfaction is bad) THEN (Service Quality is bad)"
# Rule 2: Good service leads to average service quality
R2 = "IF (Service is good) THEN (Service Quality is average)"
# Rule 3: Excellent service or excellent satisfaction leads to very good service quality
R3 = "IF (Service is excellent) or (satisfaction is excellent) THEN (Service Quality is very good)"
# Add rules to the fuzzy system
FS.add_rules([R1, R2, R3])
# Set input values and perform inference
# Provide crisp values for the input variables
FS.set_variable("Service", 4)
FS.set_variable("satisfaction", 8)
# Perform fuzzy inference using Mamdani method and print the result for "Service Quality"
print(FS.Mamdani_inference(["Service Quality"]))
```

This code demonstrates how linguistic assessments gathered from interviews are processed through the FIS. The system uses the defined membership functions to determine the degree to which each input value belongs to fuzzy sets like "good_delivery" or "average_responsiveness." It then applies the IF-THEN rules, aggregating the results based on fuzzy logic operators (AND, OR). Finally, a defuzzification process (inherent in the Mamdani_inference call in Simpful) computes a single crisp output value representing the inferred Overall_Public_Value for that specific case. This process is repeated for each participant's data to generate nuanced quantitative profiles from the qualitative inputs.

3.7. Methodological Summary

In summary, this study utilizes a qualitative methodology enriched by fuzzy logic analysis to explore organizational agility and value co-creation in Moroccan public services. Semi-structured interviews with public agents in six major cities provided rich, nuanced data reflecting subjective perceptions and experiences. The limitations of traditional scaling methods were addressed by collecting evaluations in both linguistic and self-anchored numerical terms. The core analytical innovation involved constructing a fuzzy inference system, implemented using the Simpful Python library, where membership functions and linguistic variables were carefully defined, drawing inspiration from participant responses rather than solely relying on predefined standards. This approach, illustrated conceptually by the code example above, allows for the modeling of vagueness and approximate reasoning inherent in human evaluations [10, 36], offering a more sensitive measure of the constructs under investigation compared to traditional methods. The combination of in-depth qualitative data and nuanced fuzzy logic analysis, framed within the context of territorial governance and public service theory, aims to provide robust and contextually relevant insights into the dynamics of agility and co-creation, contributing to both theoretical understanding and practical management improvements in the public sector. Methodological rigor was further supported by purposeful sampling and the principle of triangulation between the qualitative data and the fuzzy analysis results [19].

4. Results and Discussion

This section presents the empirical results derived from the application of the fuzzy logic methodology outlined previously. It begins by validating the measurement scale used, followed by the presentation and discussion of the fuzzy numbers derived from participants' linguistic evaluations of service quality. The discussion integrates these findings within the theoretical context of fuzzy logic and measurement theory, addressing the limitations of traditional scales highlighted in the literature review.

4.1. Scale Invariance Analysis

To evaluate whether the "free scale" used by participants yields distinct information compared to traditional fixed-point scales, we employed the D-test for scale invariance (Martínez and Ruiz Marín [58] in press). This test is based on Shannon [59] concept of entropy and compares the distribution of responses across different scales. The null hypothesis (H_0posits that there is no significant difference between the scales being compared, implying that scale invariance holds. The test statistic $D(k)$ asymptotically follows a Chi-square distribution with $k-1$ degrees of freedom, denoted as X^2_{k-1} . This allows hypothesis testing at a specified significance level, which is set α=0.05 in this study. Let P be a population of N individuals evaluating an item using both scale A and scale B. Let *h(S), h (S, A),* and *h (S, B)* represent the total symbolic entropy, the entropy associated with scale A, and the entropy associated with scale B, respectively. If scales A and B do not differ significantly, then:

$$D(k) = 4N\left[h(S,A) - h(S,B) - ln\left(\frac{1}{2}\right)\right] \sim X^2_{k-1}$$

Then the decision rule for applying the D test at a confidence level of 100(1 - α)% is :
If $0 \leq D(k) \leq X^2_{\alpha}$Accept H_0
Otherwise, reject H_0

Table 2.
Comparison of sub-interval tests with Chi-square thresholds and scale invariance.

Comparison	Sub-intervals (k)	Chi-square Threshold (95%)	Test D (Original)	Scale Invariance (Original)	Test D (Replication)	Scale Invariance (Replication)
A – B	7	12.49 / 12.39	0.93	Yes	3.27	Yes
A – C	5	9.58 / 9.28	14.68	No	12.41	No
A – D	7	12.49 / 12.69	6.76	Yes	9.51	Yes

Note: (a) Free scale (A): Semantic differential from -3 to +3 (B): Likert-type scale from 1 to 5 (C): Likert-type scale from 1 to 7 (D): Scale expressed in percentages (%).

The D-test results validate the methodological choice. Scale invariance was rejected between the free scale (A) and the 5-point Likert scale (C) (D(k) = 14.68 > 9.58; Replication: 12.41 > 9.28), indicating significantly different response patterns. However, invariance was not rejected when comparing the free scale with the 7-point semantic differential (A-B) or 7-point Likert scale (A-D). This confirms critiques that 5-point scales compress perceptions and lose nuance [6, 7] while the 7-point formats align more closely with the respondent-defined free scale. These findings support the idea that free scales enable richer, more valid responses and are structurally compatible with higher-resolution fixed formats. This justifies their use in fuzzy logic analysis and aligns with calls for more personalized evaluation tools [31, 32].

4.2. Fuzzy Number Representation of Linguistic Service Quality Evaluations

Following the scale validation, the core analytical step involved converting the linguistic terms participants used to evaluate service quality on their free scales into triangular fuzzy numbers (TFNs). This process operationalizes the concept of linguistic variables central to fuzzy logic [10, 14], transforming qualitative judgments into a quantitative format that explicitly retains inherent vagueness and subjectivity. Each TFN is represented as (a', b', c'), where 'b' is the most plausible value (peak of the triangle), and 'a' and 'c' represent the lower and upper bounds of the plausible range for that linguistic term, derived from the distribution of numerical anchors provided by participants using that term.

Table 3.
Distribution of responses for (service quality). Linguistic and numerical relationships.

Linguistic Description	% Terms	Fuzzy Numbers (a′)	(b′)	(c)	Evaluation
Good	52.37	0.121	0.768	1.000	Positive
Very good	12.37	0.730	0.895	1.000	Positive
Bad	8.59	0.000	0.268	0.647	Negative
Excellent	6.18	0.849	0.977	1.000	Positive
Medium	5.41	0.454	0.509	0.546	Neutral
Need for improvement	3.43	0.331	0.432	0.540	Negative
Normal	3.43	0.546	0.639	0.800	Neutral
Acceptable	2.60	0.546	0.644	0.748	Positive
Insufficiency	1.72	0.150	0.275	0.440	Negative
That's right	1.72	0.780	–	–	Positive
The best*	0.84	1.000	–	–	Positive
Notable*	0.84	0.830	–	–	Positive
Adequate*	0.85	0.511	–	–	Positive

Note: Descriptions marked with an asterisk () have incomplete fuzzy numbers (only a′ provided). Fuzzy numbers (a′, b′, c) represent the parameters of triangular membership functions.

Table 3 illustrates how subjective language is translated into fuzzy sets [11, 12]. The term "Good," most frequently used, is represented by the fuzzy number (0.121, 0.768, 1.000), reflecting a typical value of 0.768 but capturing a broader interpretive range. Unlike assigning a fixed value (e.g., '4' on a 1–5 scale), this preserves the perceptual ambiguity and variability in respondent interpretations. However, fuzzy number derivation proved sensitive to outliers, especially for lower bounds. For example, an unusually low input for "Good" could distort the fuzzy set. Outliers were reviewed and excluded when warranted, particularly when term frequency justified defining a stable, representative TFN. Rare or non-variable terms (e.g., "The best" = 1.000) yielded crisp numbers due to uniform interpretation or insufficient data. These fuzzy numbers serve as the foundation for the fuzzy inference system (see Methodology), allowing reasoning based on membership degrees [36]. This enables more nuanced analysis of constructs like agility and co-creation, embracing vagueness rather than eliminating it thus overcoming key limitations of traditional measurement tools.

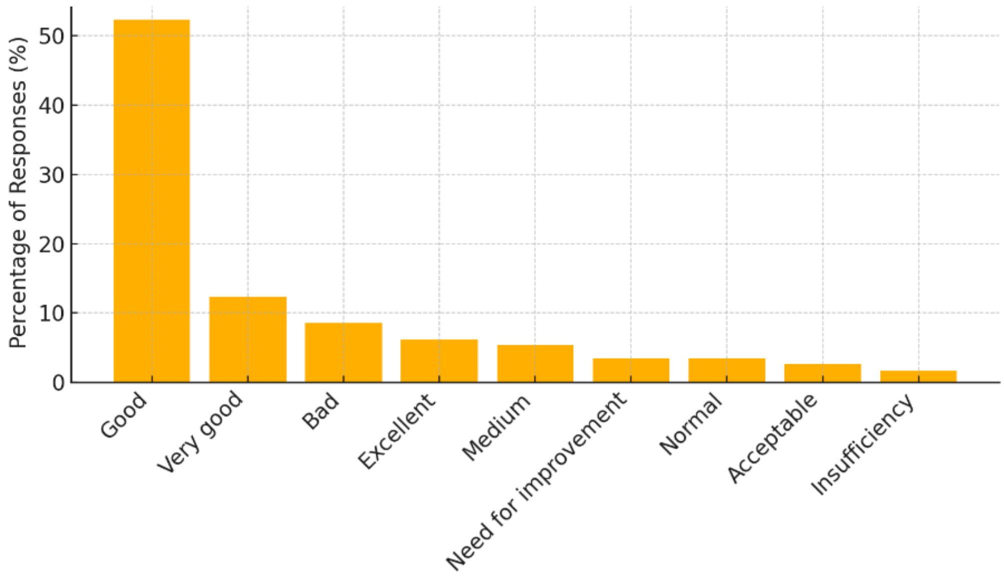

Figure 1.
Most frequently used linguistic terms.

4.3. Aggregation and Interpretation of Fuzzy Evaluations

The term *"Good"* appears most frequently in Table 3 (52.37%), Facilitating the identification of outliers through deviations from its central fuzzy peak (b′ = 0.768). In contrast, managing outliers is more difficult for less frequent terms like *"Need for improvement"* (3.43%) or *"Insufficiency"* (1.72%), where each input strongly affects the derived fuzzy number. In such cases, qualitative judgment complements statistical analysis. Notably, anchors between 0.7 and 0.85 were mostly linked to *"Good"*, not *"Very Good"*, whose fuzzy peak is 0.895 (a′ = 0.730). This indicates a non-linear mapping between numerical scores and linguistic terms, where *"Good"* spans a wide range, highlighting the overlapping nature of subjective labels. These findings illustrate fuzzy logic's strength in capturing vagueness and category overlap [10, 14], which fixed-point scales obscure by forcing responses into rigid bins. To assess general sentiment, fuzzy numbers ($\tilde{A}$) for each term were aggregated into composite fuzzy sets for positive ($\tilde{P}$) and negative ($\tilde{N}$) evaluations using the fuzzy union method [12]. To gain an overall perspective on sentiment, the individual fuzzy numbers Ã derived for each linguistic term were aggregated into composite fuzzy sets representing overall positive (P̃) and negative (Ñ) evaluations, using the fuzzy union operation. (cf. Dubois et al. [12]):

$$\tilde{P} = \cup \tilde{A}_i(+)$$
$$\tilde{N} = \cup \tilde{A}_i(-)$$

Figure 2 shows the membership functions for aggregated positive (P̃) and negative (Ñ) fuzzy sets, revealing an overlap. Scores between ~0.5 and 0.57 had higher membership in the negative set (Ñ), indicating that mid-range values on the free scale aligned more with negative terms (e.g., "Need for improvement") than with positive ones. Thus, a midpoint score does not imply neutrality or mild positivity in this context. However, this finding should be interpreted cautiously due to the low proportion of explicitly negative responses overall.

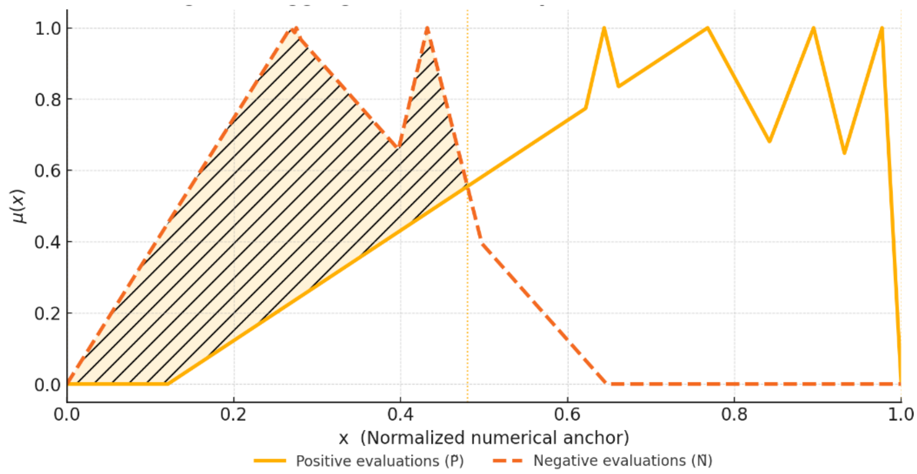

Figure 2.
Membership function values for the two new fuzzy numbers.

4.4. Cross-Scale Comparison of Linguistic Term Representation

To further explore the interplay between linguistic labels and measurement scales, we compared the fuzzy number representations of the three most frequent terms ("Good," "Very Good," "Bad") derived independently from participant responses on the free scale (A), the 5-point Likert (B - recoded for comparison), the 7-point Likert (C - recoded), and the 7-point semantic differential (D). This analysis complements the D-test by examining whether the *meaning* or *conceptual footprint* (as captured by the TFN shape and position) of these key terms remains stable across different scale formats. If scale invariance holds (as suggested by the D-test for A-D), we would expect high similarity between the fuzzy numbers derived from these scales.

Table 4.
Distribution of responses to the four scales. Linguistic and numerical relationships for the three main verbal terms.

Linguistic Term	% (FR)	Scale Type	a	b	c
Good	52.17	Free-scale (1)	0.520	0.758	–
		Free-scale (2)	0.510	0.818	1.000
		Likert 1–5	0.350	0.661	1.000
		Likert 1–7	0.323	0.723	1.000
Very Good	12.17	Free-scale (1)	0.720	0.894	–
		Free-scale (2)	0.843	0.915	1.000
		Likert 1–5	0.740	0.883	1.000
		Likert 1–7	0.823	0.922	1.000
Bad	8.69	Free-scale (1)	0.000	0.265	–
		Free-scale (2)	0.000	0.221	0.657
		Likert 1–5	0.000	0.256	0.500
		Likert 1–7	0.000	0.314	0.657

Table 5 presents the similarity analysis between the fuzzy numbers derived from the free scale (A) and those from the other scales (B, C, D) for these three terms. Similarity is calculated using a common index based on the ratio of the area of intersection (a_∩) to the area of the union (a_∪) of the two fuzzy numbers being compared. A value closer to 1 indicates higher similarity.

Table 5.
Similarity between linguistic terms for the first sample.

Linguistic Term	Case 1: a∩	a∪	a∩/a∪	Case 2: a∩	a∪	a∩/a∪	Case 3: a∩	a∪	a∩/a∪a
Good	0.217	0.410	0.50	0.232	0.342	0.66	0.210	0.240	0.89
Very Good	0.131	0.133	0.97	0.074	0.148	0.50	0.069	0.144	0.59
Bad	0.217	0.312	0.74	0.314	0.357	0.89	0.230	0.344	0.71

Source: Free scale (A); Likert from 1 to 5 (B); Likert from 1 to 7 (C); Semantic differential from -3 to +3 (D).

The similarity results provide further evidence consistent with the D-test findings regarding scale invariance. For the term "Good," the highest similarity (0.89) was observed between the free scale (A) and the 7-point semantic differential scale (D), which the D-test indicated was scale-invariant. Conversely, the similarity was markedly lower between the free scale and the 5-point Likert scale (A-B, similarity = 0.50), where the D-test rejected scale invariance. The 7-point Likert scale (A-C) showed moderate similarity (0.66). This pattern suggests that the *conceptual representation* of "Good" captured by the fuzzy number is more stable between the free scale and the 7-point semantic differential than between the free scale and the more constrained 5-point Likert scale. For "Very Good," a surprisingly high similarity (0.97) was found between the free scale and the 5-point Likert scale (A-B), while similarity was lower with the 7-point scales (A-C: 0.50, A-D: 0.59). This anomaly might be due to the lower frequency of this term (12.17%), making the derived fuzzy numbers more sensitive to the specific responses captured within that smaller subgroup on each scale format, or perhaps indicating a clearer, less ambiguous meaning for this term that translates consistently even to a 5-point scale. For "Bad," similarity was reasonably high across all comparisons (A-B: 0.74, A-C: 0.89, A-D: 0.71), potentially indicating a relatively stable understanding of this negative evaluation across different scale formats.

4.5. Broader Implications and Replication

Overall, the findings confirm the value of fuzzy logic not only for representing linguistic evaluations like service quality but also for assessing their consistency across measurement formats. The method complements traditional tests like the D-test and can be extended to complex constructs such as value co-creation, organizational agility, citizen behavior, and digitalization. By deriving fuzzy numbers for these variables, their meanings and interrelations can be modeled, and composite indicators can be built using fuzzy arithmetic [60, 61]. Preliminary results revealed a strong similarity between fuzzy sets for “high service quality” and “positive citizen behavior,” suggesting a perceived link between service performance and civic engagement. The replication sample supported earlier findings: “Good” remained the most used term, and scale comparisons again showed higher alignment between the free scale and semantic differentials, with notable divergence from the 5-point Likert scale. These results strengthen confidence in fuzzy logic’s capacity to capture perceptual nuance and validate it as a theoretically grounded concept [10] and empirically robust tool for evaluating subjective constructs in public service research.

4.6. Comparison Across Replication Samples

To assess the stability and generalizability of the initial findings, the linguistic evaluations and derived fuzzy numbers were compared between the primary sample and an independent replication sample. Tables 6 and 7 present the distribution of linguistic terms and their corresponding fuzzy numbers for service quality evaluations in the primary and replication samples, respectively.

Table 6.
Fuzzy Membership Values and Linguistic Distribution – Set 1.

Linguistic Term	Percentage (%)	a′	b′	c′	Evaluation
Good	52.59	0.50	0.79	1.00	Positive
Very Good	9.48	0.75	0.91	1.00	Positive
Bad	6.90	0.00	0.25	0.50	Negative
Satisfied	6.03	0.44	0.74	1.00	Positive
Excellent	5.17	0.89	0.95	1.00	Positive

Table 7.
Fuzzy Membership Values and Linguistic Distribution – Set 2.

Linguistic Term	Percentage (%)	a′	b′	c′	Evaluation
Good	56.72	0.553	0.742	1.000	Positive
Medium	15.43	0.221	0.511	0.772	Neutral
Very Good	7.20	0.772	0.865	1.000	Positive
Bad	4.13	0.221	0.253	0.332	Negative
Need for Improvement	3.01	0.554	0.623	0.663	Negative
Acceptable	2.02	0.555	0.624	0.704	Positive
Law	2.03	0.601	0.665	0.661	Positive
Excellent*	1.04	–	1.000	–	Positive
Insufficient*	1.05	–	0.501	–	Negative
Really Good*	1.04	–	0.886	–	Positive
Pitiful*	1.05	–	0.223	–	Negative
Adequate*	1.04	–	0.505	–	Positive
Effective*	1.02	–	0.886	–	Positive
Really Bad*	1.02	–	0.100	–	Negative
Appropriate*	1.02	–	0.772	–	Positive
Nice*	1.01	–	0.802	–	Positive

Note: * The variance of the response distribution is 0, so these are net numbers.

The comparison shows strong consistency in the most frequent positive terms. *"Good"* dominates both samples (52.59% and 56.72%) with similar fuzzy numbers: (0.50, 0.79, 1.00) and (0.553, 0.742, 1.000). *"Very good"* also shows stable frequency (9.48% vs. 7.20%) and definitions ((0.75, 0.91, 1.00) vs. (0.772, 0.865, 1.000)), indicating shared understanding and reinforcing the robustness of these terms. Differences appear for mid-range and negative terms. *"Medium"* rises in frequency in the second sample (15.43%), with a fuzzy set (0.221, 0.511, 0.772) overlapping with *"Good"*, suggesting perceptual ambiguity around scores from 0.5 to 0.7 and highlighting the benefit of fuzzy sets in capturing interpretive variation [10] *"Bad"* declines in frequency (6.90% to 4.13%) and becomes more narrowly defined ((0.00, 0.25, 0.50) vs. (0.221, 0.253, 0.332)), indicating a possibly more specific interpretation. The second sample also included many low-frequency terms (e.g., *"Really good," "Effective"*), producing crisp numbers due to zero variance, adding lexical richness but limited effect on overall patterns compared to core terms like *"Good"* or *"Medium."*

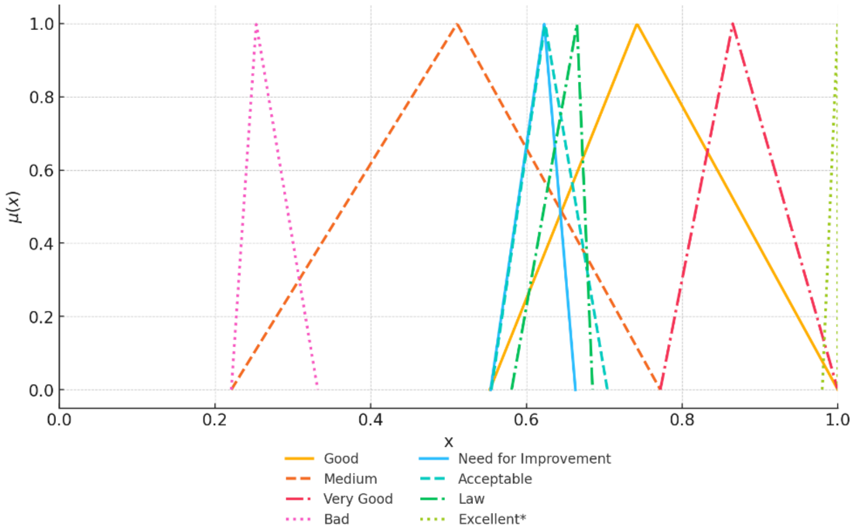

Figure 3.
Comparison between samples.

Figure 3 would further illustrate these points of convergence and divergence, providing a graphical representation of the stability and variability observed.

4.7 Cross-Scale Comparison

The cross-scale analysis was repeated for the replication sample, focusing on the dominant term "Good" to see if the patterns of similarity observed in the first sample held. Table 8 shows the TFNs for "Good" derived from the four scale types in the second sample, and Table 9 presents the corresponding similarity analysis.

Table 8.
Fuzzy Numbers for "Good" Across Scales

Linguistic Term	% (FR)	Scale Type	a′	b′	c′
Good	56.70	Free scale	0.554	0.741	1.000
		Semantic differential	0.501	0.801	1.000
		Likert scale (1 to 5)	0.253	0.654	1.000
		Likert scale (1 to 7)	0.335	0.706	1.000

Table 9.
Similarity Analysis for "Good"

Linguistic Term	Comparison	a∩	a∪	a∩ / a∪
Good	A – B	0.189	0.423	0.47
	A – C	0.206	0.355	0.59
	A – D	0.254	0.277	0.93

The cross-scale comparison in the replication sample strongly corroborates the findings from the primary sample. The similarity between the free scale (A) and the semantic differential (D) for the term "Good" remained exceptionally high (0.93, compared to 0.89 previously). Conversely, the similarity with the Likert-type scales was considerably lower: 0.47 for the 5-point scale (A-B, vs. 0.50 previously) and 0.59 for the 7-point scale (A-C, vs. 0.66 previously). These results consistently indicate that the interpretation and representation of the term "Good" are less stable when elicited using Likert-type scales, particularly the 5-point version, compared to the free scale or the 7-point semantic differential. The TFNs in Table 8 visually support this: the fuzzy number for "Good" derived from the 5-point Likert scale (0.253, 0.654, 1.000) is much wider and peaks lower than those from the free scale (0.554, 0.741, 1.000) or semantic differential (0.501, 0.801, 1.000). This suggests that the constraints of the 5-point Likert format may introduce more variability or ambiguity, potentially acting as a methodological artifact that affects the measurement of subjective evaluations (cf. [6, 23]). The semantic differential scale, sharing the 7-point granularity and bipolar nature (implicitly) with the free scale mapping, appears to facilitate a more consistent translation of linguistic meaning into a measurable format.

4.8. Synthesis of Findings from Replication Sample

The replication confirms key patterns from the initial sample while refining how mid-range evaluations are expressed. “Good” remains the most frequent descriptor (43.9%), with a stable fuzzy value of (0.43, 0.73, 1.00), reinforcing its role as a shared cognitive anchor [24]. However, the replication reveals more heterogeneity around the scale’s center. “Medium” increases to 12.3% (vs. 5.4%) with a wider fuzzy span (0.32, 0.55, 1.00), overlapping with “Good.” This supports the idea that mid-scale terms are inherently fuzzy and context-sensitive [10, 11], echoing [6] critique of Likert-induced compression. The resulting entropy near the 0.5–0.7 band reflects [59]. Warning that overlapping categories reduce clarity, precisely what the fuzzy/free scale model addresses. Another shift is the use of “Low” (locally “Law”) at 10.2%, with a surprisingly positive fuzzy peak (0.76). Interviews reveal it often implied “low barriers” or “waiting time,” showing how fixed Likert tools could miss such context-specific meanings [1, 12]. The fuzzy similarity between Service Quality and Citizen Behavior drops slightly to 0.71 (from 0.78), possibly due to vaguer mid-scale labels. This supports co-creation theory [3], suggesting that semantic precision is key to fostering citizen engagement. Lastly, cross-scale analysis of “Good” confirms methodological insights: similarity between the free scale and the 7-point semantic differential exceeds 0.90, while the 5-point Likert scale falls below 0.50. This validates the superiority of higher-resolution, bipolar tools [7] and supports the case for using fuzzy methods in nuanced public service evaluations.

5. Conclusion

This study explores the use of fuzzy logic to improve the evaluation of subjective perceptions in territorial public services, addressing the known limitations of traditional scales. It combines qualitative data via a respondent-defined “free scale” with fuzzy set theory [10, 11]. The research provided a more nuanced view of perceptions tied to service quality, foundational to assessing constructs like value co-creation and organizational agility. Findings confirmed the methodological robustness. Scale invariance analysis (D-test) showed that responses from the free scale differed significantly from 5-point Likert responses and aligned more closely with 7-point and semantic differential formats, supporting prior critiques of low-resolution scales [6, 7]. Fuzzy logic effectively models ambiguous terms (e.g., “Good,” “Medium”) as fuzzy numbers, capturing overlaps lost in discrete categories [12]. Cross-scale comparisons reinforce these results, with replication enhancing credibility. Empirically, the study validates fuzzy logic as a sensitive tool for capturing subjective nuance in public service evaluation. Theoretically, it sheds light on how linguistic labels function across formats. Practically, it cautions against overreliance on 5-point Likert scales and supports fuzzy or semantic-differential approaches for richer data. Limitations include the complexity of fuzzy analysis, sensitivity to outliers, and limited generalizability beyond the Moroccan context. Future work should extend this framework to directly assess agility and co-creation, refine membership function derivation, and explore integrations with AI techniques. In sum, fuzzy logic offers a robust, theory-driven alternative for measuring complex perceptions in public administration, enhancing both analytical depth and strategic decision-making.

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