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Study of viscous fluid turbulent stationary flow in the transition section of the inlet section of a circular cylindrical pipe

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Abstract

Because of the fluid's viscosity, the velocity field of a fluid passing through a pipe deforms. The velocity profile is rearranged to a fully developed velocity profile of the completely developed area as a result of the viscosity of the incoming fluid particles adhering to the pipe's fixed wall. An essential component of hydropneumatic automated equipment is the hydrodynamic entrance area, where the velocity profile is produced or developed and stays constant in the completely developed region. The exact operation of hydraulic units depends on its accurate design, which necessitates a thorough investigation of the hydrodynamic phenomena taking place in the hydrodynamic entrance region. A boundary problem was developed based on the boundary layer equations, and the Prandtl method for accounting friction forces in turbulent flow was used to determine the friction forces that arise between the layers. The hydrodynamic characteristics of the events taking place in the transition area were ascertained by developing a method for integrating the boundary problem. The resulting analytical answers were used in computerized experimental investigations, and velocity change graphs were created to summarize the findings. The resulting graphs make it possible to identify the pattern of speed changes in hydrodynamic entrance region and obtain quantitative indicators necessary for the design of hydraulic automatic unitst.

Keywords: Hydrodynamic entrance region, Stationary flow, Turbulent regime, Velocity.

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1. Introduction

Studies of velocity distribution patterns in closed channel transitions (entrance and exit, sudden widening and narrowing of the cut, etc.) are issues of significant interest. In addition to the fact that particles near stationary walls perform a decelerating motion, while particles near the axis acquire acceleration and perform an accelerating motion, conditions are created for a change in the velocity distribution in the live section. The area of velocity rearrangement, where the fluid velocities are rearranged to the velocity distribution pattern of the stabilized area of the closed channels, is called the transition area. The results of the study of hydrodynamic phenomena occurring in transition areas determine the key issues of accurate design of transition areas of fluid-carrying channels. Therefore, the problem under discussion is relevant and has important practical significance.

The study of hydrodynamic phenomena occurring in fluid-carrying channels is based on the equations of motion of incompressible fluids, for the possible integration of which assumptions are made that limit the limits of their applicability. Very often, the proposed mathematical model of the problem reveals the picture of the ongoing physical phenomenon with acceptable accuracy. This makes it possible to correctly construct a given transition section, ensuring the smooth, uninterrupted operation of the structure. The study of turbulent stationary hydromechanical phenomena in the transition sections of pressure pipes is one of the most complex problems in hydromechanics, where the change in quantities, in addition to the position of a point in the live section, also depends on the coordinate of the path.

2. Literature Review and Problem Statement

Based on the results of theoretical studies of the unsteady motion of an incompressible fluid in a round pipe with an arbitrary change in the kinematic viscosity coefficient over time [1] patterns of change in velocity at the inlet section were identified. The calculation results were compared with the results of numerical integration, the difference is 1%. A similar problem for laminar motion for a Newtonian fluid was investigated in Reci, et al. [2]. A discrepancy between analytical and numerical solutions was revealed.

An analysis of the dynamic laminar flow of a viscous fluid in a pipe was performed in Wiens and Etminan [3]. A solution was proposed, in which the wave velocity along the length changes. The effects of the change in friction stresses depending on the oscillation frequency were taken into account.An approximate method has been proposed that can be used for high-frequency calculations where limited numerical accuracy can introduce errors. A study of transient processes with a change in wave velocity along the length was conducted. Analytical solutions of laminar flow were obtained using the method of characteristics [4]. However, the laminar regime is rarely encountered in practice. Usually, the movement of viscous fluids occurs in a turbulent regime. In axisymmetric pipes, depending on the change in viscosity and pressure gradient, a method for solving a boundary value problem for identifying patterns of hydrodynamic parameters of laminar flow was developed [5]. The general solution is obtained using the finite Hankel transform method. In Sarukhanyan, et al. [6] the unsteady laminar flow of a viscous incompressible fluid at the inlet section of a round cylindrical pipe under general initial and boundary conditions was considered. Based on the research results, patterns of changes in speeds and pressure along the length of the transition section depending on time were revealed. Graphs of changes in the indicated physical quantities were constructed. An analysis of the stability of the accelerating laminar motion of a fluid in a pipe has been carried out [7]. A gradual increase in the flow velocity causes periodic instability. The causes of the increase in flow instability and engineering measures to overcome them have been investigated. In the area of sudden expansion, turbulent foci arise, the size and number of which depend on the Reynolds number [8]. Experimental studies have analyzed the mechanisms of their formation and development. Oscillatory plane-parallel thrust motion of a Newtonian fluid in the inlet area by the method of numerical integration study [9]. The calculation conducted to determine the length of the entrance section in a circular pipe [10]. The effects of the entrance velocity and pressure gradient on the length of the transition section have been revealed. At the site of sudden expansion of the pipe section, abrupt changes in the hydrodynamic parameters of the fluid occur. Experimental studies of the phenomena occurring in the area of sudden shear expansion under turbulent conditions have been studied [11]. Under turbulent mixing conditions, the energy losses of the fluid increase sharply due to the increase in the frictional stresses arising between the fluid layers. To reduce hydraulic losses, it is necessary to reduce or neutralize the effect of turbulence, which can lead to a reduction of hydraulic losses by up to 90% [12]. In conditions of sudden, symmetric and asymmetric widening of the section, numerical integration of nonlinear inhomogeneous differential equations obtained by quantitative estimates of the members of the Neiva-Stokes equations has been carried out [13].

New scaling expressions have been introduced for the stress components arising in the flow layers in the turbulent flow domain. It has been proposed to divide the stress field into four layers. A comparative analysis of the single-layer and proposed four-layer stress fields has been performed, according to which the single-layer stress model can be applied with acceptable accuracy in engineering calculations [14]. Using Prandtl's formula for the mixing path length in a smooth boundary layer, a smooth boundary turbulent flow simulation was performed [15], which resulted in the following velocity distribution law in the boundary layer. However, these generalizations are not applicable to cylindrical or smooth pipes. As a result of the application of the latest experimental x-probe, it has been proven that not all turbulent stresses approach the asymptotic state [16]. It has been shown that the turbulent shear stress approaches the flow direction faster than the normal direction. Luchini [17] showed that the mismatch between flows can be attributed to the effect of the pressure gradient and that the application of the logarithmic law is fully applicable in engineering calculations. Graphs of the change in parameters as a function of time were created after studies of the patterns of change in the hydrodynamic parameters of the flow were conducted in the transition area of the entrance to a circular cut under conditions of accelerating laminar motion of a viscous fluid [18]. A technique for integrating the boundary value issue was proposed for the situation of laminar non-stationary flow of a viscous fluid under conditions of variable fluid viscosity and pressure gradient [19]. In Gücüyen, et al. [20] the equations of motion of an incompressible fluid were modeled in regions of abrupt changes in live shear motion, which led to the revelation of patterns of changes in the flow's hydrodynamic characteristics and the provision of quantitative estimates. The patterns of changes in the hydrodynamic parameters of the flow under conditions of turbulent motion of a viscous fluid have been studied [21] the reliability of which has been confirmed by experimental studies [14].

In Liu, et al. [22] the results of studies of the transition period and turbulence are analyzed. However, despite the achievements, the results of studies on identifying the emerging turbulent stress remain relevant. Numerical modeling of turbulent flow after a sharp increase in flow rate was conducted. It was demonstrated that turbulent flow with a low Reynolds number gradually transitions to a laminar-turbulent regime. In a laminar flow, localized turbulent spots form, which grow and merge with each other, and eventually the flow becomes completely turbulent [23]. In Jozsa [24] a method for controlling turbulent flow with the aim of reducing hydraulic resistance was developed. This is achieved by using uniform and traveling wave oscillations of the wall, which reduces turbulent frictional resistance. Numerical modeling of accelerated flow demonstrated that applied accelerations can significantly dampen oscillations [25]. In Chovniuk, et al. [26] a general method for the numerical solution of the problem of unsteady flow of a viscous incompressible fluid in flat channels of arbitrary shape was developed. The mathematical model of the flow is based on the two-dimensional Navier-Stokes equations and the Poisson equation for pressure, which are solved using the finite difference method.

3. The Aim and Objectives of the Study

The aim of the study is to identify the characteristics of the pressure steady-state turbulent motion of an incompressible fluid at the inlet section of a cylindrical pipe of circular cross-section.

The following tasks are considered within the framework of this study:

- To create a mathematical model for studying the structure of hydrodynamic parameters of a turbulent flow, to develop a method for integrating a boundary value problem and to determine the pattern of change in hydrodynamic parameters, - to plot graphs of change in axial velocities in the direction of movement, to identify conditions for determining the length of the transition section.

4. Materials and Methods

4. 1. Choosing a Calculation Scheme

Let us assume that a turbulent motion of a viscous fluid occurs in an infinitely long cylindrical tube with radius R, in the initial section of which, where the origin of the oz axis is located, the velocity distribution is given by an arbitrary law, that $u = \varphi(r)$ when z=0. (Figure 1).

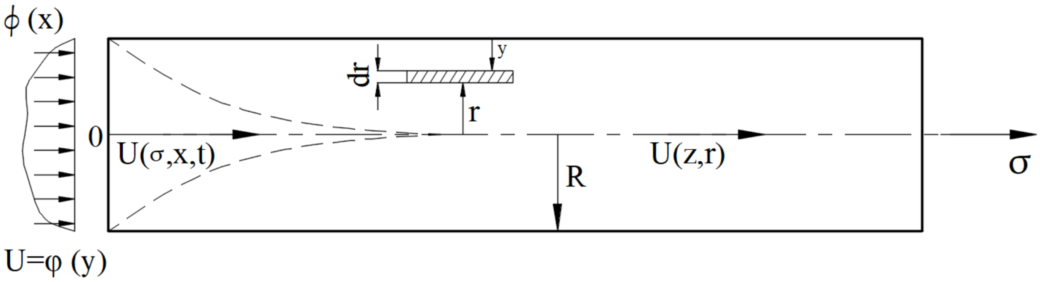

Figure 1.
Calculation diagram of the hydrodynamic entrance region.

Under these conditions, axisymmetric, isothermal motion of a viscous fluid occurs. At the entrance cut of the pipe, the velocity of the fluid moving along the velocity profile $u = \varphi(r)$ on the pipe wall becomes zero, and a deformation of the velocity profile occurs, which propagates along the length of the pipe for a certain distance. A boundary layer forms near the pipe walls, where the velocity gradient $\frac{du}{dn}$ becomes extremely large, causing the friction forces to take on very large values regardless of the viscosity coefficient. The boundary layer gradually extends from the pipe walls to encompass the entire pipe. Therefore, studies at the transition site must be performed using boundary layer equations:

To study the transition section of a cylindrical pipe with a circular cross-section, we use the boundary layer equations, which in the cylindrical coordinate system have the following form [27, 28].

$$U_z \frac{\partial U_z}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \frac{1}{\rho \cdot r}\left(r \cdot \frac{\partial \tau}{\partial r} + \tau \right), \qquad (1)$$

$$\frac{\partial U_z}{\partial z} + \frac{1}{r}\frac{\partial (U_r \cdot r)}{\partial r} = 0 \,. \qquad (2)$$

In order to integrate the resulting nonlinear inhomogeneous differential equation, let us make an assumption, according to which we replace the coefficient $\frac{\partial U_z}{\partial z}$ of the term with the average velocity of the live section, and we will have [27, 29]

$$U_z = U_0 = \frac{2}{R^2}\int_0^R \varphi(r) r \cdot dr . \tag{3}$$

After this reception, the study of the transition site is brought:

$$U_0 \frac{\partial U_z}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \frac{1}{\rho \cdot r}\left(r \cdot \frac{\partial \tau}{\partial r} + \tau \right), \tag{4}$$

$$\frac{\partial U_z}{\partial z} + \frac{1}{r}\frac{\partial (U_r \cdot r)}{\partial r} = 0 : \tag{5}$$

to the integration of equations with the following initial and boundary conditions.

$$U_z = 0, U_r = 0, \text{ at } r = R,\ z > 0 , \tag{6}$$

$$U_z = \varphi(r), \text{ at } z = 0,\ 0 \le r \le R , \tag{7}$$

Equations 4 and 5 indicate that the pressure is constant $\frac{\partial P}{\partial r} = 0$ at each point in the hydrodynamic entrance region which implies that the pressure is the same at all points of the effective-cross section and only varies while moving from one part to the next $P = P(z)$.

The shear stresses arising between the fluid layers in the turbulent regime depend on the distance from the fixed wall and the velocity gradient in the direction perpendicular to the flow. The shear stresses arising in a turbulent flow are determined according to the Prandtl assumption [27].

$$\tau = \rho \ell^2 \left(\frac{dU}{dy} \right)^2 \tag{8}$$

by the formula, where ℓ is the mixing path length which is equal to

$$\ell = \varepsilon . y \tag{9}$$

where y is the distance of the layer from the fixed wall, ε is the coefficient of proportionality, which is taken equal to 0.4 [27].

Taking into account the calculation basis of the turbulent shear stress, the turbulent motion Equation 1 at the transition section of the inlet cut, corresponding to the coordinate, will have the following form:

$$U_0 \frac{\partial U_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - \frac{1}{\rho}\left(\frac{\partial \tau}{\partial y} - \frac{\tau}{R - y} \right) \tag{10}$$

Let's introduce independent variables.

$$U(\sigma, x) = \frac{U_z}{U_0}, \quad \frac{y}{R} = x, \quad \frac{P}{P_0} = p, \quad \sigma = \frac{z}{R}, \quad -\frac{p_0}{\rho R U_0^2}\frac{\partial p}{\partial \sigma} = F(\sigma) \tag{11}$$

Taking into account formula (8), equation (10) with dimensionless variables will take the following form:

$$\frac{\partial U(\sigma, x)}{\partial \sigma} = F(\sigma) - \varepsilon^2 \left[\frac{\partial}{\partial x}\left(x \frac{\partial U}{\partial x} \right)^2 - \frac{x^2}{1 - x}\left(\frac{\partial U}{\partial x} \right)^2 \right]. \tag{12}$$

The boundary conditions for the integration of Equation 12 with dimensionless variables will be:

$$U = 0, \text{ at } x = 0,\ \sigma > 0 , \tag{13}$$

$$U(0, x) = \phi(x), \text{ at } \sigma = 0,\ 0 \le x \le 1, \quad \sigma = 0,\ x = 1 \tag{14}$$

The solution of the formulated boundary problem will allow us to reveal the pattern of changes in axial velocities in the transition section of the cylindrical pipe inlet.

4.2. Statement of the Problem and Formulation of the System of Differential

Let us find the solution of Equation 12 in the form of a sum [29].

$$U(\sigma, x) = U_1(\sigma, x) + U_2(\sigma, x) . \tag{15}$$

Here $U_1(\sigma, x)$ is the solution to the homogeneous Equation 16:

$$\frac{\partial U_1(\sigma,x)}{\partial \sigma} = -\varepsilon^2\left[\frac{\partial}{\partial x}\left(x\frac{\partial U_1}{\partial x}\right)^2 - \frac{x^2}{1-x}\left(\frac{\partial U_1}{\partial x}\right)^2\right], \tag{16}$$

and $U_2(\sigma,x)$ is the solution of the inhomogeneous Equation 12.
Solving Equation 16 in the following form

$$U_1(\sigma,x) = \exp\left(-\varepsilon^2\sigma\right)V(x) \tag{17}$$

we have

$$V(x) = \exp\left(-\varepsilon^2\sigma\right)\cdot\left[\frac{d}{dx}\left(x\frac{dV(x)}{dx}\right)^2 - \frac{x^2}{1-x}\left(\frac{dV(x)}{dx}\right)^2\right] \tag{18}$$

Let us represent Equation 18 in the form of a system:

$$\begin{cases} W(x) = x\dfrac{dV(x)}{dx}, \\ \dfrac{1}{a}V(x) = \dfrac{d}{dx}\left(W^2(x)\right) - \dfrac{W^2(x)}{1-x} \end{cases} \tag{19}$$

where $a = \exp\left(-\varepsilon^2\sigma_0\right)$.
The solution to the first equation of the system of Equation 19 according to Tikhonov and Samarskii [29] will be:

$$W(x) = C_1 + \int_1^x \xi V'(\varsigma)d\varsigma \tag{20}$$

Substituting *W(x)* into the second equation of Equation 19 for determining the function, we get

$$V'(x) + xV''(x) = xV'(x) \tag{21}$$

The solution to Equation 21 is

$$V(x) = C_1\int_{0.01}^x \frac{e^\xi}{\xi}d\xi + C_2. \tag{22}$$

Considering the boundary condition $V(0)=0$ of the problem, we get $V(0)=0$. Therefore

$$V(x) = C_1\int_{0.01}^x \frac{e^\xi}{\xi}d\xi \tag{23}$$

Let us look for the solution of Equation 12 in the form

$$U_2(x,t) = \varphi(\sigma)V(x) \tag{24}$$

which will make Equation 12 take the following form:

$$V(x)\left[\frac{\partial\varphi(\sigma)}{\partial\sigma} + \frac{\varepsilon^2}{a}\varphi^2(\sigma)\right] = F(\sigma). \tag{25}$$

Integrate both sides of the equation in the interval [0.01,1].
Denoting the below expressions

$$b = \int_{0.01}^1 V(x)dx, \quad \gamma = \frac{0.99F(\sigma)}{b} \tag{26}$$

we get

$$\frac{\partial\varphi(\sigma)}{\partial\sigma} + \frac{\varepsilon^2}{a}\varphi^2(\sigma) - \gamma = 0. \tag{27}$$

The solution to Equation 27 is

$$\varphi(\sigma) = \sqrt{\frac{a\gamma}{\varepsilon^2}}Tanh\left[\sqrt{a\varepsilon^2\gamma}\,\frac{\sigma}{a} + \sqrt{a\varepsilon^2\gamma}C_0\right]. \tag{28}$$

Substituting the obtained solutions into Equation 15 we get the general solution to the problem

$$U(\sigma,x) = \left\{\exp\left(-\varepsilon^2\sigma\right) + \sqrt{\frac{a\gamma}{\varepsilon^2}}Tanh\left[\sqrt{a\varepsilon^2\gamma}\,\frac{\sigma}{a} + \sqrt{a\varepsilon^2\gamma}C_0\right]\right\}V(x) \tag{29}$$

From the boundary condition (14) of the problem it follows that:

$$U(0,x)=\left\{1+\frac{\sqrt{a\varepsilon^2\gamma}}{\varepsilon^2}Tanh\left[\sqrt{a\varepsilon^2\gamma}C_0\right]\right\}V(x)=\phi(x). \qquad (30)$$

Considering the value of the function $V(x)$ from Equation 23 we determine the values of the coefficients $V(x)$ and γ

$$b=\int_{0.01}^{1}V(x)dx=\int_{0.01}^{1}\left[C_1\int_{0.01}^{x}\frac{e^{\xi}}{\xi}d\xi\right]dx=4.205C_1 \qquad \gamma=\frac{0.235F(\sigma)}{C_1} \qquad (31)$$

The values C_0, C_1 of the coefficients are determined by the boundary and initial conditions of the problem: integrating Equation 30 in the interval [0.01;1], we will have:

$$1+\frac{\sqrt{a\varepsilon^2\gamma}}{\varepsilon^2}Tanh\left[\sqrt{a\varepsilon^2\gamma}C_0\right]=\frac{\int_{0.01}^{1}\phi(x)dx}{b} \qquad (32)$$

Let us consider a special case of the problem when the velocity of the entering fluid is constant, therefore $U(0,x)=A_0$, we will have

$$1+\frac{\sqrt{a\varepsilon^2\gamma}}{\varepsilon^2}Tanh\left[\sqrt{a\varepsilon^2\gamma}C_0\right]=\frac{0.99}{b}A_0 \qquad (33)$$

When $\sigma=0$ we have a=1, let's assume β =0.1 and we get

$$Tanh\left[\sqrt{a\varepsilon^2\gamma}C_0\right]=0.652\sqrt{\frac{C_1}{F(\sigma)}}\left(\frac{0.235A_0}{C_1}-1\right) \qquad (34)$$

The last equation has real roots when

$$\sqrt{a\varepsilon^2\gamma}C_0=-1 \qquad (35)$$

Based on this condition, we determine the values of the constant C1 for different values of $F(\sigma)$: C_1=0.4956 when $F(\sigma)$=0.1, C_1=1.8058 when $F(\sigma)$=1. Substituting the values C_1 of the constant into Equation 29 we obtain the final solution to the problem when F(σ)=1, A_0=1

$$U(\sigma,x)=\left[\exp(-\beta\sigma)-1.142Tanh(1.142\beta\sigma-1)\right]\left(1.806\int_{0.01}^{1}\frac{e^{\varsigma}}{\varsigma}d\xi\right) \qquad (36)$$

This equation reveals the velocity distribution regularity in the entrance transition section of a cylindrical pipe, under conditions of stationary turbulent flow. The tangential stresses arising between the fluid layers corresponding to this velocity pattern will be

$$\frac{\tau}{\rho RU_0^2}=\left(1.805(\exp(-\beta\sigma)-0.689Tanh(1-0.689\beta\sigma)\exp x\right)^2 \qquad (37)$$

The obtained equations make it possible to construct graphs of the change patterns in axial velocities and tangential stresses arising between fluid layers under stationary flow conditions and to draw conclusions.

5. Graphs of Changes in Hydrodynamic Parameters of a Turbulent Steady Flow at the Hydrodynamic Entrance Region of a Circular Cylindrical Pipe

Based on the results of numerical calculations, graphs of velocity and tangential stress changes were constructed across the cross-section and along the length of the transition section. Figures 2–5 show cases where the incoming fluid velocity is constant. The graphs are constructed using Equations 36 and 37.

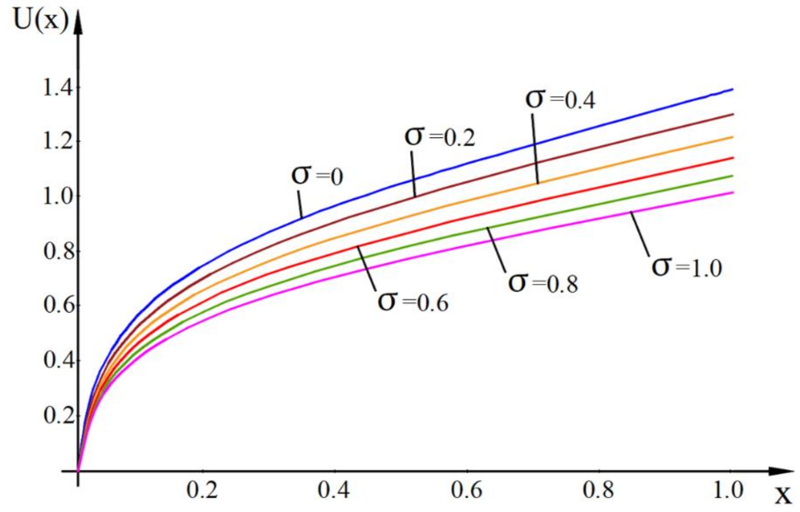

Figure 2.
Axial velocity change during steady flow of a viscous fluid at $\sigma \to (0{,}0.2{,}0.4{,}0.6{,}0.8{,}1)$, $A0 = 1$. $\beta = 0.1$, B_0=1.

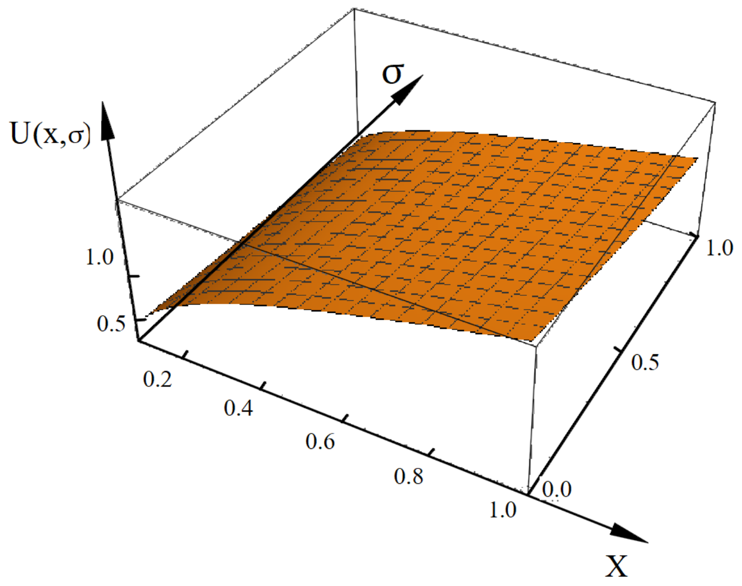

Figure 3.
Spatial view of axial velocity change in the case of stationary flow of viscous fluid.

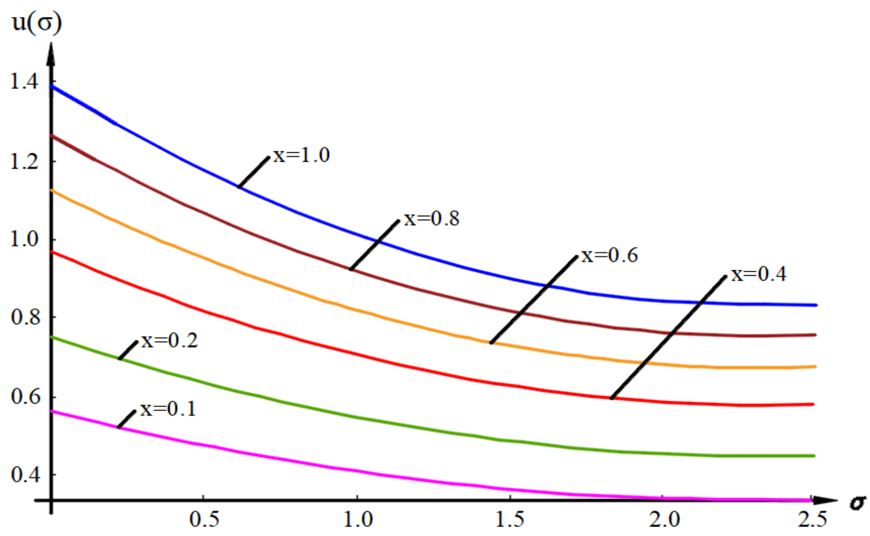

Figure 4.
Axial velocity change at the hydrodynamic entrance region $(\sigma, 0{,}2{,}5)$, $x \to (0.1{,}0.2{,}0.4{,}0.6{,}0.8{,}1)$, A_0=1, B_0=1, $\beta = 0.1$.

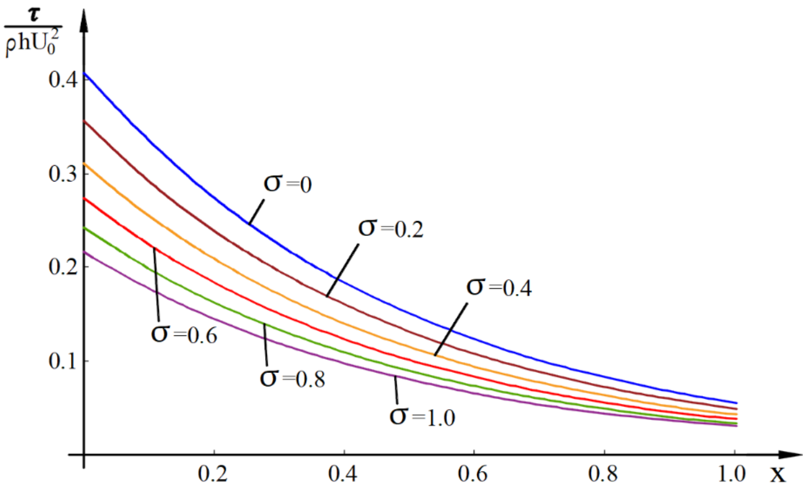

Figure 5.
Change of tangential stresses in the hydrodynamic entrance region $(x, 0,1), \sigma \rightarrow (0,0.2,0.4,0.6,0.8,1), A_0 = 1, \beta = 0.1, B_0 = 1$.

Figures 6 to 8 show graphs of changes in the hydrodynamic parameters of a viscous fluid during steady-state flow, when the fluid velocity at the entrance is zero. In this case, the change in hydrodynamic parameters is determined solely by the pressure gradient. The regularity of velocity change will be

$$U(\sigma, x) = [\exp(-\beta\sigma) - 1.313 Tanh(1.313\beta\sigma + 1)]\left(-1.366 \int_{0.01}^{1} \frac{e^{\varsigma}}{\varsigma} d\xi\right) \tag{38}$$

and tangential stresses

$$\frac{\tau}{\rho R U_0^2} = [-1.366(\exp(-0.1\sigma) - 1.313 Tanh(0.1313\sigma + 1))(\exp(1-x))]^2 \tag{39}$$

Using Equations 38 and 39 graphs of axial velocity changes along the cross-section and along the length of the entrance region are constructed at fluid zero velocity at the entrance.

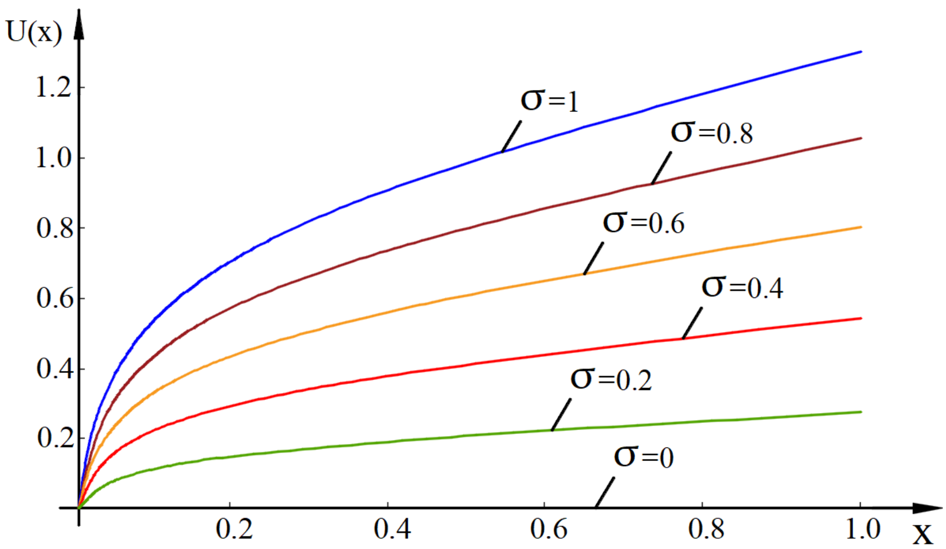

Figure 6.
Axial velocity change at the entrance when the velocity of the entering fluid is equal to zero $(x, 0.01,1)$, $\sigma \rightarrow (0,0.2,0.4,0.6,0.8,1)$, $A_0=0$, $B_0=1.0$.

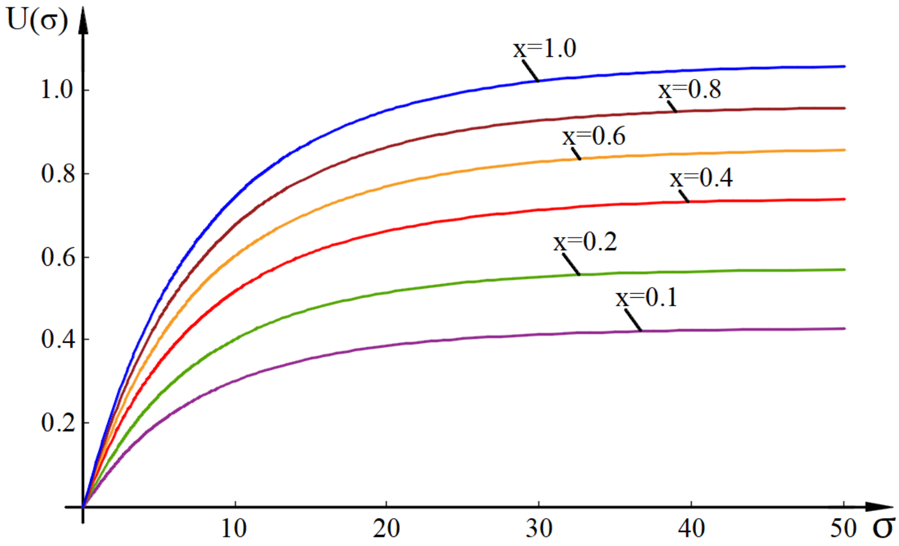

Figure 7.
Axial velocity change in the hydrodynamic entrance region. $x \rightarrow (0.1, 0.2, 0.4, 0.6, 0.8, 1), (\sigma, 0, 50)$,

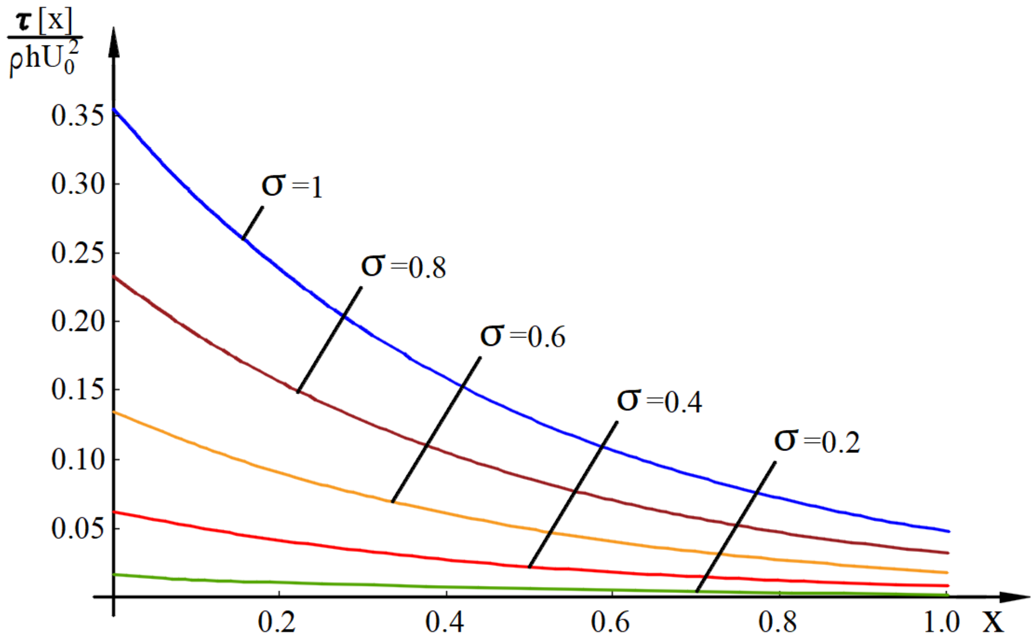

Figure 8.
Changes of tangential stresses in the cross-sections of the hydrodynamic entrance region $(x, 0, 1)$, $\sigma \rightarrow (0, 0.2, 0.4, 0.6, 0.8, 1)$.

The obtained graphs illustrate the regularity of in hydrodynamic parameters change in the entrance region. They make it possible to construct entrance region of automatic control units of hydraulic equipment ensuring their precise and smooth operation.

6. Discussion of the Findings on the Development of a Viscous Fluid Steady Flow in the Hydrodynamic Entrance Region of a Round Pipe

The laws of viscous fluid motion in the transition section of the cylindrical pipe inlet were implemented using the fundamental equations of classical hydromechanics. The tangential stresses arising between the fluid layers were taken into account using the formula proposed by Prandt the boundary conditions of the problem were defined. A boundary problem has been formulated. A method for integrating the boundary problem has been developed, as a result of which the patterns of axial velocities (36), (38) and tangential stress distributions (37), (39) have been determined at any section of the transition section. In accordance with the obtained patterns, computer experimental studies were carried out, and graphs of changes in axial velocities and tangential stresses were constructed in the cross-sections of the transition section and along its length. The results obtained are fully applicable in engineering calculations of inlets of various hydrosystems. Considering the specificity and importance of this problem, in order to ensure smooth and uninterrupted operation of hydraulic control systems, the improvement of the research of this problem is related to the calibration of the kinematic coefficient of fluid viscosity. However, the introduction of complex models of the kinematic viscosity coefficient can lead to a more complex boundary problem, the integration of which can lead to insurmountable difficulties. Analysis of the results of numerical calculations and obtained graphs allowed to determine the length of the initial section. The deviation of the axial speed in the transition section at x=1 should not exceed 1% of the unsteady speed of the stabilized section.

7. Conclusion

1. A method for studying regularities of changes in the hydrodynamic parameters of a viscous fluid in the hydrodynamic entrance region of a cylindrical pipe has been proposed. The study was conducted using the equations of motion of a viscous fluid, in which the tangential stresses arising between the fluid layers were calculated using the mixing path introduced by Prandtl. As a result, a boundary problem was formed, the integration of which determined the patterns of changes in the hydromechanical parameters of the flow in the transition section, in the case of general boundary conditions.

2. Based on the general solutions to the problem, computer experimental studies were carried out, as a result of which graphs of the patterns of changes in axial velocities and tangential stresses arising between the layers were constructed, which make it possible to identify the dynamics of the ongoing phenomena and draw conclusions. The obtained solutions and constructed graphs make it possible to correctly construct individual nodes of hydromechanical equipment.

References

[1] I. Daprà and G. Scarpi, "Unsteady flow of fluids with arbitrarily time-dependent rheological behavior," *Journal of Fluids Engineering,* vol. 139, no. 5, p. 051202, 2017. https://doi.org/10.1115/1.4035637

[2] A. Reci, A. Sederman, and L. Gladden, "Experimental evidence of velocity profile inversion in developing laminar flow using magnetic resonance velocimetry," *Journal of Fluid Mechanics,* vol. 851, pp. 545-557, 2018. https://doi.org/10.1017/jfm.2018.512

[3] T. Wiens and E. Etminan, "An analytical solution for unsteady laminar flow in tubes with a tapered wall thickness," *Fluids,* vol. 6, no. 5, p. 170, 2021. https://doi.org/10.3390/fluids6050170

[4] A. Sarukhanyan, Y. Vardanyan, P. Baljyan, and G. Vermishyan, "Pattern identification of the non-stationary laminar flow of a viscous fluid in the round pipe inlet section," *Eastern-European Journal of Enterprise Technologies,* vol. 2, no. 7 (122), pp. 33-42, 2023. https://doi.org/10.15587/1729-4061.2023.278001

[5] A. Sarukhanyan, G. Vermishyan, and H. Kelejyan, "Plane-parallel laminar flow of viscous fluid in the transition zone of the inlet section," *Journal of Architectural and Engineering Research,* vol. 4, pp. 75-85, 2023. https://doi.org/10.54338/27382656-2023.4-008

[6] A. Sarukhanyan, A. Vartanyan, G. Vermishyan, and V. Tokmajyan, "The study of hydrodynamic processes occurring on transition of sudden expanding of hydraulic section of plane-parallel full pipe flow," *TEM Journal,* vol. 9, no. 4, pp. 1494-1501, 2020. https://doi.org/10.18421/TEM94-23

[7] A. Kannaiyan, S. Natarajan, and B. Vinoth, "Stability of a laminar pipe flow subjected to a step-like increase in the flow rate," *Physics of Fluids,* vol. 34, no. 6, p. 064102, 2022. https://doi.org/10.1063/5.0090337

[8] B. Lebon, J. Peixinho, S. Ishizaka, and Y. Tasaka, "Subcritical transition to turbulence in a sudden circular pipe expansion," *Journal of Fluid Mechanics,* vol. 849, pp. 340-354, 2018. https://doi.org/10.1017/jfm.2018.421

[9] G. Shajari, M. Abbasi, and M. K. Jamei, "Entrance length of oscillatory flows in parallel plate microchannels," *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science,* vol. 235, no. 19, pp. 3833-3843, 2021. https://doi.org/10.1177/0954406220968125

[10] F. T. Najim, "Effect of inlet velocity on the entrance length of laminar and turbulent flow in a circular pipe," *Babylonian Journal of Mechanical Engineering,* vol. 2024, pp. 81-90, 2024. https://doi.org/10.58496/BJME/2024/010

[11] M. Wong, L. C. Sheng, C. N. Azwadi, and G. Hashim, "Numerical study of turbulent flow in pipe with sudden expansion," *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences,* vol. 6, no. 1, pp. 34-48, 2015.

[12] J. Kühnen *et al.*, "Destabilizing turbulence in pipe flow," *Nature Physics,* vol. 14, no. 4, pp. 386-390, 2018. https://doi.org/10.1038/s41567-017-0018-3

[13] T. Hawa and Z. Rusak, "Viscous flow in a slightly asymmetric channel with a sudden expansion," *Physics of Fluids,* vol. 12, no. 9, pp. 2257-2267, 2000. https://doi.org/10.1063/1.1287610

[14] X. Chen, F. Hussain, and Z.-S. She, "Quantifying wall turbulence via a symmetry approach. Part 2. Reynolds stresses," *Journal of Fluid Mechanics,* vol. 850, pp. 401-438, 2018.

[15] B.-H. Sun, "Closed form solution of plane-parallel turbulent flow along an unbounded plane surface," *Preprints,* vol. 2021, p. 2021110008, 2022. https://doi.org/10.20944/preprints202111.0008.v4

[16] R. Baidya, J. Philip, N. Hutchins, J. Monty, and I. Marusic, "Distance-from-the-wall scaling of turbulent motions in wall-bounded flows," *Physics of Fluids,* vol. 29, no. 2, p. 020712, 2017. https://doi.org/10.1063/1.4974354

[17] P. Luchini, "Universality of the turbulent velocity profile," *Physical review letters,* vol. 118, no. 22, p. 224501, 2017. https://doi.org/10.1103/PhysRevLett.118.224501

[18] L. Y. Ainola and Ruustal, "Flow development at the inlet section of a round pipe during accelerating fluid movement," in *Proceedings of the Tallinn Polytechnic Institute, Tallinn*, 1985.

[19] A. E. Vardy and J. M. Brown, "Laminar pipe flow with time-dependent viscosity," *Journal of Hydroinformatics,* vol. 13, no. 4, pp. 729-740, 2011. https://doi.org/10.2166/hydro.2010.073

[20] E. Gücüyen, R. T. Erdem, and Ü. Gökkuş, "Numerical modelling of sudden contraction in pipe flow," *Sigma Journal of Engineering and Natural Sciences,* vol. 37, no. 3, pp. 903-916, 2019.

[21] Z.-S. She, X. Chen, and F. Hussain, "Quantifying wall turbulence via a symmetry approach: a Lie group theory," *Journal of Fluid Mechanics,* vol. 827, pp. 322-356, 2017. https://doi.org/10.1017/jfm.2017.464

[22] L. Liu, S. N. Gadde, and R. J. Stevens, "Universal wind profile for conventionally neutral atmospheric boundary layers," *Physical review letters,* vol. 126, no. 10, p. 104502, 2021.

[23] S. He and M. Seddighi, "Turbulence in transient channel flow," *Journal of Fluid Mechanics,* vol. 715, pp. 60-102, 2013. https://doi.org/10.1017/jfm.2012.498

[24] T. I. Jozsa, "Analytical solutions of incompressible laminar channel and pipe flows driven by in-plane wall oscillations," *Physics of Fluids*, vol. 31, no. 8, p. 083605, 2019. https://doi.org/10.1063/1.5104356
[25] M. Kharghani and M. Pasandidehfard, "Turbulence structures in accelerated flow over a flat plate with non-zero pressure gradient," *Journal of Applied Fluid Mechanics*, vol. 15, no. 2, pp. 311-324, 2022. https://doi.org/10.47176/jafm.15.02.32337
[26] V. Chovniuk, A. Kravchuk, I. Moskvitina, and I. Pefteva, "Numerical simulation of unsteady flow of a viscous incompressible liquid in plane channels of arbitrary shape of heat exchangers," *Ventilation Illumination and Heat Gas Supply*, vol. 34, pp. 41–55, 2020. https://doi.org/10.32347/2409-2606.2020.34.41-55
[27] H. Schlichting and K. Gersten, *Boundary-Layer theory*. Berlin: Springer-Verlag Berlin Heidelberg, 2017.
[28] S. M. Targ, *Main problems of the theory of lamiar flows*. Moscow: Gostekhizdat, 1951.
[29] A. N. Tikhonov and A. G. Samarskii, *Equations of mathematical physics*. Moscow: Nauka, 1999.