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Toward an optimal risk retention strategy in high-risk industries: A case for Saudi cement firms

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Abstract

This study investigates the optimal risk retention strategy for cement companies in the Kingdom of Saudi Arabia, focusing on the period from 2012 to 2024. Using a quantitative approach, we model the frequency and severity of pure risks faced by the industry, applying Poisson and Gamma distributions alongside compound loss modeling to estimate the Maximum Probable Yearly Aggregate Loss (MPY). The results indicate that the optimal risk retention level for Saudi cement firms is 29.4%, suggesting that a hybrid risk management strategy—combining self-insurance and commercial insurance—offers the most cost-effective solution. By retaining a portion of the risk and transferring the remainder to insurers, firms can optimize their risk financing costs while maintaining protection against catastrophic losses. This approach is consistent with recent studies, including those by Nocco and Stulz [1] and Foster [2] which highlight the value of combining self-insurance and market insurance to balance cost control and risk mitigation. Ultimately, this research provides actionable insights for risk managers in the cement industry, enabling them to adopt strategies that enhance financial resilience and operational continuity in the face of an increasingly complex risk landscape.

Keywords: Cement industry in KSA, Hybrid Insurance Strategy, Risk Management Optimization, Risk retention, MPY.

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1. Introduction

In the field of risk management, decision-makers often face a fundamental trade-off between retaining risk internally and transferring it to external insurers. Retaining a larger share of risk can significantly reduce insurance premiums, aligning with a cost-minimization approach. Conversely, transferring more risk can protect firms against large, infrequent losses, reflecting a loss-avoidance strategy. The choice between these strategies becomes particularly critical in industries with high capital intensity and operational exposure. The Saudi cement industry plays a vital role in national economic development, supporting infrastructure, housing, and industrial growth. However, this sector is increasingly exposed to various operational, environmental, and financial risks, including equipment breakdown, supply chain disruptions, liability exposure, and regulatory challenges. These risks can severely affect production efficiency, profitability, and long-term sustainability. As such, cement manufacturers must adopt robust risk management strategies that not only reduce the likelihood of losses but also minimize their financial impact when they occur.

To effectively manage these exposures, firms can employ either risk control mechanisms—such as safety improvements and maintenance systems—or risk financing strategies, including insurance and self-insurance. A key question for risk managers, therefore, is determining the optimal level of risk retention: how much risk the firm should retain on its balance sheet versus how much it should be transferred to an insurer. This decision is central to aligning risk management with financial objectives, especially when navigating the trade-off between premium costs and potential uninsured losses.

This research aims to quantitatively assess the optimal retention level of pure risks within the Saudi cement sector. Specifically, it investigates how risk managers can determine the most cost-effective balance between self-insurance and commercial insurance, considering the industry's unique risk characteristics. As emphasized by Nocco and Stulz [1] determining an appropriate retention level is a cornerstone of enterprise risk management (ERM), which helps firms define and operate within their risk appetite. Poorly calibrated retention decisions can leave firms vulnerable to financial distress during catastrophic events or result in excessive insurance spending that undermines profitability. In capital-intensive sectors like cement manufacturing, such misalignment can lead to suboptimal financial outcomes or operational setbacks.

Extant literature supports the notion that optimal risk retention contributes to corporate value by accounting for firm-specific factors such as risk tolerance, capital reserves, and loss frequency [3, 4]. Within the Saudi context, understanding and modeling these dynamics is essential for developing a sustainable insurance strategy tailored to local economic and industrial realities.

2. Background

The cement industry in Saudi Arabia plays a pivotal role in the nation's economic development, contributing to the construction of essential infrastructure and supporting the rapidly growing population. Saudi Arabia is one of the largest producers of cement not only in the Gulf region but also in the broader Middle East [5]. The Kingdom is home to seventeen cement companies, with major firms including Saudi Cement, Southern Province Cement, Yamama Cement, and Qassim Cement. This sector's growth has been driven by the continuous demand from large-scale projects such as NEOM, the Red Sea Project, and numerous housing developments aimed at meeting the needs of a burgeoning population [6]. The cement industry is intrinsically linked to the national economy, as it supports critical construction and infrastructure projects that are essential for sustainable economic growth.

However, like other capital-intensive industries, the cement sector is vulnerable to a wide array of pure risks, which can significantly impact operational efficiency, profitability, and sustainability. These risks stem from both internal operational challenges and external environmental factors. As a result, effective risk management strategies are essential for minimizing potential financial and operational setbacks. At the organizational level, risk managers must assess the trade-offs between risk retention and risk transfer, with decisions on how much risk to retain directly influencing both cost structures and exposure to potential losses [3].

In this context, Saudi cement firms are exposed to several distinct types of risks. These include:

1. *Environmental and regulatory risks*: Cement production is energy-intensive and generates significant carbon emissions, contributing to climate change and air pollution [7]. Regulatory pressures to reduce environmental impact, alongside sustainability constraints, compel firms to adopt cleaner technologies and practices. Non-compliance could result in fines and reputational damage [8].
2. *Health and safety risks*: The cement manufacturing process exposes workers to hazardous conditions, including exposure to dust, which can lead to long-term respiratory diseases and accidents on-site [9]. In addition, workplace accidents can result in severe injuries or fatalities, highlighting the importance of comprehensive safety protocols and insurance coverage.
3. *Product liability risks: Defective cement products*, whether due to manufacturing flaws or improper application, may lead to significant legal claims and costly compensations for damage caused in large-scale construction projects [10].
4. *Cybersecurity and asset theft risks*: With the increasing digitalization of the industry, cement companies are increasingly vulnerable to cyberattacks, including hacking and data breaches, which can disrupt operations and compromise sensitive information [11]. Moreover, theft of physical assets, such as equipment and raw materials, can result in financial losses.
5. *Operational risks*: The cement industry relies on stable supply chains for raw materials, energy, and transportation. Supply chain disruptions, energy shortages, or delays in the delivery of raw materials can lead to

production downtimes or increased operational costs. The maintenance and upgrading of equipment are additional concerns, as outdated machinery can lead to inefficiencies and higher operational costs [6].

6. *Economic risks:* The demand for cement is closely tied to construction activities and overall economic growth. Economic downturns can lead to reduced infrastructure investment, which in turn reduces cement demand and negatively affects profitability [12]. Similarly, price volatility in raw materials and energy can increase operational costs, impacting the overall financial health of cement companies.
7. *Political and social risks*: Cement firms are also susceptible to political instability, such as labor strikes, community opposition to projects, or policy changes. These risks can disrupt production and delay projects, particularly in regions facing social unrest or political transitions

Given these diverse and interconnected risks, the role of the risk manager in the cement industry is crucial for ensuring that risks are effectively mitigated. The risk manager must decide whether to self-insure (i.e., retain risks) or transfer them to insurers through commercial insurance. In cases of partial retention, determining the optimal retention level is a key consideration. This decision should be based on a careful analysis of cost-benefit trade-offs, considering factors such as the firm's risk appetite, financial capacity, and potential loss exposure [1].

The aim of this research is to explore how cement industry risk managers in Saudi Arabia can determine the optimal retention level for pure risks. Specifically, this study seeks to identify the most efficient balance between self-insurance and insurance coverage, thereby minimizing total costs while ensuring adequate protection against severe losses. The optimal retention level will vary based on firm-specific factors, such as financial strength, risk profile, and the likelihood of catastrophic losses [3].

3. Research Motivation

The premiums paid by cement companies in Saudi Arabia to mitigate pure risks have historically been significantly higher than the compensations received from insurers. From 2012 to 2024, the disparity between the insurance premiums and the compensations suggests a potential inefficiency in risk management practices. This discrepancy raises questions about the optimal allocation of risk between self-insurance and commercial insurance, especially when the proportion of compensations to premiums is consistently low.

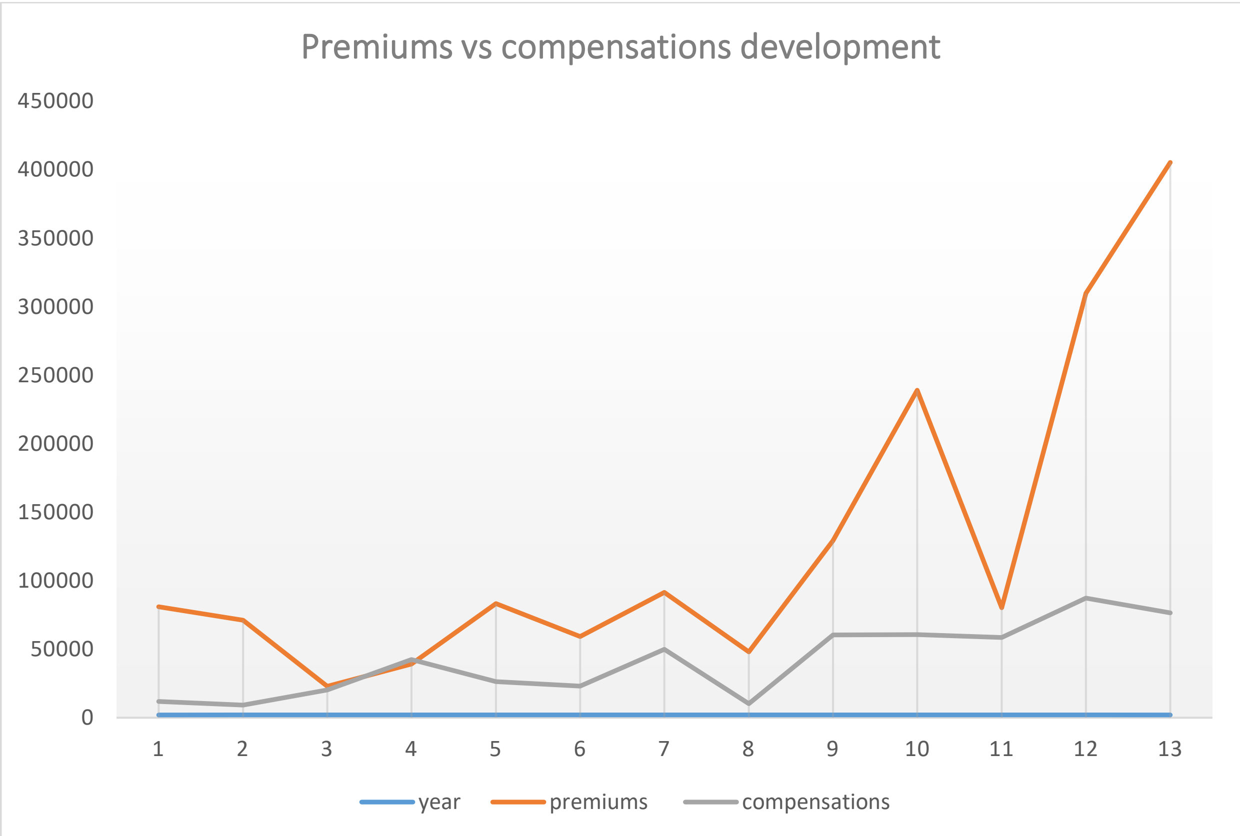

Figure 1.
Premiums development versus compensations received, The cement industry from 2012 to 2024.
Source: The authors based on data from the cement industry in KSA.

The chart illustrates the trends in premiums paid and compensations received by a representative cement company in Saudi Arabia over the study period. As shown, the percentage of premiums paid compared to compensations received has remained relatively small for most years. For example, in 2012, 2013, and 2020, the percentage of compensations was only 14.8%, 12.9%, and 15.7%, respectively. Only in 2014, 2015, and 2022 did the compensations exceed the premiums paid, with figures reaching 88.6%, 109%, and 72.8%, respectively. This inefficiency in risk financing presents an opportunity to explore more cost-effective risk management strategies. The primary motivation of this research is to determine the optimal level of risk retention for cement companies in Saudi Arabia. This can be achieved through the development of a quantitative model that will help risk managers determine the ideal balance between self-insurance and commercial insurance.

The central question guiding this study is: How can cement companies in Saudi Arabia determine the optimal level of risk retention? Specifically, this research aims to examine the risk appetite of these firms by evaluating the trade-off between the cost of risk retention and the cost of purchasing insurance. By modeling these decisions quantitatively, the research seeks to assist risk managers in making more informed, cost-efficient decisions regarding risk financing.

4. Literature Review

The determination of optimal risk retention levels has been widely discussed in risk management literature, with a variety of studies exploring how risk managers balance the trade-offs between self-insurance (retaining risk) and commercial insurance (transferring risk). The debate centers on identifying the most cost-effective and strategic approach, considering various factors such as company size, risk appetite, insurance premium costs, and potential catastrophic losses.

Smith [13] explored the decision-making process between purchasing insurance or applying risk control techniques. He emphasized that risk control—such as risk identification, analysis, and mitigation—does not require perfection, but instead demand prioritization. Risk managers must focus on key risks based on severity, frequency, and the ability to absorb losses, alongside the cost of managing these risks. This highlights that risk management is not a one-size-fits-all process but a dynamic system that requires flexibility and constant reevaluation, which aligns with the view presented by Bulut Karageyik and Şahin [14]. In a similar vein, Bulut Karageyik and Şahin [14] used a quantitative model to explore how variance, expected profit, and expected shortfalls of insurer's risk can guide the determination of optimal retention levels. They found that minimizing variance and expected shortfall while maximizing profitability is crucial in determining the most suitable retention level. Their findings build on Smith [13] emphasis on cost-benefit analysis, suggesting that companies should strive to balance their risk retention strategy with profitability goals. However, unlike Smith [13] who largely focused on risk control strategies, Bulut Karageyik and Şahin [14] work introduced a mathematical framework for measuring these trade-offs, offering a more data-driven approach.

On the other hand, Holly and Greszta [15] argued for a resurgence of self-insurance models like mutual and captive insurance. They noted that these models provide a more cost-effective and adaptable alternative to traditional insurance, which resonates with Smith [13] idea of prioritizing risks and finding the best strategies for handling them. Holly and Greszta [15] study, however, provides more context, demonstrating that mutuals and captives are particularly advantageous when firms seek to fill gaps left by demutualization, offering more control over risk retention. Krummaker [16] who focused on the demand for property insurance in Germany, similarly highlighted how firm size affects insurance decisions, noting that larger firms tend to have lower insurance needs because they often self-insure a significant portion of their risks. This aligns with Holly and Greszta [15] emphasis on self-insurance and shows a consistent pattern where larger firms are more likely to retain risks due to their ability to absorb them. Strupczewski, et al. [17] expanded on this by finding that, as insurance premiums increase, the level of risk retention rises. This observation aligns with the findings of Krummaker [16] who reported that smaller firms, due to their limited resources, are more inclined to self-insure, especially when insurance premiums are high. Thus, these studies illustrate a consistent link between premium costs and the level of risk retention. Additionally, Strupczewski, et al. [17] highlighted that companies with a higher likelihood of facing risks are more likely to retain some of their risks to mitigate the financial impact of rising premiums, which complements Smith [13] argument that companies must focus on managing risks according to their severity and likelihood.

In contrast, Wieczorek-Kosmala [18] questioned whether risk retention strategies were given enough attention in the literature, compared to risk transfer mechanisms like commercial insurance. Wieczorek-Kosmala [18] survey of risk managers revealed that risk retention strategies were often underemphasized, despite their importance in certain contexts. This critique contrasts with Hong and Kim [19] who advanced a multi-period framework to explore self-insurance and savings decisions, asserting that self-insurance is a viable and efficient option when risk managers consider utility maximization over multiple periods. Hong and Kim [19] approach to self-insurance involves a multi-dimensional framework incorporating both income and health-related factors, suggesting that these decisions are more complex than previously thought. In contrast to Wieczorek-Kosmala [18] who focused on self-insurance as a largely passive decision, Hong and Kim [19] positioned it as a dynamic, intertemporal choice, influenced by risk aversion and preferences. Similarly, Nocco and Stulz [1] focused on the role of Enterprise Risk Management (ERM) and Value at Risk (VaR) in determining a firm's risk appetite, suggesting that these tools can help risk managers decide the optimal retention level. Their framework reinforces the quantitative models proposed by Bulut Karageyik and Şahin [14] although Nocco and Stulz placed greater emphasis on risk management techniques that help define the company's risk tolerance. Their findings suggest that companies must use a data-driven approach to quantify their risk exposure, which is consistent with Oladunni and Okonkwo [20] who also used empirical data to demonstrate that the optimal retention level influences how insurance claims are managed. These studies highlight the growing recognition of quantitative tools as essential for determining retention levels. Further supporting this trend, Foster [2] illustrated that self-insurance is not always a choice between retention and transfer but rather a strategy that combines both. Foster's work aligns with Bourgeon and Picard [21] who described the complementary relationship between self-insurance and market insurance. Both studies argued that these strategies are not mutually exclusive but can be combined depending on the nature of the risk and the firm's ability to absorb losses. Bourgeon and Picard emphasized that self-insurance targets manageable risks, while insurance remains essential for handling extreme risks. This approach enables companies to optimize risk retention and reduce costs, a concept that resonates with Wieczorek-Kosmala [18] who acknowledged the strategic role of risk retention when carefully aligned with insurance mechanisms.

Finally, Sunaryo, et al. [22] underscored the role of risk retention in financial institutions, showing how it contributes to market stability by minimizing moral hazard and improving accountability. Their study further supports the notion that risk retention, when properly managed, can enhance industry resilience. This complements the work of Sehhat and Kalyani [23] who argued that larger firms, which face greater operational risks, tend to retain more of their risks. They concluded that industries with greater operational complexity require more property insurance, which, in turn, influences how much corporate insurance is purchased. These studies reinforce the idea that industry-specific characteristics play a crucial role in determining the optimal retention level.

5. Data and Limitations

The dataset covers the period from 2012 to 2024 including the premiums paid, compensations received, and the number of losses occurred each year by a cement company operating in the Kingdom of Saudi Aribia. The research mainly focuses on mitigating the pure risks that the cement industry face throughout 2012 to 2024 and hence determines the optimal retention level

6. Methodology, Hypotheses and Results

An appropriate retention rate for cement companies within the Saudi insurance market is determined by utilizing probability distributions to estimate a suitable retention limit for the insurance line. This process requires consideration of the maximum possible aggregate annual loss that these companies might incur. The method of estimating the maximum possible aggregate annual loss depends on both the frequency and severity of losses—specifically, the probability distribution of loss frequency and loss severity.

Since the objective is to derive the distribution of aggregate losses, the process involves three main stages, as outlined by Thomas [24]:

1. Estimating the first four central moments of the loss frequency data.
2. Estimating the first four central moments of the loss severity data.
3. Estimating the first four central moments of the aggregate loss distribution**.**

Determining the Loss Severity and Frequency for Cement Companies in the Saudi Insurance Market During the Study Period. The starting point in applying the proposed quantitative model for determining the appropriate retention rate is the identification of the loss severity and frequency experienced by the companies during the study period. These two variables represent the most influential factors affecting retention rate

Table 1.
Number of Claims and Loss Severity throughout the Period 2012–2024 for Cement Companies in the Kingdom of Saudi Arabia.

Years	Losses numbers	Compensations*(1000)
2012	23	11954
2013	35	9218
2014	39	20312
2015	61	42563
2016	27	26391
2017	58	23021
2018	63	49902
2019	28	10197
2020	49	60319
2021	72	60771
2022	43	58492
2023	64	87300
2024	91	76604

Source: The authors based on data from cement companies operating in Saudi Aribia.

6.1. Identifying the Probability Distribution for Claim Frequency Data

To determine the appropriate probability distribution governing the claim frequency data, the data were input into the EasyFit software. The analysis indicated that the Poisson distribution is the most suitable model.

Statistical Hypothesis for the Distribution of Claim Frequency:

- *Null Hypothesis* (H_0): The data follow a Poisson distribution.
- *Alternative Hypothesis* (H_1): The data do not follow a Poisson distribution.

Table 2.

Kolmogorov-Smirnov.	
Sample Size Statistic P-Value	13 0.33846 0.07857
α	0.05
Critical Value	0.36143
Reject?	No

6.2. Claim Frequency Distribution Analysis

To determine the appropriate probability distribution for the number of claims reported by cement companies in the Saudi insurance market, claim frequency data for the period 2012–2024 were analyzed using the EasyFit software. The software identified the Poisson distribution as the best fitting model for the data [25, 26].

Acording to the results, the maximum likelihood estimate for the Poisson distribution parameter was: $\lambda = 50.231$To statistically validate this selection, a goodness-of-fit test was conducted. The observed p-value was 0.07857, which is greater than the 5% significance level ($\alpha = 0.05$). Therefore, the null hypothesis (H_0)—that the claim frequency data follow a Poisson distribution—was not rejected, indicating a good fit.

6.3. Analysis of Loss Severity Distribution

In addition to modeling claim frequency, identifying the appropriate probability distribution for loss severity is essential for accurately estimating aggregate losses. Using compensation data from cement companies operating in the Saudi insurance market between 2012 and 2024, *EasyFit software* was employed to assess the distributional characteristics of claim amounts.

The analysis revealed that the *Gamma distribution* provided the best statistical fit for the loss severity data. This distribution is well-suited for modeling non-negative, skewed data, which is typical of insurance claims. The *Gamma distribution* is widely recognized in actuarial literature for its flexibility and its applicability to a range of severity scenarios, especially when modeling highly variable or right-skewed data [27-29].

To validate this result, a goodness-of-fit test was conducted with the following hypotheses:

- *Null Hypothesis (H_0):* The compensation data follow a Gamma distribution.
- *Alternative Hypothesis (H_1):* The compensation data do not follow a Gamma distribution.

Table 3.

Gamma	
Kolmogorov-Smirnov	
Sample Size Statistic P-Value	13 0.16892 0.79493
□	0.05
Critical Value	0.36143
Reject?	No

Continuing the analysis of loss severity, the results obtained from EasyFit software provide further statistical evidence supporting the suitability of the Gamma distribution for modeling claim amounts. The output presented in the previous table, the observed p-value was 0.79493, which is significantly greater than the 5% significance level ($\alpha = 0.05$). This leads to the acceptance of the null hypothesis, indicating that the actual compensation data are consistent with the Gamma distribution.

The estimated parameters of the Gamma distribution, as reported by EasyFit, are:

$\alpha=2.4588$(shape parameter), $\beta=16802.0$(scale parameter)

These parameter estimates further confirm the flexibility of the Gamma distribution in capturing the variability and skewness present in the claim severity data [26, 30].

Below is the frequency histogram overlaid with the Gamma probability density function (PDF), illustrating the close alignment between the empirical distribution of the data and the fitted Gamma model:

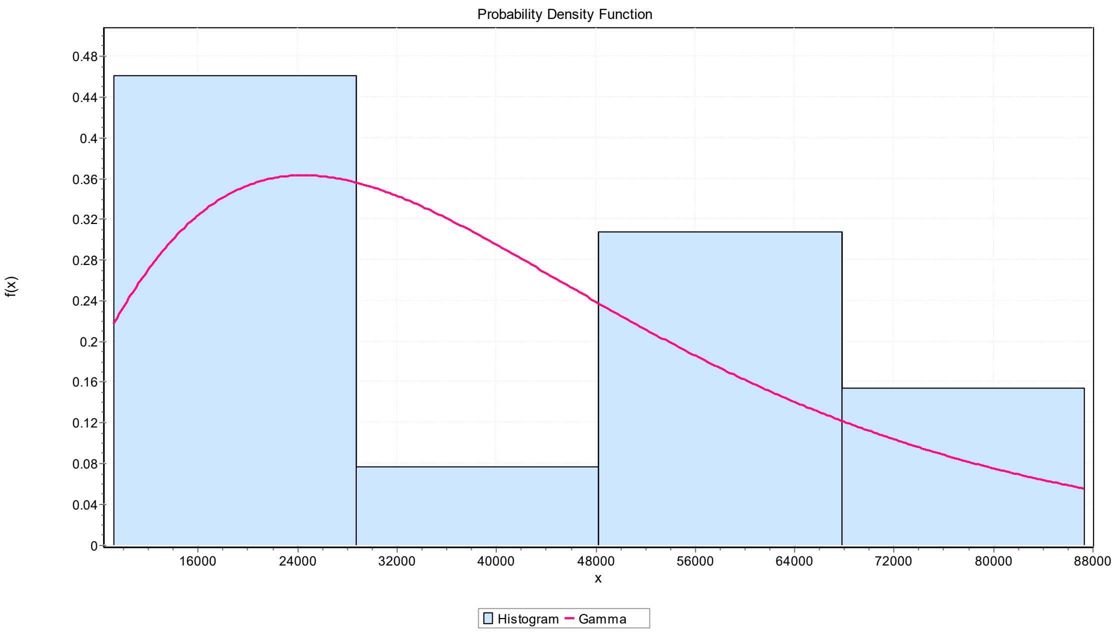

Figure 2.
The Histogram of Loss Severity Data with Fitted Gamma Distribution Curve.

6.4. Estimation of the First Raw Moments (About Zero) for the Poisson Distribution

To support the development of the aggregate loss distribution, it is necessary to compute the raw moments (moments about zero) of the Poisson distribution, which was previously determined to model the frequency of claims in the cement sector.

The probability mass function (PMF) of the Poisson distribution is defined as:

$$f(x) = \frac{\lambda . e^{-\lambda}}{x!}; x = 0,1, \dots .$$

Where λ is the average rate of claims, which also equals the variance of the distribution

Thus, the first raw moment (mean) of a Poisson distribution

The first raw moment μ_1' is calculated as the expected value:

$$\mu_1' = \sum_{x=0}^{\infty} x f(x)$$

$$\mu_1' = \sum_{x=0}^{\infty} x \frac{\lambda e^{-\lambda}}{x!}$$

$$\mu_1' = \lambda e^{-\lambda} e^{\lambda}$$

$$\mu_1' = \lambda$$

The second raw moment (mean) of a Poisson distribution is

$$\mu_2' = \sum_{x=0}^{\infty} x^2 f(x)$$

$$\mu_2' = \sum_{x=0}^{\infty} (x(x-1) + x) \frac{\lambda^x e^{-\lambda}}{x!}$$

$$\mu_2' = \lambda^2 e^{-\lambda} \sum_{x=2}^{\infty} x \frac{\lambda^{x-2}}{(x-2)!} + \lambda$$

$$\mu_2' = \lambda^2 e^{-\lambda} e^{\lambda} + \lambda$$

$$\mu_2' = \lambda^2 + \lambda$$

The third raw moment (mean) of a Poisson distribution is

$$\mu_3' = \sum_{x=0}^{\infty} x^3 f(x)$$

$$\mu_3' = \sum_{x=0}^{\infty} (x(x-1)(x-2) + 3x(x-1) + x) \frac{\lambda^x e^{-\lambda}}{x!}$$

$$\mu_3' = \lambda^3 + 3\lambda^2 + \lambda$$

The Fourth raw moment (mean) of a Poisson distribution is

$$\mu_4' = \sum_{x=0}^{\propto} x^3 f(x)$$

$$\mu_4' = \sum_{x=0}^{\propto} (x(x-1)(x-2)(x-3) + 6x(x-1)(x-2) + 7x(x-1) + x)\frac{\lambda^x e^{-\lambda}}{x!}$$

$$\mu_4' = \lambda^4 + 6\lambda^3 + 7\lambda^2 + \lambda$$

Using *PTC Mathcad Express Prime 10*, the first four raw moments (moments about zero) of the Poisson distribution were calculated as follows

The first raw moment $\lambda = 50.231$ $\mu_1' = n1 = 50.23$

The second raw moment $\mu_2' = \lambda^2 + \lambda$ $\mu_2' = n2 = 2.573 * 10^3$

The third raw moment $\mu_3' = \lambda^3 + \lambda^2 + \lambda$ $\mu_3' = n3 = 1.344 * 10^5$

The fourth raw moment $\mu_4' = \lambda^4 + 6\lambda^3 + 7\lambda^2 + \lambda$ $\mu_4' = n4 = 7.144 * 10^6$

6.5. Calculation of the First Four Raw Moments of the Gamma Distribution

To compute the first four raw moments (moments about zero) of the Gamma distribution, we begin with its probability density function (PDF), Klugman, et al. [26]; Lee and Lin [28] & Papoulis and Pillai [31].
which is given by:

$$f(x) = \frac{\alpha^{\beta}}{\;} e^{-\alpha\beta} x^{\beta-1} \ : \quad 0 \le x \le \propto$$

- α is the shape parameter
- β is the scale parameter
- Γ(α) is the gamma function evaluated at α

The first four raw moments of the Gamma distribution can be derived using the general formula for the raw moments as follows:

$$\mu_r' = \frac{\Gamma(r+\beta)}{\Gamma(\beta)\ \alpha^r}$$

By substituting r=1,2,3,4 into the raw moment function, the first four raw moments of the Gamma distribution can be calculated as follows

The first raw moment $\mu_1' = \frac{\Gamma(\beta+1)}{\Gamma(\beta)\alpha^1}$ $\mu_1' = \frac{\beta}{\alpha}$

The second raw moment $\mu_2' = \frac{\Gamma(\beta+2)}{\Gamma(\beta)\alpha^2}$ $\mu_2' = \frac{\beta(\beta+1)}{\alpha^2}$

The third raw moment $\mu_3' = \frac{\Gamma(\beta+3)}{\Gamma(\beta)\alpha^3}$ $\mu_3' = \frac{\beta(\beta+1)(\beta+2)}{\alpha^3}$

The fourth raw moment $\mu_4' = \frac{\Gamma(\beta+4)}{\Gamma(\beta)\alpha^4}$ $\mu_4' = \frac{\beta(\beta+1)(\beta+2)(\beta+3)}{\alpha^4}$

Using Mathcad, the first four raw moments (moments about zero) of the Gamma distribution were computed as follows**:**

□=16802.0=shape & □=2.4588=scale

The first raw moment $\mu_1' = \frac{\beta}{\alpha}$ $\mu_1' = s1 = 6.833 * 10^3$

The second raw moment $\mu_2' = \frac{\beta(\beta+1)}{\alpha^2}$ $\mu_2' = s2 = 4.67 * 10^7$

The Third raw moment $\mu_3' = \frac{\beta(\beta+1)(\beta+2)}{\alpha^3}$ $\mu_3' = s3 = 3.191 * 10^{11}$

The fourth raw moment $\mu_4' = \frac{\beta(\beta+1)(\beta+2)(\beta+3)}{\alpha^4}$ $\mu_4' = s4 = 2.181 * 10^{15}$

6.6. Calculation of the First Four Compound Moments

The first four compound moments describe the characteristics of the aggregate annual loss distribution, which arises from the convolution of two stochastic components: the claim frequency distribution and the claim severity distribution. In this study, claim frequency is assumed to follow a Poisson distribution, while claim severity is modeled using a Gamma distribution.

This compound modeling approach is widely used in actuarial science and insurance analytics to assess aggregate risk and determine appropriate retention levels and reinsurance strategies [32, 33]. Specifically, the total loss distribution is

constructed using the convolution of the frequency and severity distributions, allowing for the derivation of key statistical measures such as the mean, variance, skewness, and kurtosis of total annual losses [34, 35].
The following notation will be used for the moments of the claim frequency variable N:
$M_1 = mn_1 . ms1$
$M_1 = \mathbf{3.432}$
$M_2 = (ms_1)^2 . mn_2 + mn_1 . ms_2$
$M_2 = \mathbf{1.225*10^{11}}$
$M_3 = (ms_1)^3 . mn_3 + mn_1 . ms_3 + 3\, ms_1 . ms_2 . mn_2$
$M_3 = 4.536*10$
$M_4 = (ms_1)^4 . mn_4 + mn1 . ns_4 + 4\, ns_1 . ns_3 . mn_2 + 6(ms_1)^2 . ms_2 . (mn_1 . mn_2 + mn_3) + 3(ms_2)^2 . [(mn_1)^2 - mn_1 + mn_2]$
$M_4 = 1.908*10^2$
Where:
M1M_1M1 denotes the first moment of the aggregate loss distribution,
M2M_2M2 denotes the second moment of the aggregate loss distribution,
M3M_3M3 denotes the third moment of the aggregate loss distribution,
M4M_4M4 denotes the fourth moment of the aggregate loss distribution

The four central moments will be used to compute the skewness and kurtosis coefficients, which are then applied in determining the Pearson coefficient. This facilitates the identification of the appropriate distribution for the aggregate loss, resulting from the combination of the discrete and continuous distributions, through the application of Karl Pearson's differential equation: -

$$K = \frac{\beta_1(\beta 2 + 3)^2}{4[(2\beta_2 - 3\beta_1 - 6)(4\beta_2 - 3\beta_1)}$$

The skewness coefficient is: $\beta_1 = \frac{M_3}{(M_2)^{1.5}}$
The measurement of kurtosis is: $\beta_2 = \frac{M_4}{(M_2)^2}$

6.7. Calculation of Skewness and Kurtosis from the Compound Moments

After obtaining the first four central moments of the aggregate loss distribution, both skewness and kurtosis of the aggregate loss can be calculated using the following relationships, implemented via Mathcad:

Skewness coefficient $\beta_1 = \left(\frac{M3}{(M2)^{1.5}}\right) = 1.058$

Kurtosis coefficient $\beta_2 = \frac{M4}{(M2)^2} = 1.272$

After obtaining the measures of skewness and kurtosis, the value of MPY (Moment Pearson's Y) is estimated using the compound moments of the probability distributions.

6.8. Estimating the Maximum Probable Yearly Aggregate Loss (MPY)

The Maximum Probable Yearly Aggregate Loss (MPY) refers to the largest total loss that the subject of risk may face during the year with a certain probability. It is important to note that the aggregate loss depends on both the number of claims and the expected severity of each loss. Therefore, to estimate the expected aggregate loss, it is necessary to identify the appropriate probability distributions for both claim frequency and claim severity, as they are considered random variables. To obtain the total loss, it is necessary to combine the frequency distribution with the severity distribution to derive a mathematical expression that can be used to estimate the distribution of aggregate losses [36].

Expected Value of the Aggregate Loss Distribution $\mu_s = \overline{x}_1 . \overline{x}_2$

Variance of the Aggregate Loss Distribution $\sigma_s^2 = \sigma_1^2 . (\overline{x_2})^2 + \sigma_2^2 . \overline{x_1}$

Where:
The expected value of the probability distribution of the number of claims is $\overline{x_1}$
The expected value of the probability distribution of the severity of losses is $\overline{x_2}$
The variance of the probability distribution of the number of claims is σ_1^2
The variance of the probability distribution of the severity of losses is σ_2^2

There are several methods for estimating the Maximum Probable Yearly Aggregate Loss (MPY), such as the method of approximating to a normal distribution. However, since many losses exhibit skewness in their severity distribution, this makes the normal approximation inaccurate for data with skewed distributions. Therefore, in this application, we will use Chebyshev's inequality as follows [37].

$$MPY = \mu_S + \sigma_S . K$$

$$K = \sqrt{\frac{1}{\alpha}}$$

For determining the value of MPY, the data is skewed to the right, as evidenced by the skewness coefficient being greater than zero (i.e., positive), the *Chebyshev approximation method* will be applied as follows: Yang, et al. [38]

$$MPY = \mu_S + \sigma_S . K$$

$$K = \sqrt{\frac{1}{\alpha}} = \sqrt{\frac{1}{0.05}} = 4.472$$

$\mu_s = M1 = 3.432*10^5$

$\sigma_S^2 = M_2 = 1.225*10^{11}$

$$MPY = M1 + K\sqrt{M2} \ \& \quad MPY = 1.908 \times 10^6$$

6.9. Pearson's Method (Karl Pearson)

The appropriate probability distribution is determined by relying on the Mathcad program. The value of K depends on the measures of skewness and kurtosis, as Pearson developed eleven distributions, known as Pearson curves or the Pearson family. These curves are defined based on the values of skewness and kurtosis, and these values are then substituted into the following equation:

$$K = \frac{\beta_1 . (\beta_2 + 3)^2}{4. (2\beta_2 - 3\beta_1 - 6). (4\beta_2 - 3\beta_1)} \therefore K = -0.381$$

It is found that the value of Pearson's equation **K** is negative. Therefore, the probability distribution curve for the aggregate losses of cement companies in the Saudi insurance market during the study period follows the first shape of the Pearson curves, and the probability density function takes the following form: Lahcene [39].

$$f(x) = c(1 + \frac{x}{a_1})^{m_1}(1 - \frac{x}{a_2})^{m_2}; -a_1 \prec x \prec a_2 \& m1, m2 \succ 0$$

Where: -

X: A continuous random variable representing the total value of losses.

a1: The minimum loss, which will be replaced by zero.

a2: The maximum loss, representing the Maximum Probable Yearly Aggregate Loss (MPY).

c: A constant value.

m1, m2: The parameters of the distribution for the probability density function.

The probability density function may take the following form:

$$f(x) = c(x + a_1)^{m_1}(1a_2 - x)^{m_2}; -a_1 \prec x \prec a_2 \& m1, m2 \succ 0$$

Since the probability density function is unknown, it will be transformed into a known probability density function through substituting the value of a1 with zero, the probability density function becomes:

$$f(x) = c(x)^{m_1}(a_2 - x)^{m_2}; 0 \prec x \prec \ a_2 \& m1, m2 \succ 0$$

$$f(x) = c(x)^{m_1} a_2{}^{m_2}(1 - \frac{x}{a_2})^{m_2}$$

Consider that $y = \frac{x}{a_2} \therefore x = ya_2$

Finding the Jacobian transformation $|J|$ for this quantity or the transformation coefficient.

$$|J| = \left|\frac{dx}{dy}\right| = a_2$$

Then, the probability density function for the random variable y is found in the following form:

$$f(y) = f(x)\left|\frac{dx}{dy}\right|$$

$$f(y) = cx^{m1}(a_2)^{m_1}(1 - \frac{x}{a_2})^{m_2} a_2$$

By substituting x = ya2, the probability density function for the random variable y is obtained.

$$f(y) = c(ya_2)^{m1}(a_2)^{m_1}(1 - \frac{ya_2}{a_2})^{m_2} a_2$$

$$f(y) = c(a_2)^{m1+m2+1}(y)^{m1}(1 - y)^{m_2}; 0 \leq y \leq 1$$

The probability density function for the random variable y takes the form of a Beta function, and thus the constants of this function resemble those of the Beta function**.**

$$c = \frac{1}{\beta(m_1 + 1, m_2 + 1)} = \frac{\Gamma(m_1 + m_2 + 2)}{\Gamma(m_1 + 1)\Gamma(m_2 + 1)}$$

Finding the probability distribution f(x)

$$f(x) = f(y)\left|\frac{dy}{dx}\right|$$

$$f(x) = \frac{\Gamma\,(m_1 + m_2 + 2)}{\Gamma\,(m_1 + 1)\Gamma\,(m_2 + 1)} (\frac{x}{a_2})^{m1} (1 - \frac{x}{a_2})^{m_2} \frac{1}{a_2}; 0 \le x \le a_2$$

By using the transformation x = ya2, the parameters of the probability distribution for the function f(x) can be found. Finding the first moment around zero.

$$M11 = 10^5 * 3.432$$
$$M22 = 1.225 * 10^{11}$$
$$M33 = 4.536 * 10^{16}$$

$$M1 = a_2 (\frac{m_1 + 1}{m_1 + m_2 + 2})$$

$$M2 = {a_2}^2 \frac{(m_1 + 1)(m_2 + 1)}{(m_1 + m_2 + 3)(m_1 + m_2 + 2)^2}$$

–

$$M3 = {a_2}^3 \left[\frac{(m_1 + 1)(m_1 + 2)(m_1 + 3)}{(m_1 + m_2 + 4)(m_1 + m_2 + 3)(m_1 + m_2 + 2)}\right]$$
$$-3\left[\frac{(m_1 + 1)(m_1 + 2)}{(m_1 + m_2 + 3)(m_1 + m_2 + 2)^2}\right] \cdot \left[\frac{(m_1 + 1)}{(m_1 + m_2 + 2)}\right]$$
$$+2\left[\frac{(m_1 + 1)}{(m_1 + m_2 + 2)}\right]^3$$

The results from the Mathcad program provide the parameter values for the annual aggregate loss probability distribution as follows:

$a2 = 6*10^5$ & $m1 = 4$ & $m2 = 6$

Therefore, the probability density function for the distribution **f(x)** is obtained, and by substituting the parameters of the Beta distribution:

$$f(x) = \frac{\Gamma(m_1 + m_2 + 2)}{\Gamma(m_1 + 1)\Gamma(m_2 + 1)} (\frac{x}{a_2})^{m1} (1 - \frac{x}{a_2})^{m_2} \frac{1}{a_2}$$

$$f(x) = \frac{\Gamma(m_1 + m_2 + 2)}{\Gamma(m_1 + 1)\Gamma(m_2 + 1)} (\frac{x}{6 * 10^5})^4 (1 - \frac{x}{6 * 10^5})^6 \frac{1}{6 * 10^5}$$

$$f(x) = 2310 (\frac{x}{6 * 10^5})^4 (1 - \frac{x}{6 * 10^5})^6 \frac{1}{6 * 10^5}$$

Finding the retention probability: Based on the data obtained from the cement companies, where the premium value for the last year was 405,311, the retention probability is determined using the following equation

$$f(x) = 2310 (\frac{x}{6 * 10^5})^4 (1 - \frac{x}{6 * 10^5})^6 \frac{1}{6 * 10^5}$$

$$F(405311) = \int_0^{405311} f(x)dx$$

By performing the integration to find the probability using the Mathcad program

$$F(405311) = \int_0^{405311} f(x)dx = 0.9666$$

To find the *truncated distribution function*, we use the probability density function for the aggregate losses f(x), after truncating it from the right side. This allows us to determine the appropriate retention limit. The function is truncated at the Maximum Probable Yearly Aggregate Loss (MPY), which is 1.908 × 10^6. Therefore, the truncated probability distribution function for the annual aggregate losses become as follows

$m1 = 4$ & $m2 = 6$ & $a2 = 6*10^5$ & $MPY = 1.908 \times 10^6$

$$f_1(x) = \frac{(\frac{x}{a_2})^{m1} (1 - \frac{x}{a_2})^{m_2}}{\int_0^{mpy} (\frac{x}{a_2})^{m1} (1 - \frac{x}{a_2})^{m_2} d(mpy)}$$

$$\int_0^q f_1(x)dx = 0.9666$$

To find the appropriate retention limit, denoted as q, the truncated probability distribution function for the annual aggregate losses will be equated to the retention probability 0.9666, as follows

$$\int_0^T f_1(x)dx = 0.9666$$

Then, using the *Mathcad program*, the value q = 5.676 × 10^5 was determined, which represents the loss that the cement companies can bear during the upcoming year. The value of q represents a proportion of the Maximum Probable Yearly Aggregate Loss.

q = 5.676 × 10^5

Calculating the appropriate retention limit (z) by dividing the value of q by MPY, as follows:

$$Z = q/mpy = 5.676*10^5 / 1.908 \times 10^6 = 0.2974$$

Based on the data and analysis the optimal retention level for the next year of the cement companies during the upcoming year is 0.2974.

7. Results and Conclusion

This study investigated the optimal risk retention level for the cement industry in Saudi Arabia using data from 2012 to 2024. Employing Poisson and Gamma distributions, along with compound loss modeling, we estimated loss frequency, severity, and the Maximum Probable Yearly Aggregate Loss (MPY). Through this analysis, we found that the optimal retention level is 29.4%, meaning the most efficient strategy is to retain nearly one-third of the risk and transfer the remainder (70.6%) to insurers.

These findings support a hybrid risk management approach as self-insurance and commercial insurance work in tandem to balance cost efficiency and protection against catastrophic losses. This aligns with the conclusions of Nocco and Stulz [1] and Foster [2] who emphasized that effective enterprise risk management involves determining a firm's risk appetite and optimizing retention accordingly. Similarly, Bourgeon and Picard [21] highlighted that self-insurance and market insurance can complement each other, particularly when losses vary in severity and insurers apply significant loading factors.

Thus, for Saudi cement firms operating in a risk-intensive environment, a blended strategy allows for cost control, risk diversification, and improved financial resilience, ensuring long-term sustainability in line with global best practices.

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