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Dynamics of steady-state gravity-driven inviscid flow in an open system

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Abstract

Various factors can be responsible for the flow of incompressible fluid under gravity. Torricelli's theorem gives the relationship between the efflux velocity of an incompressible, gravity-driven flow from an orifice and the height of liquid above it. The concept of the original derivation of Torricelli's theorem is limited in application because of certain inherent assumptions in the method of derivation. An alternate method of derivation is the use of Bernoulli's principle. However, its result tends towards Torricelli's flow only with some assumptions. In this study, an inherent assumption was incorporated into the conventional method of derivation to obtain an amended Torricelli's equation. This study also considers a more general approach of derivation with Bernoulli's principle, which tends to eliminate some of the limitations. The method involves the theoretical construction of gravity-driven flow from the bottom of a reservoir that is opened to atmospheric pressure. Bernoulli's equation, with the continuity equation, is applied to gravity-driven open flow. The derived equations are used to analyze the prerequisite conditions for vertical flow in an open system and the variables that affect the flow rate. It is assumed that the flow is steady, inviscid, and has one inlet port and one exit port. Findings show that the surface area ratio of discharge to upstream, which was neglected in the convectional Torricelli velocity, can influence the velocity significantly. The study shows that a high surface area ratio can be used to augment the velocity of established flow for a decreased flow height.

Keywords: Bernoulli's principle, Energy conservation, Fluid flow, Gravity-driven flow, Torricelli's law, Steady state flow.

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1. Introduction

There exist different factors that can govern the flow of a vertical, incompressible fluid under gravity. Torricelli's theorem is one law that uniquely describes the gravity induced downward flow for one upstream port and one exit port. The law gives the relationship between the efflux velocity through an orifice and the liquid's height above it. Torricelli originally derived this relationship from an experiment where a water jet issuing from a small upward-facing orifice at the bottom of a tank rises to a peak and then falls with a projectile motion [1]. He observed that the projectile motion peak is the same as the height of water in the tank. Due to the time symmetry of projectile motion, the upward velocity was taken to be equal to the downward velocity. This downward rate, however, is the speed of a body falling freely under gravity from height h and is given as $\sqrt{2gh}$. Torricelli takes the efflux velocity through the orifice to be the same as this downward value.

The limitation of Torricelli's concept is that the tank's water flow is treated as a discrete body or droplets, falling from a height as an entity rather than as a continuum. For the discrete body, its velocity is uniform within the whole entity at any point; no velocity gradient occurs within the body, and gravity law solely governs the flow rate at the terminal point. However, for a continuum flow, such as liquid flow, a velocity gradient exists, and velocity may vary at different sections of the same flow entity. Thus, in such a case, apart from the gravitational pull, the difference in velocity within the flow body can influence the exit flow rate. Although certain flow conditions may yield Torricelli's law, the derivation description does not consider all possible views. One negligence is that of the additional energy input that is applicable where the fluid velocity varies within the flow body.

Another approach used for the derivation of Torricelli's law is the energy conservation principle in gravity flow. The conservation law implies that at a steady state, the rate of potential energy lost from the column of fluid in the upstream section, $\dot{m}gh$, is equal to that of kinetic energy generated in the downstream flow, $\frac{1}{2}\dot{m}v^2$, and then solving for velocity v, as $v = \sqrt{2gh}$ [2, 3]. The theory suggests that as the liquid flows vertically down, the downstream kinetic energy harvestable at the exit equals the expended potential energy at the upstream. Again, this approach neglects the effect of the additional kinetic energy activated and present at the upstream flow. The relation is not exhaustive despite the fact that the value of the efflux velocity is considered theoretical because it applies only to inviscid fluid. The actual kinetic energy gained is less than Torricelli's theorem's corresponding theoretical values [4].

Torricelli's theorem is also derivable from the Bernoulli equation [5] with certain assumptions. The Bernoulli equation, sometimes refers to as the Bernoulli principle or law, like Torricelli's theorem, is the law of conservation of energy applied to flow [6]. However, unlike Torricelli's law, the expression is not limited to gravity flow, and the direction of flow is not necessarily downwards, towards the center of the earth. For a typical scenario, in addition to gravitational potential and kinetic energies, the Bernoulli equation adds to the energy balance, pressure energy, and a third energy component [7, 8]. The typical or simplified Bernoulli's equation gives a relationship between pressure, velocity, and elevation (potential) in a flow, with the typical assumptions that the flow is of an incompressible fluid, inviscid (frictionless) fluid, and without shaft work such as pump or turbine, between points of consideration [9, 10]. However, the Engineering or extended Bernoulli equation is used to adapt the equation to real flow by incorporating frictional loss in the form of friction head and shaft work, in the form of a pump or turbine head, where applicable [11]. Bernoulli's principle, with modifications, has wide applications in liquid, semi-liquid and gaseous flow [12, 13]. This paper considers the theoretical analysis of gravitational conduit flow, by the application of the Bernoulli principle, incorporating neglected assumptions in Torricelli's law.

2. Materials and Methods

Comparison of resulting expression from the application of Bernoulli's principle to gravity flow with the convectional Torricelli's law shows the limitations and assumptions inherent in conventional Torricelli's law. One step in the derivation of convectional Torricelli's law from the typical Bernoulli's principle is the elimination, from Bernoulli's principle, the effect of pressure component on flow. The elimination of the pressure component is achieved by using the assumption of the same magnitude of pressure acting on the upstream free surface and the downstream discharge points of consideration. This assumption is applicable to open flow, where the same magnitude of atmospheric pressure acts at the upstream and downstream sections of flow. However, it is inapplicable, for example, if the upstream section is closed to the ambient atmospheric conditions such that the pressure of gas exerted on the liquid can be varied and, as such different from the downstream discharge atmospheric pressure. Where there is an end-points difference in pressure, the balance of the difference in pressure between the upstream and downstream sections is an added factor that can influence the flow. The effect can either be positive in the downhill flow or positive in the uphill flow. When the enclosed upstream section's pressure is greater than the downstream ambient atmospheric pressure, then the effect is positive in the downhill flow. If the enclosed upstream section is lower than the ambient atmospheric pressure, that is, a partial or an absolute vacuum, the pressure balance gives a positive uphill flow against gravity and negatively affects the downhill flow velocity. When the upstream surface pressure is equal to the downstream discharge ambient pressure, the net pressure is zero and the pressure energy component does not affect the flow, and the flow simplifies the convectional Torricelli's law.

Pressure energy component is not the only factor that is neglected in the derivation of Torricelli's law from the Bernoulli principle. This can be deduced from the fact that applying the typical Bernoulli's principle to the vertical flow of inviscid fluid does not give Torricelli's law even with the elimination of the pressure component. Analysis of the Bernoulli principle on the vertical flow shows that the activated upstream kinetic energy when the flow is established is another ignored factor in Torricelli's law. The phenomenon implies that the total kinetic energy that is gained and harvested at the downstream discharge section is the sum of expended gravitational potential energy and the activated upstream kinetic energy. Thus, unlike the conventional derivation of Torricelli's law, where only the gravitational potential energy is deemed converted to downstream kinetic energy, in this case, the sum of the gravitational potential energy and the activated upstream kinetic energy is converted to the downstream discharge kinetic energy. It is only when this fact is also considered that the derivation of Torricelli's law can align with the application of Bernoulli's principle to the vertical flow of inviscid fluid. Thus, the method used involves an analysis of the flow based on the application of Bernoulli's principle to steady-state, vertical, inviscid, and open flow and determining the effect of the variables, including the hitherto neglected variables.

3. Results

3.1. Derivation of Downstream Flow Rate

The flow rate of a gravity-induced flow is derived from the conservation principle applied to a conceived gravity flow system. The flow rate is also derived from the application of Bernoulli's principle to the system. The derived flow rates are compared to Torricelli's velocity to determine its inherent limitations and assumptions.

3.1.1. Derivation of Amended Torricelli's Law from Conservation Principle

Torricelli's law's amended derivation implies that the rate of downstream kinetic energy at Point 2 equals the sum of the rate of consumption of the gravitational potential energy and the activated upstream kinetic energy at Point 1 as expressed in Equation 1 and depicted in Figure 1.

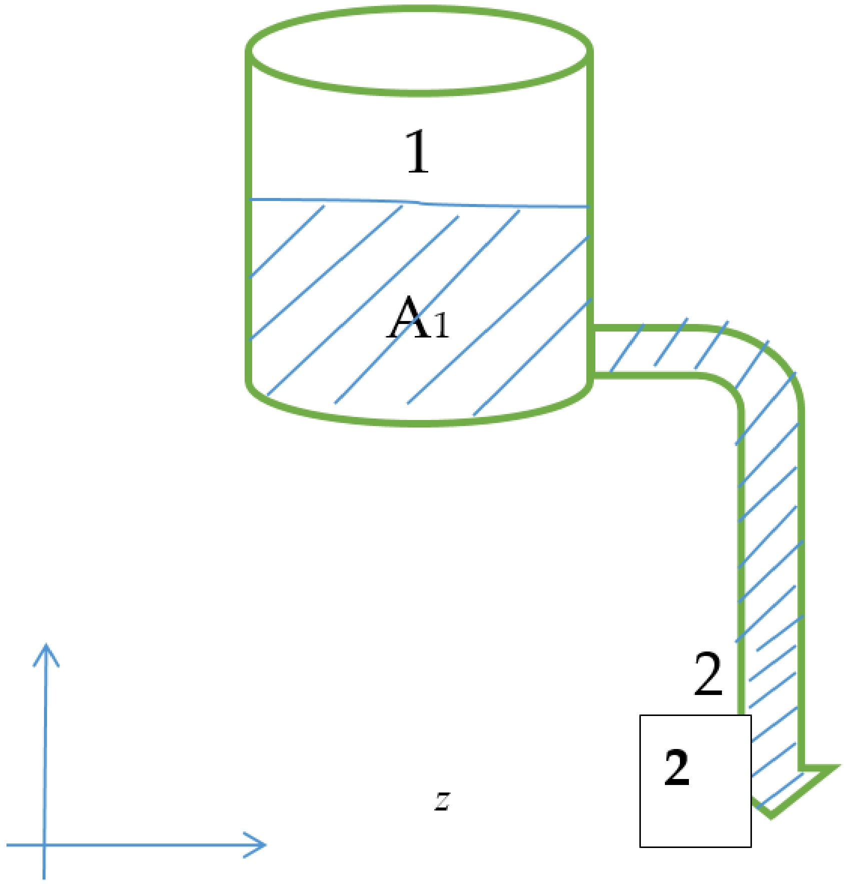

Figure 1.
Downhill flow from a reservoir.

Thus,

$$\frac{d\left(\frac{1}{2}m_2{v_2}^2\right)}{dt} = \frac{d(m_1 g\Delta z)}{dt} + \frac{d\left(\frac{1}{2}m_1{v_1}^2\right)}{dt} \quad (1)$$

Equation 1 gives the rate of kinetic energy harvestable at the discharge Region 2 as equal to the sum of the rate of change of gravitational potential energy and kinetic energy at Region 1. For a flow system maintained at a steady state of fixed height, z, giving rise to fixed upstream and downstream velocities, v_1 and v_2 respectively, Equation 1 becomes:

$$\left(g\Delta z + \frac{1}{2}{v_1}^2\right)\frac{dm_1}{dt} = \left(\frac{1}{2}{v_2}^2\right)\frac{dm_2}{dt} \quad (2)$$

By conservation principle, the total mass flow rate, $\frac{dm}{dt}$, of an incompressible fluid, is fixed at all points; thus, for single-inlet and single-exit ports $\frac{dm_1}{dt} = \frac{dm_2}{dt}$, the mass flow rate, $\frac{dm}{dt}$, can be written in terms of fluid properties and flow conditions as:

$$\frac{dm}{dt} = \rho A v = constant. \quad (3)$$

Equation 3 presents the expression for the conservation of mass, usually termed the continuity equation. Given that the subscript 1 and 2 denote the upstream (entry) and downstream (exit) positions, respectively, then the mass flow rate at the end-points, leading to the continuity equation for the incompressible flow, may be written as:

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \quad (4)$$

where ρ is the fluid's density, v is its velocity and A is the cross-sectional area.
Density is constant for incompressible flow and eliminating $\boldsymbol{v_1}$ from Equation 4 gives:

$$\boldsymbol{v_1} = \frac{\boldsymbol{A_2 v_2}}{\boldsymbol{A_1}} \quad (5)$$

Substituting $\boldsymbol{v_1}$ into Equation 2 gives:

$$\left[\frac{1}{2}{v_2}^2\right] = \left[gz + \frac{1}{2}\left(\frac{\boldsymbol{A_2 v_2}}{\boldsymbol{A_1}}\right)^2\right] \quad (6)$$

Thus, the amended Torricelli's velocity, $\boldsymbol{v_{2,a}}$, is obtained from Equation 6 as:

$$v_{2,a} = \sqrt{\frac{2gz}{1-\left(\frac{A_2}{A_1}\right)^2}} \quad (7)$$

Equation 7 presents the amended Torricelli's velocity. $\frac{\boldsymbol{A_2}}{\boldsymbol{A_1}}$ is the surface area ratio of downstream to the upstream flow, also denoted here as A_r.

The derivative of the $\boldsymbol{v_{2,a}}$ with respect to z is given as:

$$v_{2,a}{}'(z) = \frac{1}{2}\left\{\sqrt{\frac{2g}{[1-(Ar)^2]z}}\right\} \quad (8)$$

Equation 8 presents the rate of change of amended Torricelli's velocity with respect to the height of a column of water, Z, at a constant surface area ratio $\frac{\boldsymbol{A_2}}{\boldsymbol{A_1}}$. The derivative of the $\boldsymbol{v_{2,a}}$ with respect to Ar is given as:

$$v_{2,a}{}'(Ar) = \frac{Ar\sqrt{2gz}}{\left(\sqrt{1-(Ar)^2}\right)^3} \quad (9)$$

Equation 9 gives the rate of change of amended Torricelli's velocity with respect to the surface area ratio, $\frac{\boldsymbol{A_2}}{\boldsymbol{A_1}}$, at constant height of column of water, Z.

3.1.2. Derivation of Amended Torricelli's Law from Bernoulli's Equation

Equation 7 is also derivable from Bernoulli's equation with the elimination of the pressure component. $V_{2a}{}'_{(z)}$
The engineering Bernoulli's equation, applicable to viscous flow and where shaft work is involved, is given as:

$$\frac{\boldsymbol{P_1}}{\boldsymbol{\rho g}} + \boldsymbol{Z_1} + \frac{\boldsymbol{v_1^2}}{\boldsymbol{2g}} = \frac{\boldsymbol{P_2}}{\boldsymbol{\rho g}} + \boldsymbol{Z_2} + \frac{\boldsymbol{v_2^2}}{\boldsymbol{2g}} + h_f - h_p + h_t \quad (10)$$

When the Bernoulli equation is expressed in Equation 10, each of the terms of the equation is referred to as head and has the unit of energy per unit weight and the dimension of length. Thus,
Z = gravitational Potential head (due to hydrostatic pressure exerted by the column of liquid)
$\frac{\boldsymbol{P}}{\boldsymbol{\rho g}}$ = Pressure head (due to the pressure of the fluid at a point.)
$\frac{\boldsymbol{v^2}}{\boldsymbol{2g}}$ = Kinetic or Velocity head
h_f = Frictional head (head loss due to friction)
h_p = Pump head (head added by pump)
h_t = Turbine head (head extracted by the turbine)
The typical Bernoulli equation, for inviscid flow without shaft work, is given as:

$$\frac{P}{\rho g} + Z + \frac{v^2}{2g} = Constant \quad (11)$$

Equation 11 implies that the sum of the head components is constant at any point within the flow. Thus, at two different points, 1 and 2, such as end-points, within the flow system, the Bernoulli equation is written as:

$$\frac{\boldsymbol{P_1}}{\boldsymbol{\rho g}} + \boldsymbol{Z_1} + \frac{\boldsymbol{v_1^2}}{\boldsymbol{2g}} = \frac{\boldsymbol{P_2}}{\boldsymbol{\rho g}} + \boldsymbol{Z_2} + \frac{\boldsymbol{v_2^2}}{\boldsymbol{2g}} \quad (12)$$

In the conventional derivation of Torricelli's law, there is an implied assumption of open-system flow, where the upstream surface is opened to the atmosphere and the downstream discharges into the atmosphere, such that both ends are at atmospheric pressure and $P_1 = P_2$. With this assumption, the pressure heads are equal and cancel out on both sides of Bernoulli's equation in Equation 12. For the open-system flow, substituting v_1 from the continuity equation in Equation 5, the Bernoulli equation in Equation 12 simplifies the same amended Torricelli's velocity, $\boldsymbol{v_2}$, given in Equation 7.

The amended Torricelli's law in Equation 7 gives the conventional law only when $\frac{\boldsymbol{A_2}}{\boldsymbol{A_1}}$ is very small relative to 1, i.e. $\frac{\boldsymbol{A_2}}{\boldsymbol{A_1}} < 1$. The resulting simplification of Equation 7 yields the convectional Torricelli's law given as:

$$v_2 = \sqrt{2g\Delta z} \quad (13)$$

The ratio of the amended Torricelli's velocity to Torricelli's velocity, v_r, as obtained in Equation 7 and Equation 13, respectively is given as:

$$v_r = \frac{v_{2,a}}{v_2} = \frac{1}{\sqrt{1-\left(\frac{A_2}{A_1}\right)^2}} \quad (14)$$

The Ar-derivative of the amended Torricelli's law is given as:

$$v_r{'}_{(Ar)} = \frac{Ar}{\left(\sqrt{1-(Ar)^2}\right)^3} \tag{15}$$

3.2. Variation of Derived Velocity with Associated Variables

The amended Torricelli velocity derived in Equation 7 is plotted against z for Ar ranging from 0 to 0.8 in Figure 2. The z-derivative of the amended velocity is plotted against z for varying fixed Ar between 0 and 0.8 in Figure 3. Figure 4 shows the variation of the amended velocity with Ar for fixed values of z between 0m and 20m. Figure 5 depicts the variation of the Ar-derivative of the amended velocity with Ar for fixed values of z ranging between 4m and 20m. The ratio of amended velocity to actual Torricelli velocity is plotted against Ar for Ar ranging from 0 to 0.9 in Figure 6. The Ar-derivative of the ratio is plotted against Ar in Figure 7.

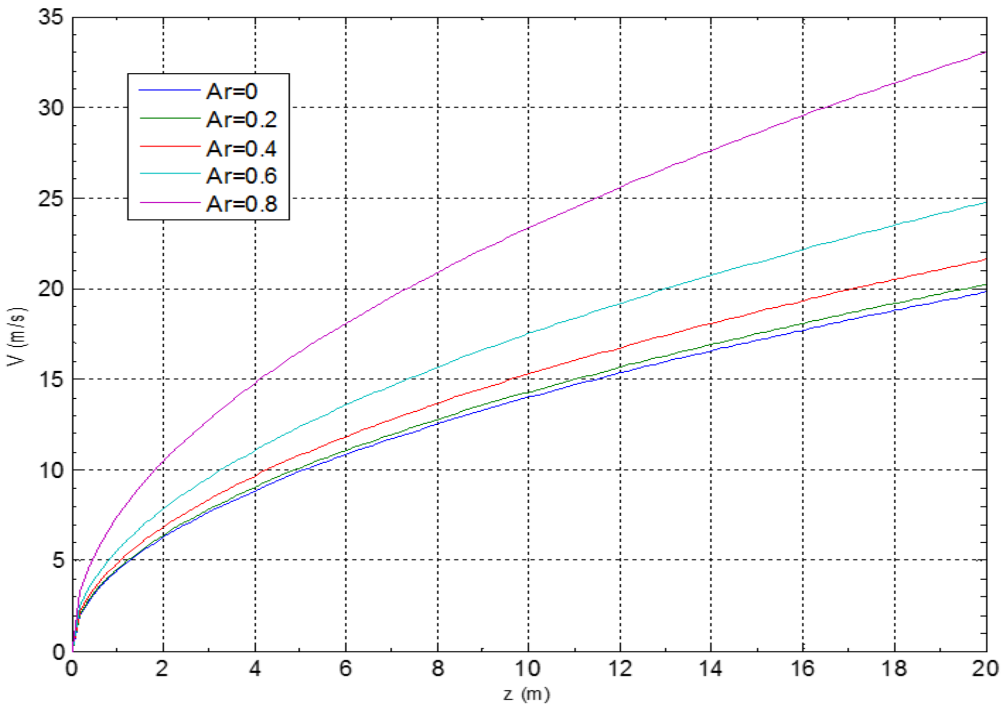

Figure 2.
Variation of amended Torricelli velocity with liquid height for Ar between 0 and 0.8.

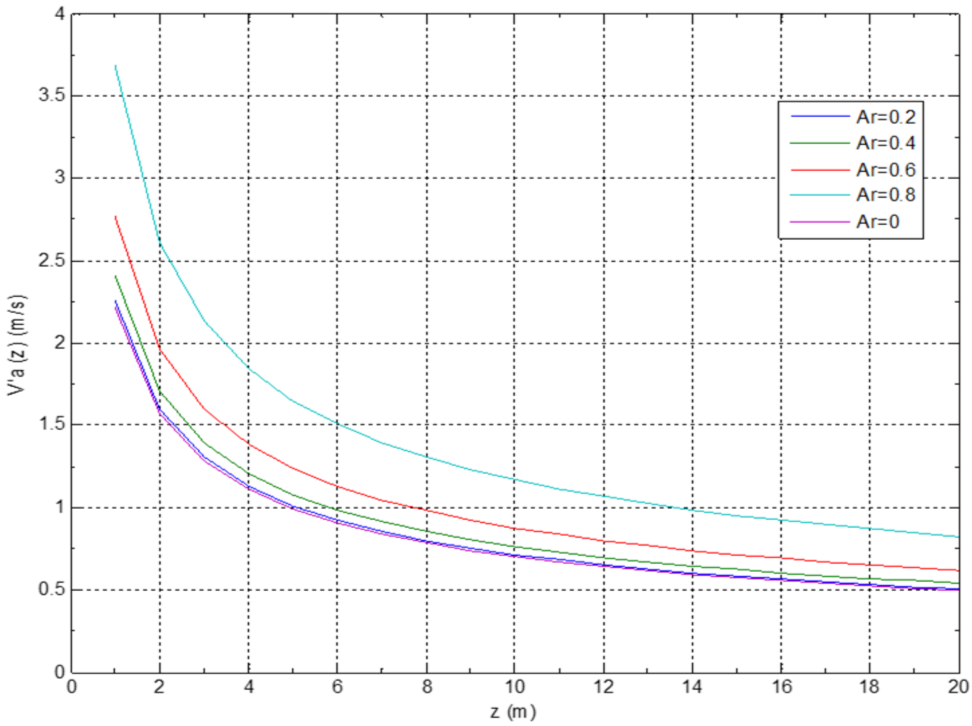

Figure 3.
z-derivative of amended Torricelli velocity for Ar ranging between 0 and 0.8.

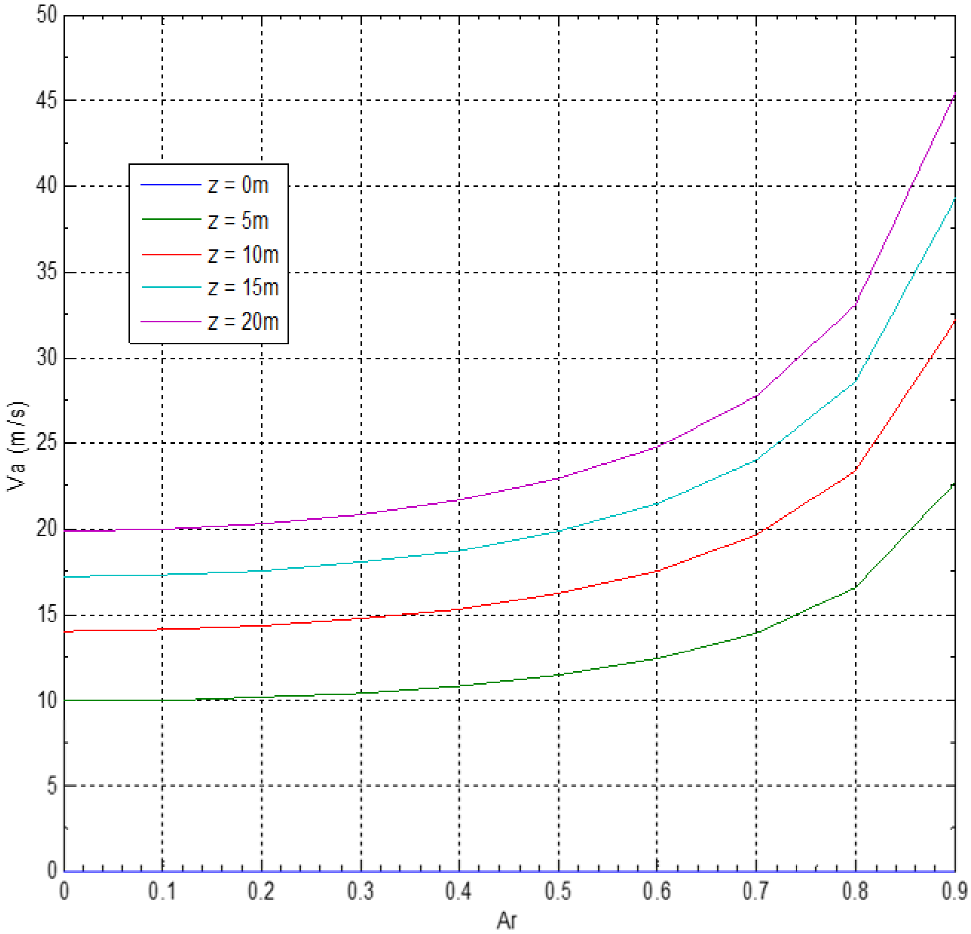

Figure 4.
Variation of amended Torricelli velocity with Ar for z ranging between 0m and 20m.

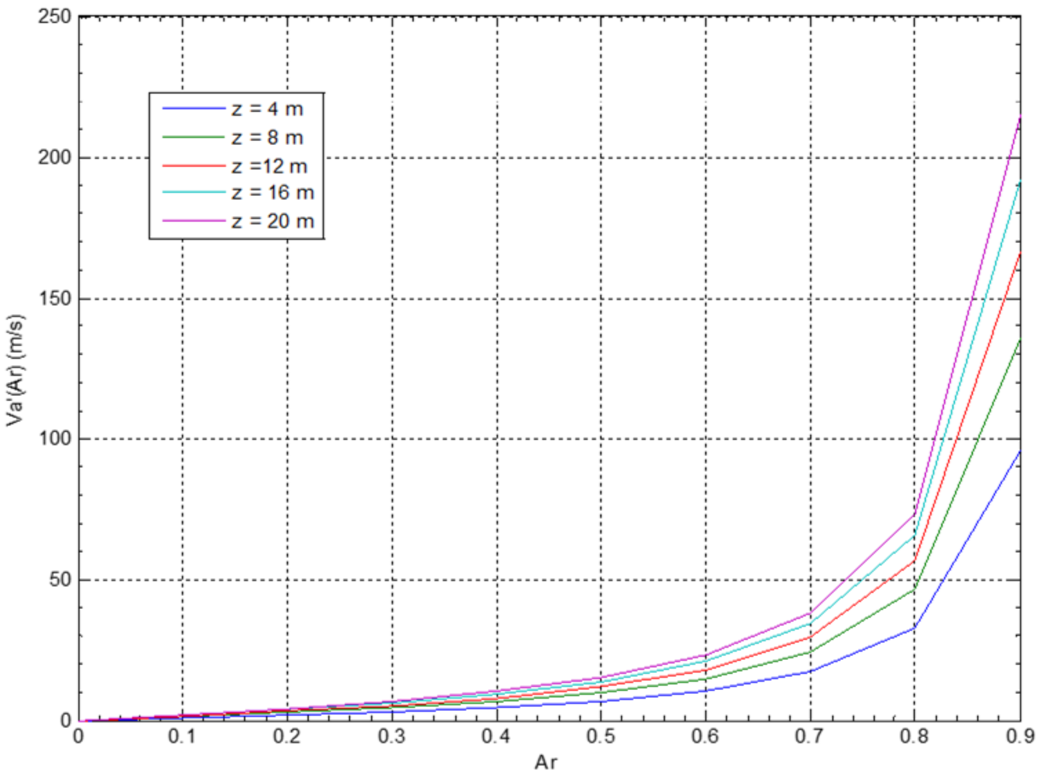

Figure 5.
Ar-derivative of amended Torricelli velocity for z ranging between 4 m and 20 m.

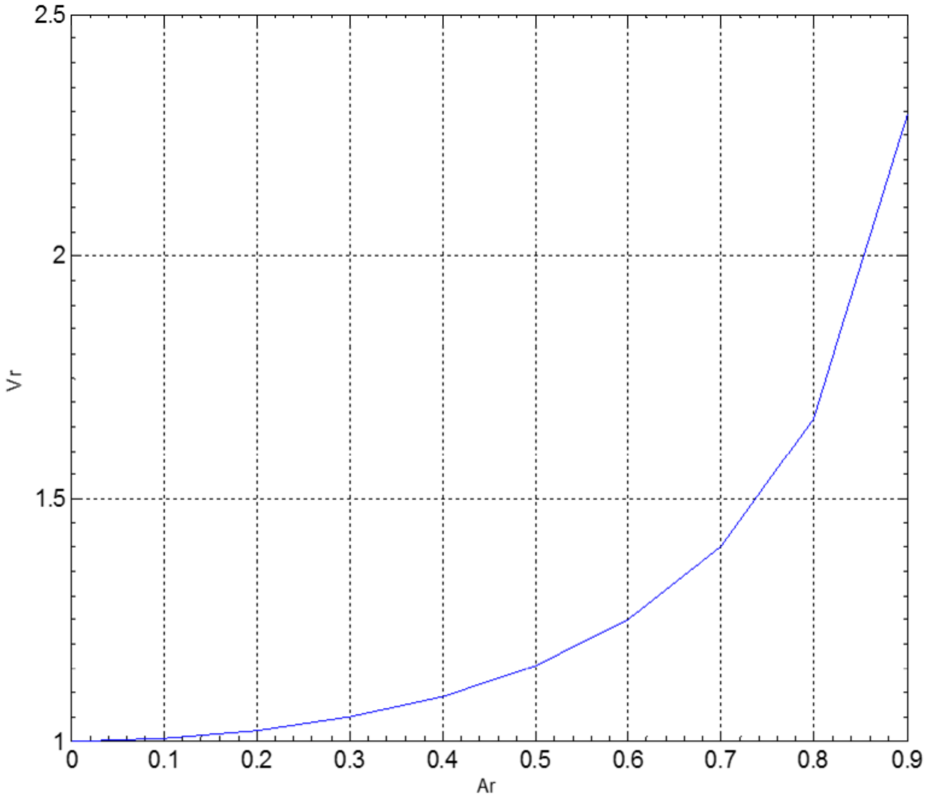

Figure 6.
Variation, with Ar ranging from 0 to 0.9, of the ratio of amended to real Torricelli velocity.

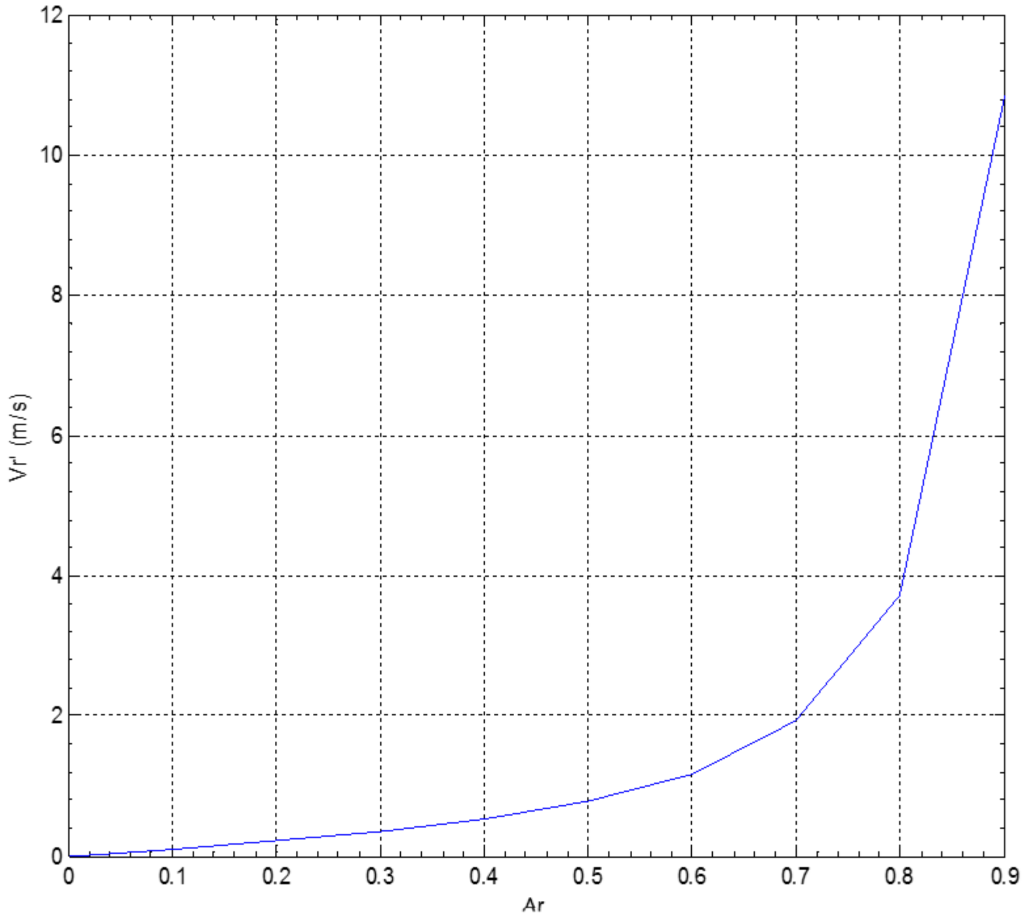

Figure 7.
Variation in Ar-derivative of the ratio of amended to real Torricelli velocity against Ar.

4. Discussion

The derived amended Torricelli velocity in Equation 7 shows that it is a function of both height of liquid, z, above the discharge, and the surface area ratio of downstream to upstream, Ar. The expression for convectional Torricelli velocity is, however, only a function of z. When Ar = 0, the amended Torricelli velocity, in Equation 7, equals the convectional Torricelli velocity in Equation 13. Equation 7 suggests that flow is valid when Ar < 1; otherwise, the equation becomes undefined. Ar < 1 implies that the downstream discharge surface area of flow must be less than the upstream surface area.

Figure 2 depicts the variation of the amended velocity with z for varying fixed Ar ranging from 0 to 0.8. The plot shows that the velocity increases with an increase in z for each fixed Ar. It is observed from the figure that the velocity is zero when z is zero for all values of Ar. The zero value of the velocity at z = 0 suggests that z is the driving force for the open flow system, and the upstream flow surface must be higher in the gravitational field than the discharge surface. Thus, the flow is gravity driven and cannot be achieved without $z > 0$ irrespective of the value of Ar. It may also be seen from the figure that the line plots of Ar = 0 and Ar = 0.2 are very close, and hence their velocity as well, and higher uniform intervals of fixed Ar give larger line plot departure respectively. This implies that when the downstream surface area is small relative to the upstream, the amended Torricelli velocity approximates the convectional Torricelli velocity; however, the difference becomes progressively significant at higher Ar. The z-derivative of the amended Torricelli velocity is plotted against z in Figure 3. Although an increase in z leads to an increase in velocity, Figure 3 illustrates that the increase in velocity decreases with a unit increase in z; hence, a higher value of z gives a correspondingly lower increment in velocity.

The variation of amended Torricelli velocity with Ar for varying fixed intervals of z between 0 m and 20m is depicted in Figure 4. The figure shows that the amended velocity increases with an increase in Ar for the fixed z = 5m, z = 10m, z = 15m, and z = 20m; however, for z = 0m, the plot is horizontal and the velocity is zero through the range of Ar. Thus, when the discharge is not lower than the upstream, flow is not established and the value of Ar makes no difference. Figure 5 depicts the Ar-derivative of amended velocity variation with Ar for different fixed values of z. At the onset, where Ar is close to 0, the figure has gentle slopes, almost horizontal, irrespective of the value of z. Therefore, Ar's effect on the amended velocity for small Ar is insignificant, and the flow then simplifies to real Torricelli velocity. The slopes become steeper as Ar drifts towards 1, implying that at high Ar, there is a significant impact of Ar on the velocity.

The ratio of the amended to convectional Torricelli's velocity, v_r, is given in Equation 14. The ratio is the factor by which the amended velocity is greater than the conventional velocity. Equation 14 shows that the ratio is only a function of Ar. Figure 6 depicts the plot of v_r against Ar. The figure is a non-linear curve with positive slopes that shows that with an increase in Ar, the factor by which the amended Torricelli velocity is greater than the convectional velocity is increased. From the figure, when the Ar is 0.9, the ratio, Vr, is about 2.3 m/s, which implies that at Ar = 0.9, the amended velocity can be increased by a factor of 2.3 over the velocity where Ar is negligible. The variation of Ar-derivative with Ar is depicted in Figure 7. The plot has gentle slopes close to Ar = 0 and the slopes become steeper towards Ar = 1. The figure implies that the factor by which the amended velocity is greater than the convectional velocity is small for values of Ar close to 0 and becomes progressively large for large values of Ar. The plot shows that the maximum value of the factor by which amended velocity is greater than convectional velocity between Ar = 0 and Ar = 0.7 is 2 m/s but becomes 3.6 m/s at Ar = 0.8 and 10.8 m/s at Ar = 0.9. The illustration shows that for a 0.1 increase in Ar, there is an average increase in factor of about 0.3 m/s between Ar = 0 and Ar = 0.7, of 3.6 m/s at Ar = 0.8 and 10.8 m/s at Ar = 0.9. This illustration shows that the increase in the factor grows exponentially as Ar increases towards 1.

5. Conclusions

The analysis of the derived conservation equation explicitly shows its validity which implies that the upstream surface must be higher than the discharge surface in the gravity-induced open flow system. Also, the discharge surface area must be smaller than the upstream surface area for an established flow; otherwise, the discharge surface naturally restricts its flow area to follow suit. The amended Torricelli's velocity increases exponentially with downstream to upstream surface area ratio, Ar, against the conventional Torricelli theorem, where the area ratio's effect on velocity is assumed to be negligible. A high Ar can be used to compensate for a drop in the velocity of the reduced height of liquid, z, as long as the flow can be established.

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