International Journal of Innovative Research and Scientific Studies, 9(9) 2026, pages: 75-88

IJIRSS
International Journal of Innovative Research & Scientific Studies

ISSN: 2617-6548

URL: www.ijirss.com

Multi-objective optimal control of an industrial absorption column via a hybrid particle swarm optimization–NSGA-II algorithm

Mourad. Selatnia[1*], Mohamed.Tadjine[2]

[1,2]*National Polytechnic School of Algiers, BP 182, Hassan Badi Avenue, El Harrach, Algiers, Algeria*.

Corresponding author: Mourad. Selatnia (*Email: mouradselatnia69@gmail.com*)

Abstract

This paper addresses the dynamic modeling, simulation, and multi-objective optimal control of an industrial packed absorption column for CO_2 removal from natural gas using methyldiethanolamine (MDEA) as a reactive solvent. A comprehensive nonlinear dynamic model is developed to describe the coupled mass and energy transfer phenomena, incorporating axial dispersion, vapor–liquid equilibrium, and temperature gradients along the column. The control objective is to optimally regulate the solvent flow rate and thermal profile in order to maximize CO_2 absorption efficiency while minimizing energy consumption. The problem is formulated under operational constraints, including a first-order dynamic constraint imposed on the outlet CO_2 concentration to ensure smooth transient behavior and avoid abrupt variations that may compromise process stability and product quality. To solve this problem, a hybrid metaheuristic optimization framework combining Particle Swarm Optimization (PSO) and the Non-dominated Sorting Genetic Algorithm II (NSGA-II) is proposed. PSO enhances convergence speed toward promising regions of the search space, whereas NSGA-II ensures a robust exploration and accurate approximation of the Pareto-optimal front. This hybrid strategy enables the simultaneous optimization of conflicting objectives while satisfying the imposed dynamic constraint. Simulation results demonstrate that the proposed PSO–NSGA-II approach achieves improved convergence performance and provides a well-distributed set of Pareto-optimal solutions, offering a balanced trade-off between energy efficiency and CO_2 removal performance. The study highlights the effectiveness of hybrid metaheuristic based optimal control in improving operational efficiency, ensuring process stability, and meeting environmental requirements in industrial gas treatment systems.

Keywords: CO_2 capture, Methyldiethanolamine (MDEA), Hybrid metaheuristic optimization, Industrial absorption column, Multi-objective optimal control, NSGA-II, Particle Swarm Optimization.

DOI: 10.53894/ijirss.v9i9.11936
Funding: This study received no specific financial support.
History: Received: 15 July 2026 / **Revised:** 26 August 2026 / **Accepted:** 31 August 2026 / **Published:** 28 September 2026

Competing Interests: The authors declare that they have no competing interests.
Authors' Contributions: All authors contributed equally to the conception and design of the study. All authors have read and agreed to the published version of the manuscript.
Transparency: The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.
Publisher: Innovative Research Publishing

1. Introduction

Methyldiethanolamine (MDEA), a tertiary alkanolamine, is widely recognized as a solvent of choice in industrial gas sweetening processes, particularly for the selective removal of hydrogen sulfide (H_2S) from sour gas streams containing carbon dioxide (CO_2). Its favorable physicochemical properties, including low vapor pressure, high thermal and chemical stability, reduced heat of absorption, and selective reactivity toward H_2S, provide significant energy savings compared to conventional primary and secondary amines such as monoethanolamine (MEA) and diethanolamine (DEA). Unlike MEA and DEA, MDEA does not form stable carbamate species with CO_2; instead, CO_2 absorption proceeds through hydration and bicarbonate formation, facilitating easier solvent regeneration and making MDEA highly suitable for large scale CO_2 capture in packed absorption columns [1, 2].

With increasingly stringent environmental regulations and rising energy costs, improving the efficiency and dynamic performance of CO_2 absorption processes has become a major research focus. Although extensive studies have addressed steady state modeling and pilot scale validation [3-5] the development of advanced dynamic optimization and control strategies for industrial-scale packed columns operating under transient conditions remains limited. To overcome these challenges, this study develops a detailed nonlinear dynamic model of an industrial packed absorption column using aqueous MDEA solutions.

The proposed model is formulated as a system of coupled nonlinear partial differential equations (PDEs) describing axial dispersion, interphase mass transfer, heat transfer, and the chemical kinetics of the CO_2–MDEA system. This distributed parameter framework enables accurate representation of the spatiotemporal evolution of concentration and temperature profiles, providing a reliable basis for dynamic simulation and optimal control design.

Building upon this modeling framework, a multi objective optimal control strategy is developed using a hybrid metaheuristic optimization approach that combines Particle Swarm Optimization (PSO) and the Non-dominated Sorting Genetic Algorithm II (NSGA-II). The control objective is to determine the optimal time varying solvent flow rate and thermal operating conditions that simultaneously maximize CO_2 removal efficiency and minimize energy consumption, while satisfying operational constraints, including a first-order dynamic constraint imposed on the outlet CO_2 concentration to ensure smooth system response and process stability.

In the proposed hybrid framework, PSO is employed to efficiently explore the search space and accelerate convergence toward high quality candidate solutions, leveraging its fast computational performance and simplicity. NSGA-II is then integrated to enhance population diversity and accurately approximate the Pareto optimal front, enabling a comprehensive analysis of trade-offs between conflicting objectives such as energy usage, absorption efficiency, and control smoothness. This hybridization exploits the complementary strengths of both algorithms, resulting in improved convergence behavior and solution quality.

Furthermore, the proposed optimization framework accounts for practical industrial considerations by incorporating measurement uncertainties through the introduction of Gaussian noise on process variables. This allows for the evaluation of control robustness under realistic operating conditions and sensor imperfections.

Simulation results demonstrate that the hybrid PSO–NSGA-II approach provides superior performance in terms of convergence speed, solution diversity, and control effectiveness compared to conventional single method approaches. The obtained Pareto front offers a set of optimal trade-off solutions, facilitating flexible decision making for process operators. The proposed methodology highlights the potential of hybrid metaheuristic based optimal control for enhancing energy efficiency, process stability, and environmental performance in industrial gas treatment systems.

This work contributes to the advancement of intelligent process control by providing a robust and flexible optimization framework for the dynamic operation of packed absorption columns. It supports the development of energy efficient and environmentally sustainable CO_2 capture technologies, in line with global efforts toward industrial decarbonization and sustainable process engineering [6-15].

2. Modelling of the Absorption Packed Column

The absorption packed column analyzed in this study is part of the In Salah Natural Gas project. The column is 8 meters in height and 4 meters in diameter, and is filled with Pall rings to enhance mass transfer through increased interfacial contact between gas and liquid phases. To improve the CO_2 removal efficiency the liquid phase, a mixture of water and MDEA flows counter currently to the upward moving gas stream. The column operates under industrial conditions of 71.5 bar and 55°C [16]. At the gas-liquid interface, the mass transfer of CO_2 from the gas phase to the liquid phase is promoted by the packing elements and is followed by a chemical reaction between CO_2 and MDEA Figure 1. The manipulated variable in this system is the flow rate of the MDEA solution, while the controlled output is the CO_2 concentration in the treated gas stream at the column outlet. This section details the mathematical formulation of the process model, based on simplifying assumptions that maintain physical relevance while reducing complexity. These include negligible gas phase resistance, fast reaction kinetics (Hatta number >5), and negligible axial dispersion in both phases. The resulting equations form the basis for the control oriented modelling presented in the subsequent sections.

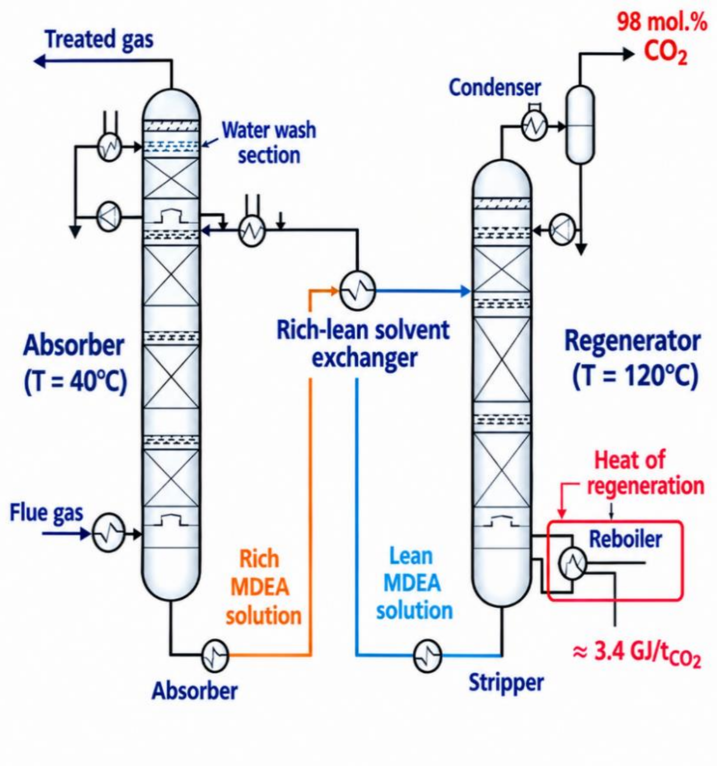


Figure 1.
The absorption packed column.

2.1. System Equations

The following assumptions are made to simplify the model [2, 5, 17]:

- There is no resistance in gas phase
- The reaction between CO_2 and MDEA is fast (Ha>5)
- Axial dispersion is negligible in the gas phase and the liquid phase
- The MDEA does not pass in gas phase

Under these conditions, the equations of the model are reduced to writing partial material balances in each phase, to which are added the relationships reflecting the boundary and equilibrium conditions.

Since we are interested in the evolution of the concentration of CO_2 in amine along the absorption column, we carried out the material balance on the CO_2 in the gas phase and in the liquid phase on one element (dz) and the material balance on the amine in the liquid phase Figure 2 [1, 2, 17].

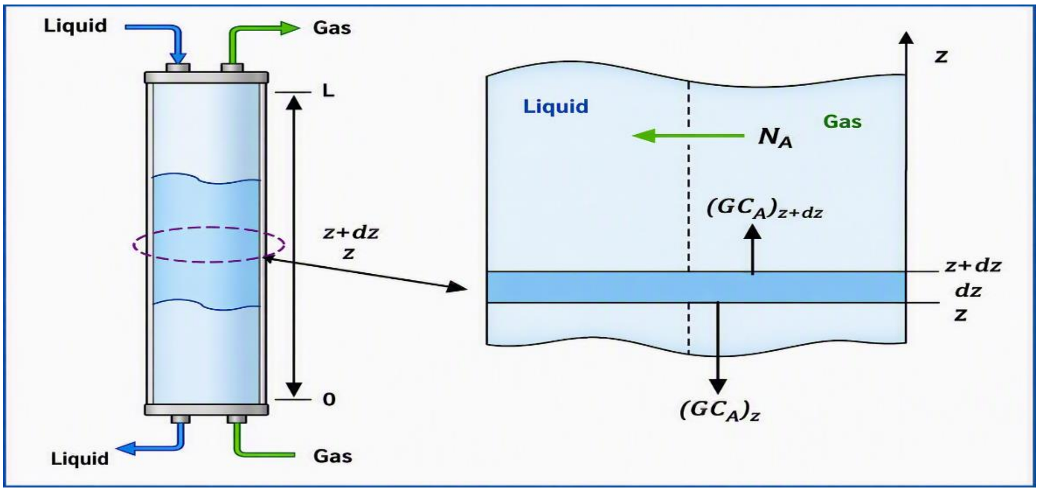


Figure 2.
Material balance on an elementary slice (dz) along the column axis.

In a steady state we can write:

2.2. State Space Model

- Material balance on CO_2 (A) in the gas phase:

Quantity of aqueous solution at input z = quantity of aqueous solution at the output(z+dz) + quantity of aqueous solution transferred from the gas phase to the liquid phase.

Consider the material balance on a slice dz in the gas phase:

$$\left(GC_{Ag}\right)_z = \left(GC_{Ag}\right)_{z+dz} + \varphi S dz + S\frac{dC_{Ag}}{dt}dz \quad (1)$$

Where G (m^3/s) is the volumic gas flow, φ the CO_2 flow transferred from the gas phase to the liquid phase, S the section of the column and C_{Ag} (mol/m^3) the CO_2 concentration in the gas phase. Given $U_g=G/S$ (m/s) the gas flow velocity, we obtain then:

$$U_g\frac{dC_{Ag}}{dz} + \varphi = -\frac{dC_{Ag}}{dt} \quad (2)$$

The chemical reaction between CO_2 and the MDEA is [3-4]:

$$CO_2 + R_1R_2NCH_3 + H_2O \rightarrow R_1R_2NCH_4{}^{+} + HCO_3{}^{-} \quad (3)$$

In the liquid phase, CO_2 reacts with mono diethanolamine (MDEA) according to formula (3), the speed r_A of this second order reaction is given by Rascol [1] and Lecomte, et al. [18]:

$$r_A = kC_{AL}C_{BL} \quad (4)$$

C_{AL} is the CO_2 concentration (A) in the liquid phase (mol/m^3) and C_{BL} the MDEA concentration (B) in the liquid phase (mol/m^3).

Where k is the reaction rate constant ($m^3/mol.s^{-1}$) [1, 17]:

$$k = 2{,}9610^5 \exp\left(-\frac{5332.8}{T}\right) \quad (5)$$

T: Temperature (K)

Material balance on CO_2 in the liquid phase give:

$$(LC_{AL}) = (LC_{AL})_{z+dz} + \varphi S dz - [kC_{AL}C_{BL}]S dz + S\frac{dC_{AL}}{dt}dz \quad (6)$$

Where L (m^3/s) is the volumic liquid flow. By taking account of (5) and noting by U_L = L/S (m/s) the mean liquid flow velocity,

$$U_L\frac{dC_{AL}}{dz} + \varphi - [kC_{AL}C_{BL}] + \frac{dC_{AL}}{dt} = 0$$

Generally, the reaction between CO_2 and MDEA is considered to be a rapid reaction (Hatta number > 5), the quantity of CO_2 present in the liquid is thus negligible, which implies that, there is no accumulation of CO_2 in the liquid phase.

$$\begin{cases} \frac{dC_{AL}}{dt} = 0 \\ \frac{dC_{AL}}{dz} = 0 \end{cases} \Rightarrow \varphi = [kC_{AL}C_{BL}] \quad (7)$$

Which means that the CO_2 transferred into the liquid phase reacts completely and instantly with the MDEA.

- Balance of the amine (B) in the liquid phase:

The material balance on the MDEA in the liquid phase, taking into account the stoichiometric coefficient of the reaction and the fact that the MDEA cannot pass into the gas phase, gives:

$$(LC_{BL})_z = (LC_{BL})_{z+dz} - [kC_{AL}C_{BL}]S dz - S\frac{dC_{BL}}{dt}dz$$

$$\Rightarrow U_L\frac{dC_{BL}}{dz} - [kC_{AL}C_{BL}] - \frac{dC_{BL}}{dt} = 0$$

We obtain:

$$U_L\frac{dC_{BL}}{dz} - \varphi = \frac{dC_{BL}}{dt} \quad (8)$$

Our absorption packed column is finally described by the following set of partial derivative equations:

$$\begin{cases} U_g \frac{dC_{Ag}}{dz} + \varphi = -\frac{dC_{Ag}}{dt} \\ U_L \frac{dC_{BL}}{dz} - \varphi = \frac{dC_{BL}}{dt} \end{cases} \quad (9)$$

The procedure to compute flow φ is given in Roizard, et al. [2] according to Rascol [1]; Lecomte, et al. [18] and Bedelbayev, et al. [19].

Finally, the boundary conditions must be defined. For the gas phase, the boundary condition corresponds to the CO_2 concentration at the bottom of the column, i.e., the inlet concentration denoted by C_{Age}. For the liquid phase, the boundary condition is given by the MDEA concentration at the top of the column represented by the inlet concentration C_{BLe}.

$$\begin{cases} C_{Ag}|z = 0 = C_{Age}, \frac{\partial C_{BL}}{\partial z}|z = 0 = 0 \\ C_{BL}|z = h = C_{BLe}, \frac{\partial C_{Ag}}{\partial z}|z = h = 0 \end{cases} \quad (10)$$

The exothermic nature of the chemical reactions taking place within the industrial absorption column leads to considerable heat release, resulting in the formation of a non-uniform temperature profile along the column height. Experimental observations indicate a temperature variation of approximately 5°C between the column inlet and outlet. This thermal gradient not only influences the local thermodynamic equilibrium but also affects the mass transfer efficiency and reaction kinetics throughout the column. Consequently, it becomes essential to formulate an energy balance that accounts for heat generation, conduction, and convective transfer in order to accurately characterize the temperature distribution and its coupled effects on the spatial evolution of species concentrations [17]:

$$\begin{cases} U_g \frac{\partial T_g}{\partial z} + \frac{a.h_{g|l}(T_l - T_g)}{[\sum_i cp^g{}_i C^g{}_i]} = \frac{\partial T_g}{\partial t} \\ -U_L \frac{\partial T_l}{\partial z} + \frac{1}{\sum_i cp^l{}_i C^l{}_i}\left[\Delta H_r r_A - a.h_{g|l}\left(T_l - T_g\right)\right] = \frac{\partial T_l}{\partial t} \end{cases} \quad (11)$$

With:

- C_i^g: concentration in gas phase at the interface (mol/m^3)
- C_i^l :concentration in liquid phase at the interface (mol/m^3)
- $h_{g/l}$: coefficient of heat transfer (convection) (J/m^2.K.s)
- T_l: Liquid temperature (K)
- cp_i^g : Specific heat in the gas phase at the interface (J/mol.K)
- Tg: Gas temperature (K)
- ΔHr: enthalpy of the reaction (J/mol)
- cp_i^l : specific heat in the liquid phase at the interface (J/mol.K)

We finally takes into account the boundary conditions for the temperature which are the temperatures for gas and the liquid at the column inlet.

$$\begin{cases} T_g \Big| z = 0 = T_{ge}, \frac{\partial T_l}{\partial z}|z = 0 = 0 \\ T_l|z = h = T_{le}, \frac{\partial T_g}{\partial z}|z = h = 0 \end{cases} \quad (12)$$

The experimental and functional tests of the packed column yielded empirical results based on the energy balance and the evolution of gas and liquid temperatures along the 8 meter column height Figure 3.

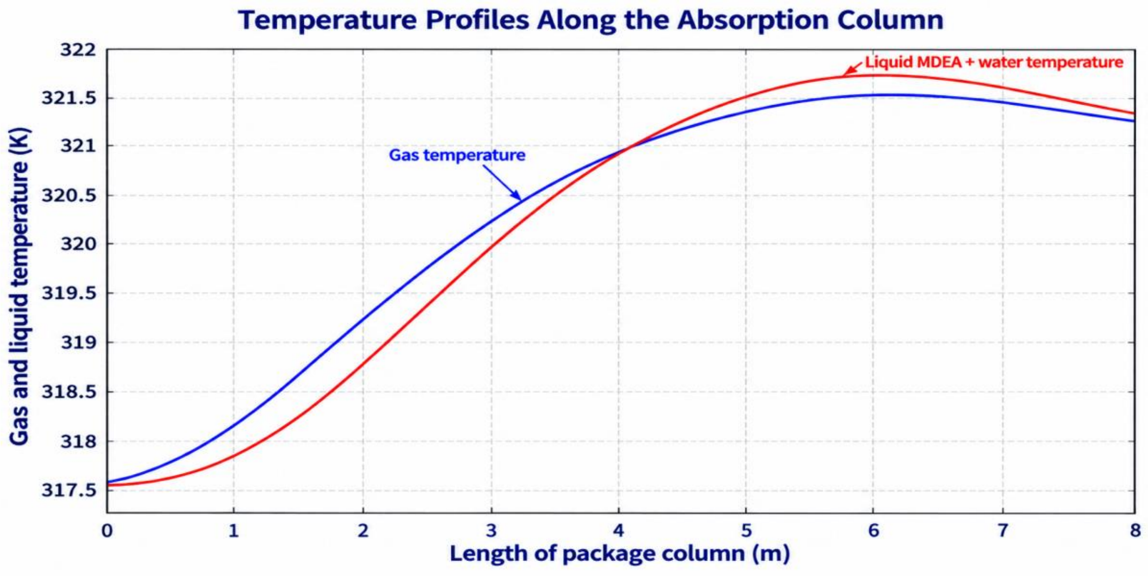


Figure 3.
Temperature Gradient of Gas and Washing liquid (MDEA+water).

The inlet temperatures are as follows;

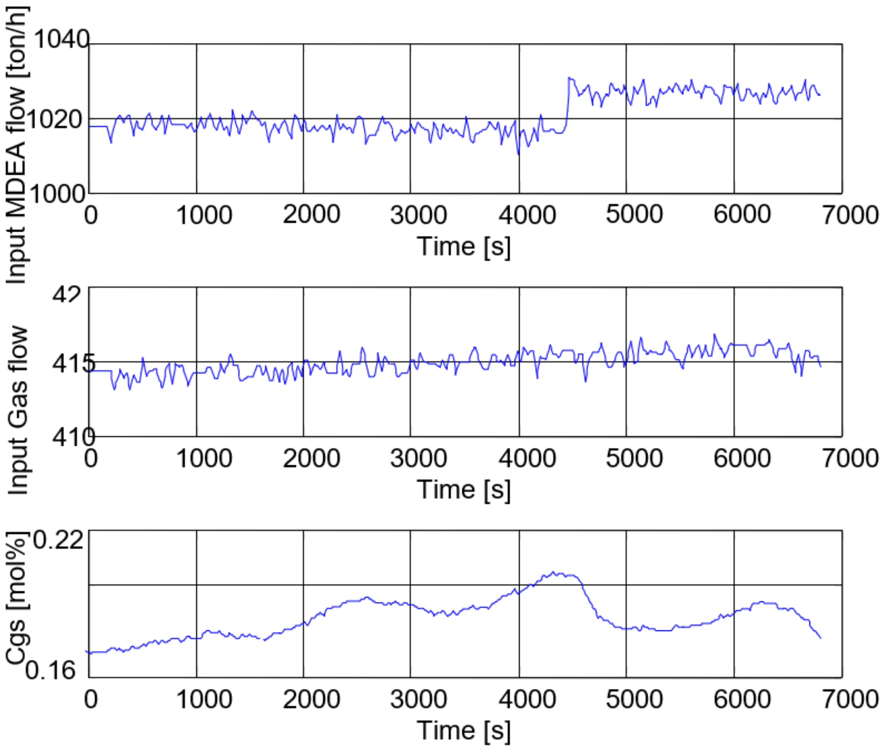
1. Gas temperature: $\mathrm{T_{ge}} = 44 + 273.15\ K$
2. Washing liquid temperature: $T_{le} = 48 + 273.15\ K$

The temperature profile along the column height, x=8 m is given by empirical relation:

$$\mathrm{T(x)} = \mathrm{T_{ge}} + \left(\frac{(\mathrm{T_{le}} - \mathrm{T_{ge}})}{9}\right) \times (x^3) \times \exp\,(-0.5 \times x) \tag{13}$$

2.3. Model Validation

A validation experiment was conducted on the industrial absorption column to assess the predictive accuracy of the proposed dynamic model. The test involved a step change in the recorded over duration of 6800 seconds; Figure 4 illustrates the time evolution of the MDEA and gas inlet flow rates, along with the CO_2 outlet concentration, as measured experimentally and as predicted by model. The results demonstrate a strong agreement between the model and experimental data, confirming the model's capability to accurately capture the dynamic behavior of the CO_2 absorption process. Furthermore, the process exhibits open loop stability, supporting the applicability of the model for advanced control strategy design.



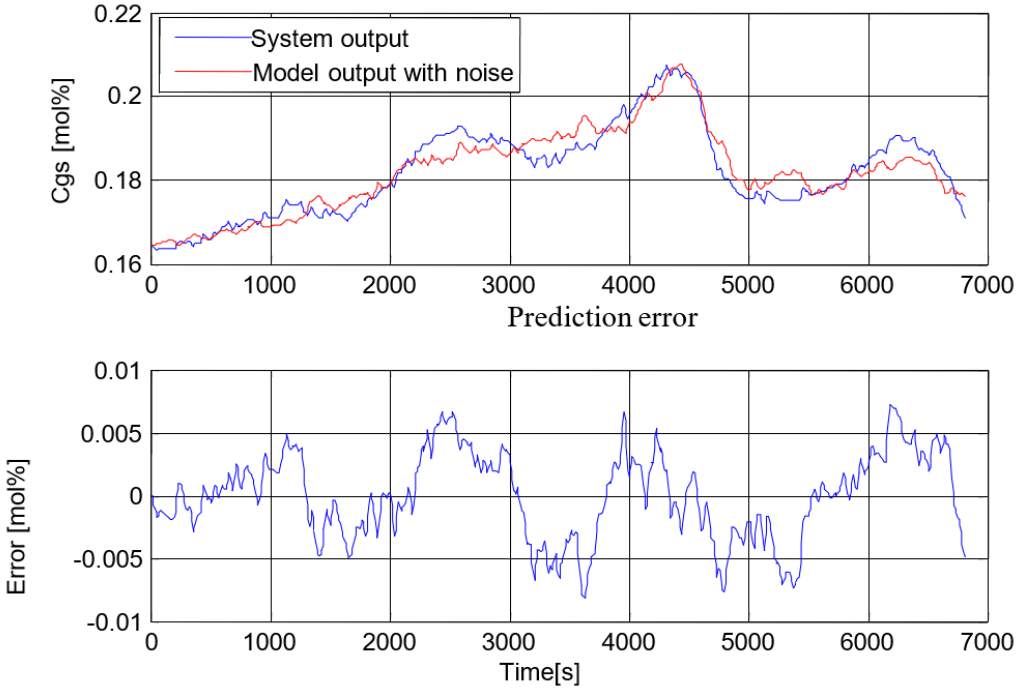


Figure 4.
Measured and simulated CO_2 concentration at the inlet and outlet of the system.

3. Optimal Control using PSO and NSGA-II Algorithms

3.1. Introduction

The integration of advanced optimal control strategies into large scale industrial absorption systems has become increasingly essential to meet the growing demands for enhanced energy efficiency, reduced environmental impact, and improved process performance. Packed absorption columns, widely used in gas purification and chemical processing, exhibit highly nonlinear and distributed dynamics arising from complex mass and heat transfer mechanisms, strong interphase interactions, and pronounced spatial and temporal dependencies. These intrinsic characteristics pose significant challenges for control design, particularly in the presence of multiple conflicting objectives and stringent operational constraints. Conventional control techniques and gradient based optimization methods often face limitations when dealing with the high dimensionality, non convexity, and multimodal nature of such problems, especially under uncertainty and performance trade-offs.

In this context, metaheuristic optimization algorithms have emerged as powerful and flexible tools for solving complex optimal control problems through global exploration of the search space. In this study, a hybrid multi-objective optimal control framework combining Particle Swarm Optimization (PSO) and the Non-dominated Sorting Genetic Algorithm II (NSGA-II) is proposed for an industrial packed absorption column. The objective is to optimize the dynamic solvent flow rate of methyldiethanolamine (MDEA), while simultaneously minimizing energy consumption and ensuring accurate regulation of the outlet CO_2 concentration under a first-order dynamic constraint.

Within the proposed framework, PSO is employed to efficiently guide the search toward promising regions of the solution space due to its rapid convergence and low computational complexity. NSGA-II is subsequently integrated to enhance solution diversity and to construct a well-distributed Pareto front, enabling an effective representation of trade-offs between conflicting objectives. This hybridization leverages the complementary strengths of both algorithms, combining the exploitation capability of PSO with the exploration and elitism mechanisms of NSGA-II.

The novelty of this work lies in the development of this hybrid PSO–NSGA-II optimization strategy coupled with a high-fidelity dynamic model of the absorption column. Through comprehensive simulation based analysis, the proposed approach is evaluated in terms of convergence performance, control accuracy, energy efficiency, and robustness under dynamic operating conditions. The resulting Pareto-optimal solutions provide valuable insights for decision making in industrial gas treatment processes. This study demonstrates the effectiveness and applicability of hybrid metaheuristic based optimal control for complex nonlinear distributed systems, highlighting its potential for improving operational performance and ensuring compliance with industrial and environmental constraints.

3.2. Problem Statement

The optimization of CO_2 removal in industrial absorption columns presents a significant challenge due to the non linearities, strong coupling between variables, and the distributed nature of the system dynamics. In this context, the problem is framed as an optimal control problem over a finite time horizon T=500 seconds, aiming to regulate the molar concentration of CO_2 in the treated gas stream, denoted as $C_{gs}(t)$ [mol%].; for modeling and control purposes, this output is represented by the state variable X(t), which evolves dynamically based on the interaction within the absorption column. The manipulated input is the flow rate of the MDEA aqueous solution, $u(t)$ [ton/h], serving as the control variable.

We aim to determine the optimal control *u(t)*, defined over the horizon $t \in [0, T]$ with T=500 sec, that minimizes the energy of control subject to linear constraint dynamics and state boundary constraints.

- First order constraint dynamics:

$$\dot{X}(t) = -aX(t) + u(t),\ \ a > 0 \ and\ X(0) = X0 = 0.25$$

$$X(T) = XT = 0.28,\ a = 0.00061 \qquad (14)$$

This problem is addressed using a hybrid metaheuristic optimization approach that combines Particle Swarm Optimization (PSO) with the Non-dominated Sorting Genetic Algorithm II (NSGA-II)."

3.3. Optimal Control Design Approach

3.3.1. A Hybrid Particle Swarm Optimization and NSGA-II Algorithm

The hybridization of Particle Swarm Optimization (PSO) with the Non-dominated Sorting Genetic Algorithm II (NSGA-II) has emerged as an effective strategy for solving complex multi-objective optimization problems. PSO, originally introduced by Kennedy and Eberhart [20] is a population-based stochastic optimization technique inspired by the social behavior of bird flocks. It is widely recognized for its fast convergence and simplicity in handling continuous optimization problems without requiring gradient information. However, standard PSO may suffer from premature convergence and limited diversity when addressing multi-objective problems.

On the other hand, the Non-dominated Sorting Genetic Algorithm II (NSGA-II), developed by Deb, et al. [21] is a well-established evolutionary algorithm designed for multi-objective optimization. It incorporates fast non-dominated sorting, elitism, and crowding distance mechanisms to ensure diversity and accurate approximation of the Pareto front. NSGA-II has been successfully applied in various chemical engineering applications, including dynamic optimization of reactors and separation processes [22].

To overcome the limitations of individual methods, hybrid PSO–NSGA-II frameworks have been proposed in the literature, combining the fast convergence capability of PSO with the diversity preservation and Pareto ranking mechanisms of NSGA-II [23, 24]. Such hybrid approaches have demonstrated superior performance in handling nonlinear, high-dimensional, and multi-objective control problems.

In this study, the optimal control problem of the CO_2 absorption process is formulated as a multi-objective optimization problem (MOOP) over a finite time horizon. The aim is to simultaneously minimize two conflicting objectives:

- Minimization of the energy cost associated with the MDEA flow rate
- Minimization of the Tracking Error with terminal penalty to ensure that the outlet CO_2 concentration reaches the desired set point

The objective functions are defined as follows:

- Objective 1: energy cost associated with solvent injection:

$$J_1(u) = \frac{1}{2}\int_0^T u^2\,(t)\ \ dt \qquad (15)$$

- Objective 2: terminal tracking error:

$$J_2(u) = \int_0^T (X(t) - X_f)^2 dt + \gamma(X(T) - X_f)^2 \qquad (16)$$

γ weighting coefficient = 0.25

The multiobjective optimization problem is thus expressed as:

$$\underbrace{min}_{u} = [J_1(u), J_2(u)]$$

$$u_{min} \leq u \leq u_{max}$$

$$\text{Subject to: } \dot{X}(t) = -aX(t) + u(t),\ \ X(0) = 0.25, a = 0.00061 \qquad (17)$$

In addition, an evolutionary optimization layer based on NSGA-II is incorporated to complement the PSO search process. This component ensures effective Pareto-based ranking of candidate solutions, promotes population diversity through crowding distance mechanisms, and enables the generation of a well-distributed set of Pareto-optimal control trajectories. As a result, the hybrid framework provides a robust multi-objective optimization strategy, allowing decision-makers to select suitable trade-off solutions between energy consumption and tracking performance according to operational requirements.

3.3.2. Hybrid PSO–NSGA-II Algorithm Mathematical Development

Step 1: Time Discretization of the Control Problem

The control input u(t)u(t)u(t) is discretized over the time horizon [0,T] into N uniform intervals:

$t_k = k\Delta t,\ \Delta t = \frac{T}{N}\quad ,\quad k = 1, \dots\dots, N$

$$u(t_k) = u_k,$$

Step 2: Discrete-Time System Dynamics
The system is approximated using an explicit Euler scheme:

$$X_{k+1} = X_k + \Delta t * \ (-a * X_k + u_k) \ , X0 = 0.25$$

3.4. Step 3: Objective Functions
Objective 1: control effort

$$J_1(u) = \frac{1}{2}\sum_{k=0}^{N-1} u^2{}_k \, \Delta t$$

This term represents:

- energy consumption,
- operational cost of the solvent flow.

Objective 2: Tracking Error with terminal penalty

$$J_2 = \sum_{k=0}^{N-1} (X_k - X_f)^2 \Delta t + \gamma\big(X_N - X_f\big)^2$$

where:

- the first term penalizes the global tracking error
- the second term enforces accurate terminal convergence
- $\gamma > \ 0$ is a weighting coefficient

3.5. Multi-Objective Optimization Problem

$$\underbrace{min}_{u} = [J_1(u), J_2(u)]$$

$$u_{min} \leq u_k \leq u_{max} \ \forall k$$

3.6. Hybrid PSO–NSGA-II Framework
The optimization is solved using a hybrid metaheuristic combining:

- Multi-Objective Particle Swarm Optimization (MOPSO) for exploration,
- NSGA-II operators (crossover + mutation + selection) for diversity and convergence

1. Particle Representation
Each particle represents a candidate control trajectory

$$u^{(i)} = [u_0{}^{(i)}, u_1{}^{(i)}, \dots \dots .., u_{N-1}{}^{(i)}]$$

Each particle has:

- position: $u^{(i)}$
- velocity: $v^{(i)}$
- personal best: $p^{(i)}$

2. PSO Update Equations

- Velocity Update

$$v_k{}^{(i)} \leftarrow w v_k{}^{(i)} + c_1 r_1\big(p_k{}^{(i)} - u_k{}^{(i)}\big) + c_2 r_2 (g_k - u_k{}^{(i)})$$

where:

- w: inertia weight,
- c_1 , c_2: acceleration coefficients,
- r_1, r_2 ~U(0,1)
- g: leader selected from Pareto archive.
- Position update

$$u_k{}^{(i)} = u_k{}^{(i)} + v_k{}^{(i)}$$

With bounds:

$$u_k{}^{(i)} = \min\ (\max\ (u_k{}^{(i)}, u_{min}), u_{max})$$

3. NSGA-II Genetic Operators

- Crossover (Arithmetic)

For two parents u^1, u^2

$$u^{(c1)} = \beta u^{(1)} + (1-\beta)u^{(2)}$$

$$u^{(c2)} = \beta u^{(2)} + (1-\beta)u^{(1)}$$

With $\beta \sim \mathrm{U}(0,1)$

- Mutation

A random component is perturbed

$$u_k \leftarrow u_k + \delta$$

Where δ is a small random variation

4. Pareto Dominance

A solution $u^{(i)}$ dominates $u^{(j)}$ if:

$$J_m{}^{(i)} \leq J_m{}^{(j)} \quad \forall \; m \; \in (1,2)$$

$$\exists \; m \; such \; that \; J_m{}^{(i)} < J_m{}^{(j)}$$

5. External Archive

An archive *A* stores Non dominate solution:

$$A = [u| \; u \; is \; Non \; dominate)$$

6. Final Control Selection

From the Pareto set:

$$P^* = (u^{(1)}, \dots \dots \dots ..., u^{(M)})$$

A compromise solution is selected:

$$u^* = \arg min J_1(u)$$

Building upon this formulation, the multi-objective optimal control problem is solved using a hybrid MOPSO–NSGA-II algorithm, which effectively combines the rapid convergence capability of Multi-Objective Particle Swarm Optimization with the robust Pareto ranking and diversity preservation mechanisms of NSGA-II. In this framework, the first objective J1 is dedicated to minimizing the control effort, thereby reducing the energy consumption associated with the solvent circulation, while the second objective J2, incorporating both an integral tracking term and a terminal penalty, ensures accurate convergence of the system state to the desired final concentration.

From a mathematical standpoint, the inclusion of the terminal penalty term $\gamma(XN-Xf)^2$ plays a crucial role in constraining the asymptotic behavior of the system, enforcing a strict adherence to the target value at the final time. This is particularly important in industrial applications, where even small steady state deviations in CO_2 concentration can lead to significant operational inefficiencies or non-compliance with process specifications.

Furthermore, the integration of measurement noise into the optimization framework significantly enhances the realism of the problem. By considering the measured output $cgs(t)=X(t)+w(t)$, where w(t) is modeled as a zero-mean Gaussian process, the objective functions are implicitly evaluated under uncertain conditions. Consequently, the optimization process does not merely seek an optimal control trajectory for an ideal deterministic system, but rather identifies solutions that remain effective and robust in the presence of stochastic disturbances.

The hybridization strategy further strengthens the algorithmic performance by balancing exploration and exploitation. The MOPSO component drives a global search of the decision space through velocity position updates guided by personal and global best solutions, while the NSGA-II operators namely crossover, mutation, and Pareto-based selection enhance population diversity and prevent premature convergence. This synergy enables the algorithm to generate a well-distributed and high quality approximation of the Pareto front, even in the presence of nonlinear dynamics and noisy measurements.

Assuming an initial MDEA solvent flow rate defined as

$$(U_{ini}) = 0.0212$$

The optimal control law expressed in the original process variables becomes:

$$U^*(t) = U_{ini} - u^*(t) \qquad (18)$$

where $U^*(t)$ represents the optimal solvent flow rate applied to the absorption column, providing an efficient operating policy that regulates the outlet CO_2 concentration while maintaining economical solvent usage.

4. Results and Discussion

- A Hybrid Particle Swarm Optimization and NSGA-II Algorithm under Gaussian Measurement Noise

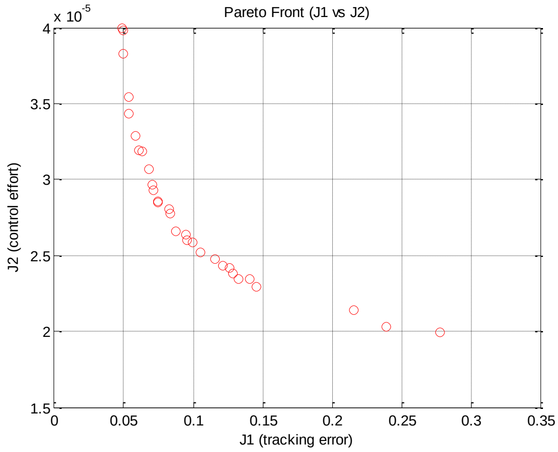


Figure 5.
Optimal Pareto Front.

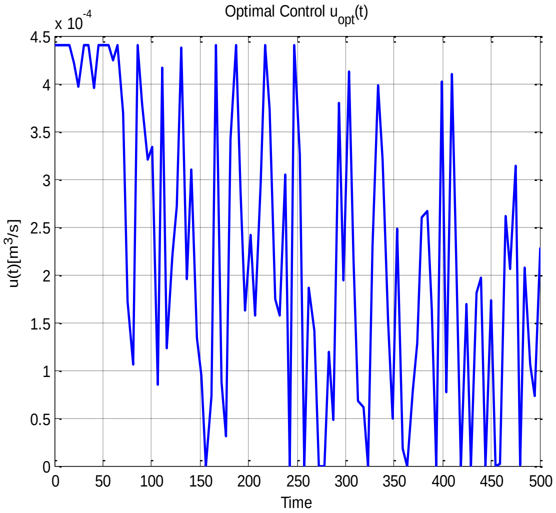


Figure 6.
Extracted optimal control MDEA flow rate from Pareto Front.

σ represents the standard deviation of the normally distributed measurement noise added to the output signal. (sigma σ = 0.001 and 0.0005).

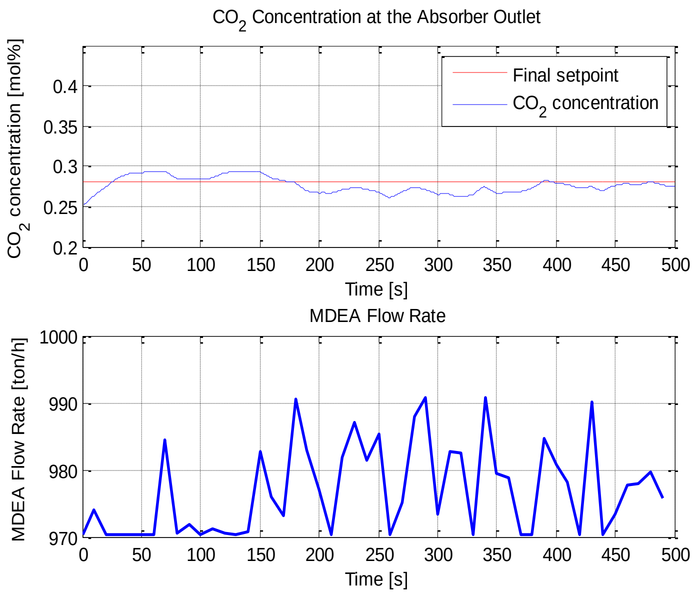


Figure 7.
Dynamic response of CO_2 outlet concentration and washing liquid flow rate to a set point shift from 0.25 mol% to 0.28 mol% with measurement noise sigma=0.001.

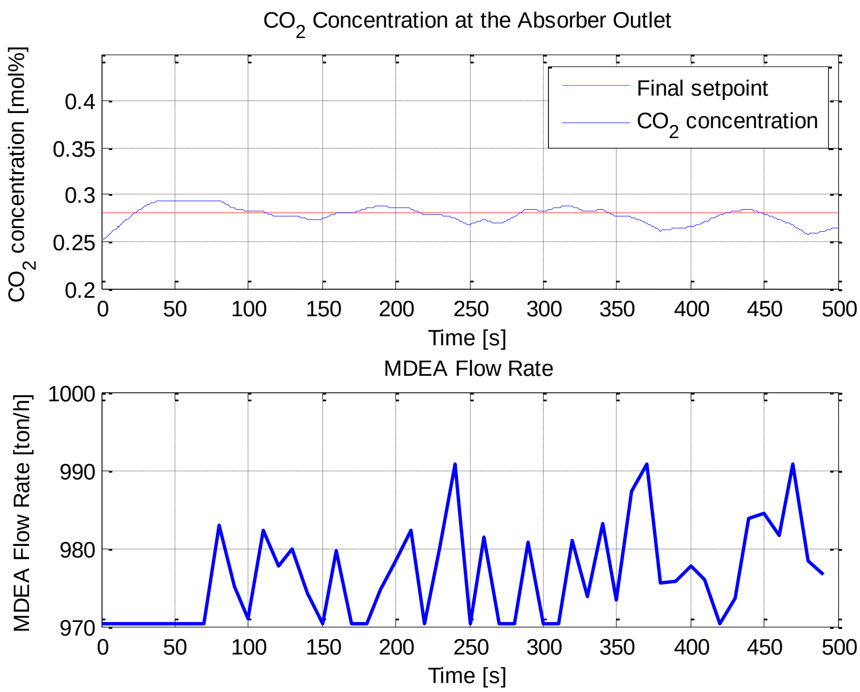


Figure 8.
Dynamic response of CO_2 outlet concentration and washing liquid flow rate to a set point shift from 0.25 mol% to 0.28 mol% with measurement noise sigma=0.0005.

The simulation results obtained using the hybrid MOPSO–NSGA-II algorithm demonstrate the effectiveness of the proposed multi objective optimal control strategy under measurement noise conditions.

Figure 5 illustrates the obtained Pareto front, which reflects the trade-off between the control effort J1 and the tracking performance J2. The well-distributed shape of the Pareto front confirms the ability of the hybrid algorithm to maintain diversity while converging toward optimal solutions. This indicates that the integration of NSGA-II mechanisms significantly enhances the exploration capabilities of MOPSO, preventing premature convergence and ensuring a wide range of feasible control trajectories.

Figure 6 presents the optimal control input (MDEA flow rate) extracted from the Pareto front. The selected control profile exhibits a smooth and physically realistic behavior, which is essential for industrial implementation. The absence of excessive oscillations confirms that the control effort minimization objective effectively regulates actuator activity while maintaining acceptable dynamic performance.

Figures 7 and 8 show the dynamic response of the CO_2 outlet concentration and the corresponding control input under two levels of Gaussian measurement noise (σ=0.001 and σ=0.0005). In both cases, the system successfully tracks the set point change from 0.25 mol% to 0.28 mol%, demonstrating the robustness of the proposed approach.

For σ=0.001, the response exhibits slight fluctuations due to the higher noise intensity. However, the system remains stable, and the concentration converges to the desired set point with minimal steady-state error. This confirms that the optimization process, which incorporates noise in the objective evaluation, leads to control solutions that are resilient to measurement uncertainties.

When the noise level is reduced to σ=0.0005, the system response becomes smoother, with reduced oscillations and faster convergence toward the reference value. This behavior highlights the sensitivity of the closed-loop performance to measurement noise, while also demonstrating that the proposed control strategy maintains high performance even under degraded measurement conditions.

Overall, the results confirm that the hybrid MOPSO–NSGA-II algorithm provides a robust and efficient framework for solving multi objective optimal control problems in industrial CO_2 absorption processes. The approach ensures an effective compromise between energy consumption and tracking accuracy, while maintaining stability and robustness in the presence of stochastic disturbances.

5. Conclusion

In this study, a hybrid MOPSO–NSGA-II algorithm has been developed to address the multi-objective optimal control problem of a CO_2 absorption process under realistic operating conditions. The control objective was formulated as a trade-off between minimizing the solvent flow rate (energy cost) and ensuring accurate tracking of the desired outlet CO_2 concentration, with the inclusion of a terminal penalty to enforce precise steady-state convergence.

The proposed hybrid framework successfully combines the fast convergence characteristics of Multi Objective Particle Swarm Optimization with the diversity preservation and Pareto ranking capabilities of NSGA-II. This synergy enables an efficient exploration of the search space while maintaining a well-distributed approximation of the Pareto front. As demonstrated by the simulation results, the algorithm provides a set of optimal control trajectories offering different compromises between control efforts and tracking performance, allowing for flexible decision-making based on operational requirements.

Furthermore, the explicit integration of Gaussian measurement noise into the optimization process significantly enhances the practical relevance of the proposed approach. The obtained control solutions exhibit strong robustness, maintaining stability and accurate convergence to the desired set point even under noisy measurement conditions. The inclusion of the terminal tracking term plays a crucial role in eliminating steady state errors, ensuring that the final CO_2 concentration strictly meets the specified target.

From an industrial perspective, the extracted optimal control profiles are smooth and physically feasible, which is essential for real-world implementation. The results confirm that the hybrid MOPSO–NSGA-II strategy effectively balances performance and energy efficiency while preserving robustness against uncertainties.

Overall, this work demonstrates that hybrid evolutionary optimization techniques constitute a powerful and flexible tool for solving complex multi objective optimal control problems in nonlinear and uncertain environments. Future work may focus on extending the approach to fully distributed PDE-based models of absorption columns, as well as developing real-time feedback control strategies based on the obtained optimal solutions.

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