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Teaching geometry in school: Digital resources to develop students' spatial thinking

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Abstract

Good school geometry education not only develops logical thinking but also the spatial imagination and thinking of students. The main means of developing spatial thinking is solving stereometric problems and especially making drawings for these problems. In current practice, teaching students how to construct images of geometric shapes is a challenging task. Weaknesses in geometric preparation are manifested primarily in the inability to accurately depict geometric figures, perform additional constructions and analyze the constructed drawing. Therefore, creating precise and correct drawings, fostering a culture of working with visual effects and developing spatial thinking skills are all vital skills to teach students in geometry courses. In this regard, digital resources and specialized mathematics programs serve as powerful learning tools. The purpose of this study is to identify the significance of digital resources in solving geometric problems for the development of students' spatial thinking. This paper discusses the relationship between the type of spatial thinking and geometric problems for reasoning and digital technologies, presents digital educational materials and resources and provides recommendations for their integration into the educational program. The research methods of this study include observation, study and generalization of teaching experience, interviewing, surveying and testing students, development of methodology and digital resources. A pedagogical experiment was conducted to test the effectiveness of the developed methodology. The results showed the effectiveness of digital resources in the learning process with a noticeable improvement in students' spatial thinking skills observed due to the introduction of digital education.

Keywords: Development, Digital education, Digital resources, Geometric problems, Geometry teaching, Learning process, School, Spatial thinking.

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1. Introduction

1.1. Background and Research Questions

In the era of educational digitalization, the preparation of adept, adaptable professionals with elevated levels of intellectual capacity is gaining prominence [1]. Spatial thinking as a constituent of human intelligence emerges as a pivotal factor for success across various academic pursuits within modern schooling. A proficient level of spatial thinking is indispensable for addressing the professional challenges encountered by graduates in their respective fields [2].

Spatial thinking holds significant importance for effectively grasping mathematical concepts. Among the subjects in the mathematical curriculum, geometry offers the greatest opportunity for fostering spatial thinking [3]. Through geometry, students can manipulate spatial structures, examine spatial properties and relationships, alter original structures and generate new ones. Both in high school and junior high, it is imperative to facilitate an environment that allows students to engage with space, fostering their unique subjective experiences while considering individual differences in their spatial thinking abilities.

The primary aspect of updating the educational process in schools revolves around a shift in the teacher's role during class preparation and delivery. The focus was on delivering instruction to a group of students collectively. However, in modern settings, conditions permit planning and preparation tailored to each individual student. The promise of digital learning environments in schools can only be completely realized with such a customized, student-centered approach to instruction [4].

In a digital society, there is a prevailing emphasis on science and knowledge with individuals engaging not only with other humans but also with electronic entities particularly artificial intelligence capable of autonomous decision-making. The advancement of digital society entails the integration of cutting-edge digital technologies and the cultivation of digital competence among individuals. Education emerges as a fundamental avenue for shaping a digital society marked by the increasing computerization of schools and the emergence of new socialization dynamics. The complexity and refinement of production processes necessitate not only highly skilled personnel but also the retraining of displaced workers. Independent activity in seeking and mastering information including knowledge as the highest form of information, takes precedence in a digital society. Consequently, this necessitates certain adjustments in the methodology of teaching geometry to align with the evolving landscape of education and technology. In this regard, the problem of developing the thinking abilities of the younger generation in the school years is of particular relevance. The development of the student's thinking is ensured by a high level of education. Objective is the dependence of learning outcomes on the characteristics of the developing personality's interaction with the world. Learning outcomes depend on the nature of the activity in which the learner is involved at one stage or another of his development. Equally important is the regularity of the correspondence of the content, forms and methods of teaching to the age and individual characteristics and capabilities of students.

The main purpose of the geometry course in elementary grades is to enrich students with geometric representations to familiarize them with the richest possible set of geometric shapes, to master basic geometric terminology and to acquire the skills and abilities to depict geometric shapes [5]. Sharing the author's point of view both in terms of the selection of geometric content in terms of the types of educational activities aimed at mastering it to develop students' thinking, it is advisable to use other active methods, forms and means of teaching and a geometry course. Realization of the developing potential of geometric content is possible only under the condition of the developing orientation of the entire mathematics course; the study was conducted in line with the methodological concept of developing geometry for schoolchildren which expresses the need for purposeful development of students' thinking in the process of studying mathematical content. We believe that when studying geometric content, the specificity of this concept will find its expression in deliberate work on schoolchildren's development of spatial thinking. Spatial thinking is defined as a specific type of mental activity that occurs in solving problems that require orientation in practical and theoretical space (both visible and imaginary).

The analysis of the theory and practice of the development of spatial thinking of students allows us to identify several contradictions in need of constructive resolution between the teaching of geometry in older adolescents and the sensitive period of its productive assimilation in younger adolescents; the need for purposeful development of spatial thinking of students and the lack of a generalized, developed and implemented in practice set of digital development tools; features of the development of the motivational sphere and the specifics of the formation of thinking (spatial thinking in particular).

The problem of this research is to identify methodological conditions that effectively contribute to the development of spatial thinking in students within the framework of digital learning. The solution to this problem is the purpose of our research.

1.2. Literature Review

It is essential to foster an environment that encourages students to explore their unique subjective experiences of interacting with space while also considering their spatial thinking abilities in both junior and high school [6]. Each student harbors a distinct set of spatial images shaped not only by academic subjects like geometry, physics and geography but also by their practical interactions with space. Teachers are tasked with crafting learning methodologies that integrate various tools for developing spatial thinking ensuring that abstract knowledge is imparted alongside sensory experiences [7]. Students can more thoroughly and deeply assimilate concepts, laws and theoretical frameworks particularly in the study of geometry, thereby facilitating a more comprehensive development of their cognitive abilities through such approaches [8].

The analysis of scientific literature, research results and the study of the experiences of leading mathematics teachers made it possible to identify the following as means of developing spatial thinking in the study of geometry [9-11]:

- A system of tasks (entertaining geometry tasks).
- Tasks for creating an image and tasks for operating an image.

- Tasks correspond to three types of spatial thinking (movement, reconstruction and composition).
- Solution tasks with the use of logical thinking techniques.
- Application of the apparatus of spherical geometry to solving problems, constructive tasks and tasks for constructing sections).
- The method of projects and the use of digital technologies.

Spatial thinking is a crucial aspect of intelligence, the cultivation of which is essential for the success of students in various subject areas in contemporary education [12]. A high proficiency in spatial thinking is particularly vital for tackling professional tasks in emerging fields such as makeup artistry, design, fashion design and related disciplines.

Geometry has the greatest potential for the development of spatial thinking, nevertheless, as the research results show among the school disciplines of the mathematical cycle [10, 13].

Textbooks used in schools often present geometry theory in an analytical manner which may not fully leverage the potential of the geometry course to enhance students' spatial thinking skills [14]. There is a scarcity of pedagogical software tools for geometry with the majority focusing primarily on developing the logical aspects of thinking. Furthermore, when educational materials are computerized for visualization, geometric images are often merely transferred from paper to electronic formats without considering the findings of psychological and pedagogical research or fully exploiting the capabilities of digital technologies [15].

The methodological challenge of fostering spatial thinking in both schoolchildren and students remains unresolved despite the considerable theoretical advancements made by psychologists and teachers [16, 17]. Furthermore, although digital technologies offer additional avenues for addressing this issue practically, they have not significantly impacted the methodology of spatial thinking development.

The rapid development of digital technologies is a characteristic feature of the modern world [18]. Once more, digital technologies are extensively employed in education with particular emphasis on geometry. Research reviews indicate that digital technology is recognized as a tool for cultivating students' cognitive engagement [19, 20].

Thus, we conclude that the use of digital technologies in education and teaching geometry in particular remains an urgent problem. A contradiction is brewing between the requirements of regulatory documents and the existing practice of using digital technologies.

The effectiveness of using digital technologies is beyond doubt but it is important to understand what we mean by this phrase. A teacher who works with a presentation in class or uses an interactive whiteboard instead of a chalkboard can be completely confident that he uses digital technologies. However, the current state of digital technologies and their constant development force us to look at this problem differently which requires further research.

An overview of new technologies and the possibilities of their application in mathematical education is promising. The problem of developing spatial thinking towards geometry using digital technologies requires further consideration.

We present the following scientific assumptions after analyzing the above literature:

H_{01}: If the methodology for the development of concepts in the geometry course at school is based on the integrated use of visual aids at each stage of the geometric concept development process, then its implementation will ensure a high level of spatial thinking and assimilation of geometric concepts among students.

H_{02}: The use of products of digital technologies in solving geometry problems has a positive effect on the development of spatial thinking in school lchildren.

2. Materials and Methods

2.1. Main Methods of Research

A comprehensive analysis of articles from reputable peer-reviewed journals was conducted to establish the significance of the topic and identify current shortcomings in the field of science.

Bibliographic sources such as the Web of Science (WOS), Scopus databases and Mendeley products were selected for this purpose. The analysis involved studying the works of both foreign and domestic researchers focusing on keywords relevant to the topic. The initial data collected through this process was used to address the research objectives of the project and establish the theoretical framework.

In addition, the methods of pedagogical control contributed to determining the indicators of spatial thinking of students, assessing the possibilities of organizing and conducting geometry lessons based on digital technologies. The method of mathematical and statistical analysis through the SPSS program made it possible to process the data obtained in the course of a pedagogical experiment to assess the level of development of spatial thinking in students.

The testing system enabled it to assess both the proficiency of spatial imagination (the capacity to form mental images) and the proficiency in manipulating spatial images.

The reliability of the differences in the experimental results before and after the experiment was verified using the Mann-Whitney criterion.

2.2. Stages of the Study, Study Participants and Experimental Assessment Methods

Experimental work was carried out which involved 200 students from different schools in Almaty to assess the effectiveness of building the educational process using spatial thinking development tools (see Table 1).

Table 1.
Detailed information about the student participating in the experiment.

Groups	Number		Percentage (%)	Total
Experimental	100		50%	200 (100%)
Control	100		50%	
Gender	Female (120)		60%	200 (100%)
	Control 58	Experimental 62		
	Male (80)		40%	
	Control 42	Experimental 38		

The pedagogical experiment was conducted in three stages:

During the initial phase, we conducted a theoretical examination of literature pertinent to the research topic, synthesized observational data and scrutinized experiences in teaching geometry at the school level. This phase involved a comprehensive review of electronic textbooks, elucidating the nuances of integrating computer-generated imagery into the educational process.

At the second stage, a search pedagogical experiment was conducted during which the development of a methodology for the development of spatial thinking was carried out.

The formative stage was carried out in the 2022-2023 academic years based on gymnasium №. 208 and №. 125 in private schools in Almaty to test the hypothesis experimentally.

Students were presented with 10 tasks categorized into three levels of difficulty corresponding to A, B, and C types of image manipulation, each rated at 1, 2 and 3 points respectively. It becomes feasible not only to assess the level of development but also to discern the predominant type of spatial thinking exhibited by the student by analyzing the outcomes of the test. Importantly, since participants were unaware of the point allocation for each task, the sequence in which tasks were chosen can serve as an indicator of the student's spatial thinking type.

3. Results

3.1. The Author's Conclusions in the Use of Digital Resources as a Means of Developing Spatial Representations

Currently, the opportunities for organizing training at different levels of education have increased with the development of digital technologies [21]. According to the guidelines provided, the task entailed developing educational content comprising an explanatory note, thematic and hourly planning as well as course training materials. These materials encompassed presentation materials for classes, resources for independent study and evaluation materials for both interim and final assessments.

Given the constraints of the tight timeframe allotted for course development, let's direct our attention to the nuances of using digital technologies and webinars specifically for digital geometry. We posit that the insights garnered from this experience could prove invaluable in crafting similar educational offerings.

The main purpose of using digital resources was to remind students of the theoretical information needed to solve problems. The educational material is systematized by topics according to the textbook geometry of high school students.

Currently, there are several dozen software environments for working with mathematical objects. All of them differ only in detail. In Kazakhstan, the most famous such environments are Live Mathematics, Mathematical Constructor, GEONExT and GeoGebra. The latter two environments are freely distributed software products which contributes to their widespread use by users. The GeoGebra program is particularly popular today [22]. The GeoGebra program allows creating all kinds of constructions from points, vectors, segments and lines to build graphs of elementary functions that can be dynamically changed by varying some parameter included in the equation.

Theoretical material is mastered completely and quickly if the need to search for relevant knowledge is realized. School textbooks on mathematics emphasize that mathematical knowledge is required for construction at all times because mathematical knowledge makes it possible to rationally carry out measurements, calculations and planning in construction. Various textbooks on basic-level geometry provide various examples explaining the need to apply mathematics when performing construction work but these connections are poorly visible at the senior level. Higher professional education should to a greater extent show examples of the use of theoretical knowledge to solve practical problems. The study of equations of second-order surfaces takes place in elective courses in mathematics and is a mandatory element in many specialties at universities. In practical classes, the simplest manipulations are carried out with these equations while the use of these equations provides great opportunities for constructing structures of various shapes.

In individual tasks, it is possible to implement mathematical modelling of simple construction tasks using the simplest structures having the shapes of a sphere, cylinder and cone. The use of digital technologies allows for visually depicting the work of the model to implement the research in practice.

Spatial representations serve as essential components of cognition and practical activities across various domains [23]. In the realm of geometry, students are required to mentally manipulate spatial objects—both three-dimensional and planar geometric figures within the context of problem-solving tasks.

Through a comprehensive analysis of scientific works, it was possible to create digital models of spatial objects that can be enlarged, rotated, moved and displayed in different colors contributing to a better understanding of the properties of objects and the development of spatial ideas about them. Let's consider what software allows for implementing the modeling process in mathematics lessons.

Tinker CAD serves as a modeling platform tailored for working with 3D objects and electronic circuits [24]. It boasts an open environment characterized by an accessible and user-friendly interface, making it suitable for children and is compatible with most contemporary computers. However, Tinker CAD facilitates the creation of 3D models using a variety of building blocks such as cubes, pyramids, spheres and cylinders. It does present limitations for certain geometric transformations necessary for solving complex problems. Notably, the platform does not support the creation of objects lacking at least one dimension such as points, line segments, straight lines, or planes. It’s possible to generate a rudimentary model of a plane by minimizing the thickness of a parallel pipe. Tinker Cad’s capabilities are limited in this regard. Go to the website tinkercad.com (see Figure 1).

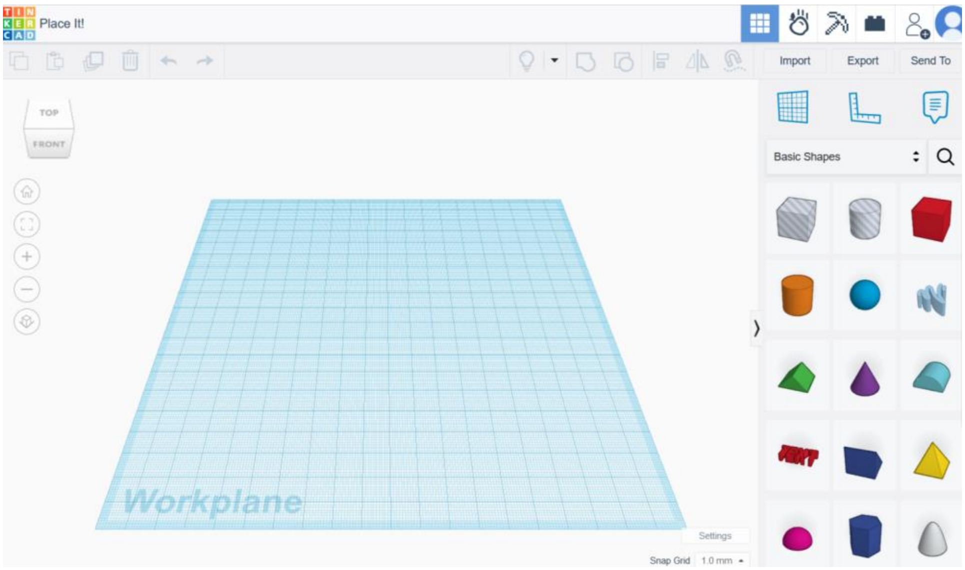

Figure 1.
Program Tinker CAD for modeling spatial representations from geometry.

Sketch Up: This program is convenient for use in modeling simple three-dimensional objects such as buildings, furniture etc. [25]. One notable feature of the program is its minimal reliance on preset windows with users primarily inputting geometric characteristics through the keyboard. This approach empowers users to directly specify and customize geometric attributes according to their preferences or requirements fostering a more hands-on and interactive modeling experience (see Figure 2).

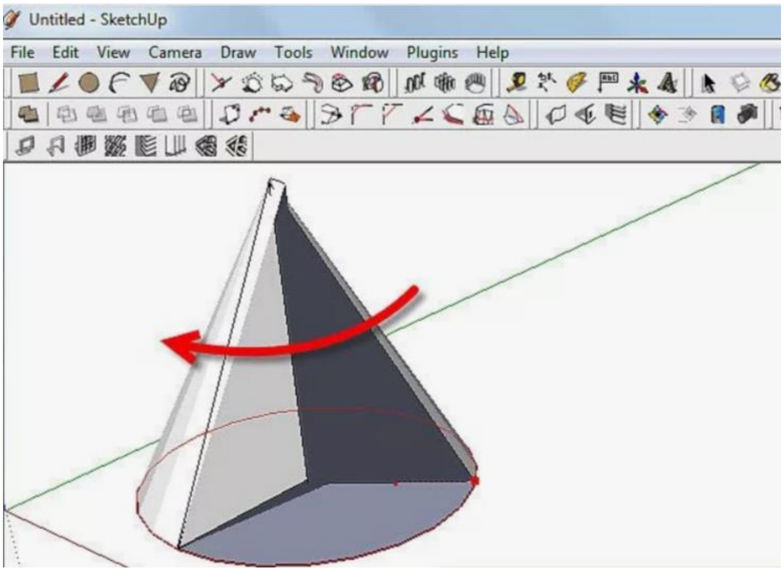

Figure 2.
Program sketch for modeling spatial representations from geometry.

Constructing 3D augmented reality is a new way of illustrating educational material. It does not interfere with the interaction of students with each other with the teacher or most importantly with the subject being studied. Interactive whiteboards schools that have been actively equipped with lately are not always in demand and are most often used to demonstrate static presentations. Augmented reality is another relevant content for working with interactive whiteboards.

At the same time, the setting of the blackboard is no different from the setting for displaying a presentation but all students need 3D glasses or a smartphone to view a three-dimensional image.

Fundamentally new in terms of interaction the Construct 3D application is a vivid example of the use of augmented reality in the field of geometry. This application uses stereoscopic head-up displays and personal interactive panels. Construct 3D allows multiple people to work in the same space and build different geometric models that overlap with the real world. This service has a fairly wide range of 3D animation, modeling and visualization functionality (see Figure 3).

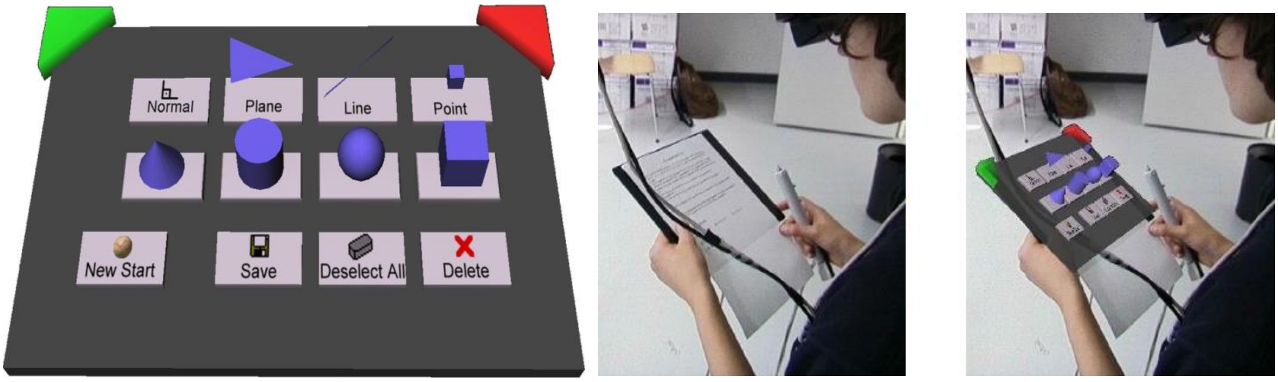

Figure 3.
Program Construct 3D for modeling spatial representations from geometry.

3.2. Examples of the use of Digital Resources in the Development of Spatial Thinking

Theoretical material is absorbed comprehensively and efficiently when learners recognize the necessity of seeking out pertinent knowledge. School textbooks on geometry emphasize that mathematical knowledge is required for spatial thinking at all times because mathematical knowledge makes it possible to rationally carry out measurements, calculations and planning. Various textbooks on basic level geometry provide various examples explaining the need to apply mathematics to spatial thinking but these connections are poorly visible at the high level.

It is possible to implement mathematical modeling of simple tasks using the simplest structures in the form of a sphere, cylinder and cone in individual tasks. The use of digital technologies allows us to visually depict the work of the model and then implement the research in practice.

The course "Modeling of Lines and Surfaces" should be based on the material of the chapter "Geometry on a sphere, cylinder and cone" adding elements of computer graphics.

The cylinder in the manual is understood as a cylindrical surface indefinitely extended in both directions, i.e. the surface obtained by rotating a straight line around a line parallel to it (see Figure 4).

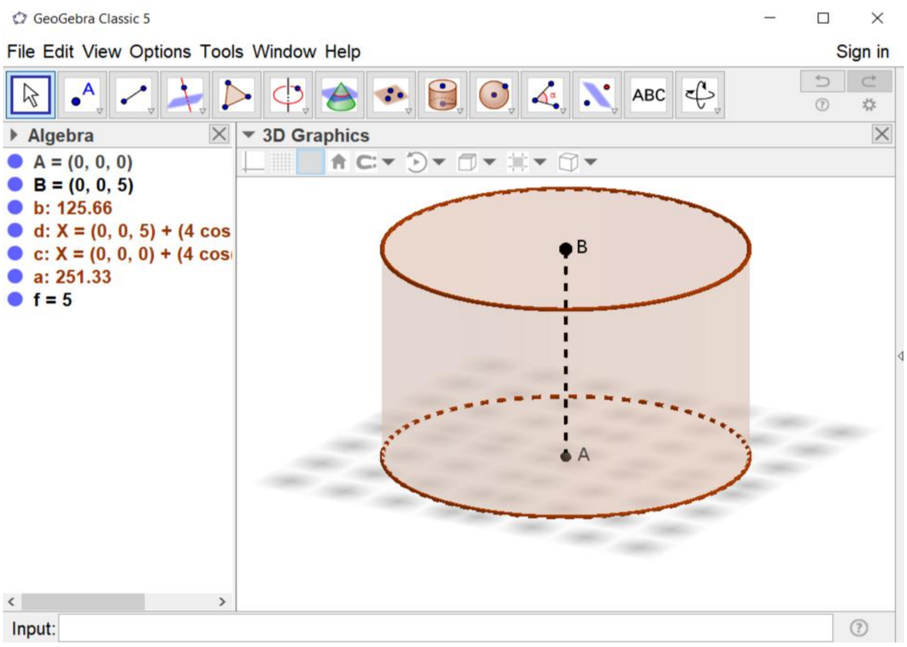

Figure 4.
View of the drawing of the cylinder according to the definition in the 3-dimensional program GeoGebra.

To prove the statement, it is enough to consider the sphere as a surface obtained by rotating a circle around a straight line passing through the center of the sphere. Let us prove the statement that a plane not parallel to the axis of the cylinder and not perpendicular to the axis of the cylinder intersects the cylinder in an ellipse (see Figure 5).

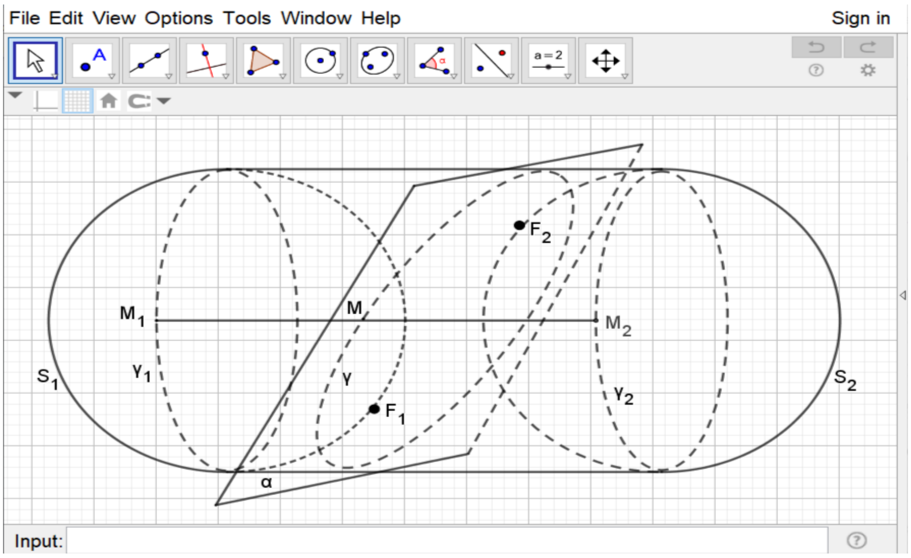

Figure 5.
A schematic diagram according to the calculation condition.

Let the plane intersect the cylinder along the line. Consider two spheres *S1* and *S2* whose radii are equal to the radius of the cylinder. Let the spheres *S1* and *S2* touch the plane, respectively, at points *F1* and *F2*, and the cylinder along the circles γ_1 and γ_2.

Through an arbitrary point *M* of the section, draw a line parallel to the axis of the cylinder, i.e. forming a cylinder γ, and let it intersect the circles γ_1 and γ_2, respectively at points *M1* and *M2.* The length of the segment *M1M2* does not depend on the choice of the point *M* in the section because the length of the segment *M1M2* is equal to the distance between the circles γ_1 and γ_2.

The segments *F1* and *MM1* are equal as tangents drawn from point *M* to sphere *S1*. The segments *MF2* and *MM2* are equal as tangents drawn from point *M* to sphere *S2*.

Therefore, $MF_1 + MF_2 = MM_1 + MM_2 = const$

The sum of the distances from an arbitrary point *M* of the line γ to two fixed points *F1* and *F2* is a constant value, so the line γ is an ellipse with foci *F1* and *F2*.

Using a computer program, we will construct an analog of Figure 6, i.e. a cylinder, a section of the cylinder with a plane and two spheres touching the cylinder. The distance between the centers of the spheres will be selected experimentally so that the drawing illustrates the contact of the spheres with the plane. Compare with the program "cylinder section by plane". Let's find the distance between the centers of two spheres if the radius R of the cylinder and the angle φ between the axis of the cylinder and the plane of section are known. Let's study the simulation of the intersection of two cylinders with perpendicular axes.

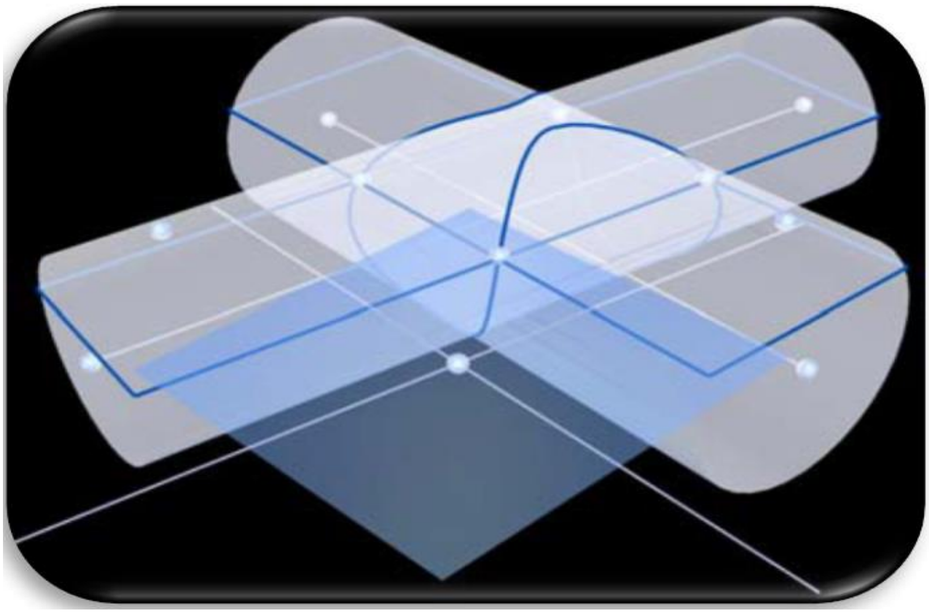

Figure 6.
The plane intersects with both cylinders and then the resulting lines formed by this intersection are intersected once more.

Consider a cylinder with the Oy axis given by the equation $x^2 + z^2 = R^2$ and a cylinder with the Oz axis given by the equation $x+^2y^2 = R_1^2$ and let $R_1<R$ (see Figure 6).

The coordinates of the points of the intersection line of the two cylinders satisfy the system.

$$\begin{cases} x^2 + z^2 = R^2 \\ x^2 + y^2 = R_1^2 \end{cases}$$

The line of intersection of two cylinders is projected onto the *oxy* plane into the circle $x^2 + y^2 = R_1^2$. Therefore, we obtain parametric projection equations from this equation.

$x = R_1 \cos t, y = R_1 \sin t$.

Substituting the value $x = R_1 cos^2 \mathrm{t}$ into the equation$x^2 + z^2 = R^2$, we find an expression for the third coordinate of the intersection points $z = \pm\sqrt{R^2 - R_1^2 cos^2 \mathrm{t}}$.

Finally, the parametric equations of the intersection line take the form:

$x = R_1 \cos t, y = R_1 \sin t$. $z = \sqrt{R^2 - R_1^2 cos^2 \mathrm{t}}$ - for the top line,

$x = R_1 \cos t, y = R_1 \sin t$. $z = -\sqrt{R^2 - R_1^2 cos^2 \mathrm{t}}$ - for the bottom line.

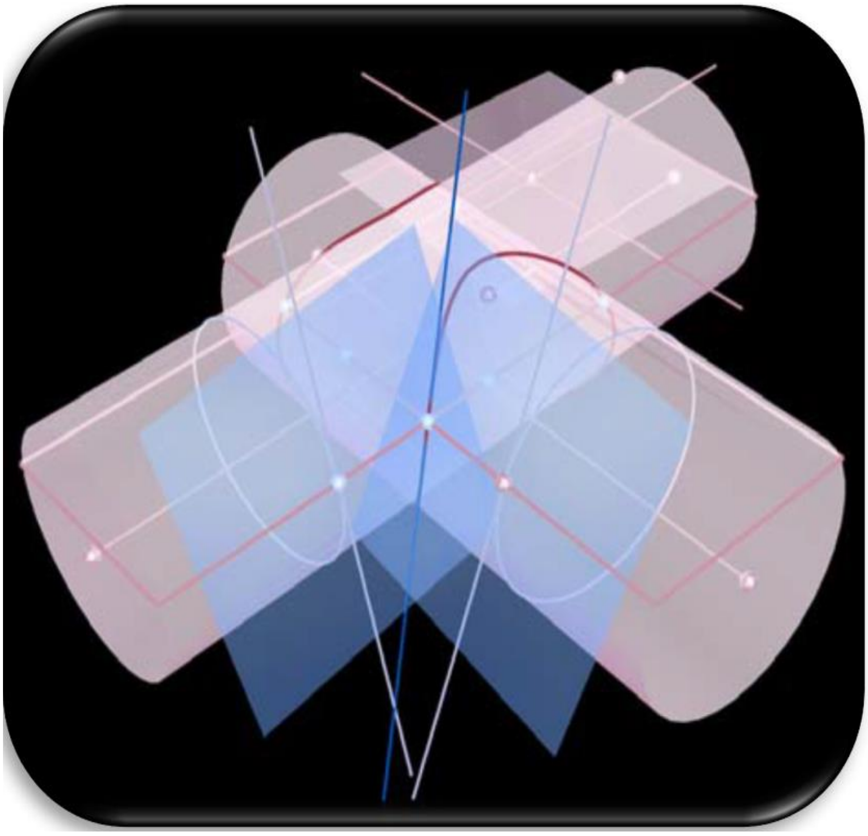

Figure 7.
A tangent (Indicated by the dark blue line) is constructed at the point of intersection.

Two closed lines of intersection of two cylinders with perpendicular axes are called bicylindrics.

The projection of the bicylindrics onto the *Oyz* plane has the equation $z^2 - y^2 = R^2 - R_1^2$ and is part of a hyperbola.

For R_1, the projection of the bicylindrics onto the *Oyz* plane has the equation $z^2 - y^2 = 0$ and is part of two intersecting lines. In this case, the bicylindric degenerates into a pair of intersecting ellipses (see Figure 7).

For independent research, it can be proposed to build a family of bicylindrics on a cylinder of fixed radius *R* provided that the radius *R1* of perpendicular cylinders varies.

A more complex project is to develop a program for modeling the intersection of a sphere of radius *R* with a cylinder of radius *R1*. If $R_1=R/2$ and one of the cylinder generators passes through the center of the sphere, the line is called the Viviani line (see Figure 8).

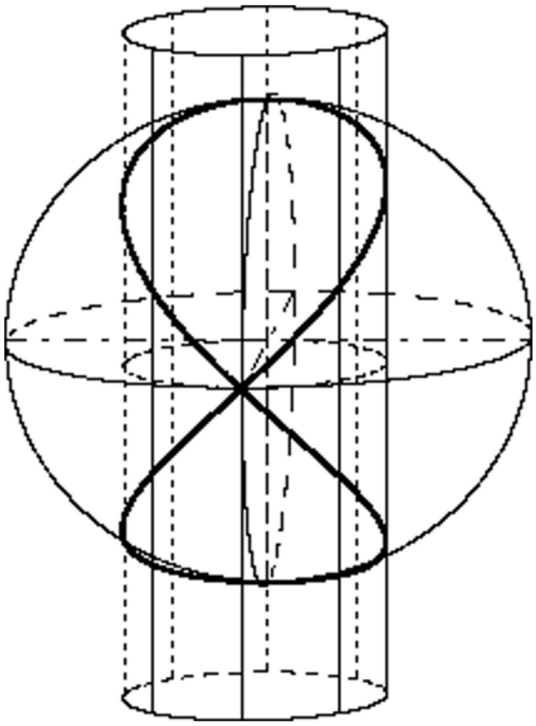

Figure 8.
The Viviani lines.

When studying different coordinate systems on the plane and in space, it is useful to consider the oblique coordinate system. If the network on the plane consists of regular triangles, then it is convenient to use an oblique coordinate system to number horizontal lines and inclined lines. If the length of the side of the smallest regular triangle is R, then for a node with coordinates (i,k) and the angular coordinates are $iR + kR\cos 60^0$; $kR\sin 60^0$.

In the educational process, we used the above digital resources using such reports in each lesson.

3.3. The Result of an Organized Learning Experiment Using Digital Resources to Develop Spatial Thinking

At the experimental stage, students completed tasks of various levels of complexity. According to the results of the tasks, it is possible to judge not only the level of development but also the prevailing type of spatial thinking of the student. When processing the data, the following results were obtained among 10th-grade students: 40% of students have a low level of development of the inventiveness factor, 45% average and 15% high. For 11th-grade students, 35% have a low level, 45% have an average level and 20% have a high level. The proposed testing system facilitated the assessment of both the development level of spatial imagination and the proficiency level in manipulating spatial images. The reliability of coincidences and differences in experimental data measured on a relational scale was verified using the Wilcoxon–Mann–Whitney criterion. At the preliminary stage, with a 5% margin of error, it was found that these groups exhibit roughly equivalent levels of spatial thinking development.

Below is the table presenting the results of a paired selective t-test evaluating the efficacy of the educational process utilizing digital education in the realm of geometry specifically focusing on tasks designed to assess students' spatial thinking levels (see Table 2).

Table 2.
Results of a paired selective t-test evaluating the efficacy of the educational process utilizing digital education in the realm of geometry.

Category	Group	Rating	M	SD	t	df	p (2-tailed)	MD
Spatial thinking	EG	Before the experiment	3.5	0.47	-9.92	99	< 0.001	-0.80
		After the experiment	4.3	0.38				
	CG	Before the experiment	3.4	0.42	-1.66	99	0.10	-0.17
		After the experiment	3.5	0.45				

Let's make a descriptive analysis based on the results of the pre-and post-training assessments of the course "geometry" by students in grades 10 and 11. The results of the control group showed that in the experimental group before the introduction of the course into the educational process and after the educational process, students showed a statistically significant increase in spatial thinking in the performance of tasks. $t(99) = -9.92$, $p < .001$ (two-tailed): if, before the experiment ($M = 3.5$, $SD = 0.47$), after the experiment, these indicators were $M = 4.3$, $SD = 0.38$. The average growth was 0.80 at a confidence interval of 95%. In the control group, however, there was no statistically significant increase in space. The average growth was 0.17 at a confidence interval of 95%.

This result confirms the validity of the H01 hypothesis. If the methodology for developing concepts for the geometry course at school is based on the integrated use of visual aids at each stage of the process of developing a geometric concept, then its implementation will provide a high level of spatial thinking and mastery of geometric concepts among students.

The table below shows the results of an independent selective t-test to assess the effectiveness of the educational process based on digital resources in the field of geometry through the task of determining the level of spatial thinking of students between experimental and control groups (see Table 3).

Table 3.
Results of an independent selective t-test to assess the effectiveness of the educational process based on digital resources in the field of geometry.

Category	Group	M	SD	Levene's test				t-test	
				F	***p***	**t**	**df**	**p (2-tailed)**	**MD**
Interest	EG	4.3	0.3	0.38	0.52	9.1	198	< 0.001	0.75
	CG	3.5	0.4						

According to the students' perspective, there was a notable statistical discrepancy between the pedagogical experiment, represented by the experimental group after completing the course ($M = 4.3$, $SD = 0.3$) and the control group ($M = 3.5$, $SD = 0.4$): $t(198) = 9.1$, $p < .001$. The disparity in average values between the two groups amounted to 0.75.

Based on the results obtained, the H02 hypothesis was also confirmed.

The reliability of the differences in the experimental results before and after the experiment was verified using the Mann-Whitney criterion. According to the data obtained before the start of the experiment, the conditions of the experimental and control groups coincided and the students of both classes had approximately the same level of spatial thinking development. And after the end of the experiment, they differ. Therefore, it can be concluded that the changes occurred due to the implementation of complex measures during the developmental experiment in the experimental classroom.

The data obtained indicate positive dynamics in the development of spatial thinking in students in the experimental class. The proportion of students with a low level of spatial thinking decreased as a percentage of students. The percentage of students with a high level of spatial thinking has increased.

4. Discussion

Currently, the role of graphic material in the assimilation of knowledge has increased. The scope of its application has expanded and new means of visualization have been introduced that is associated with the main trends in science and technology. Many images used in the educational process have become not just an auxiliary, illustrative tool that facilitates the assimilation of knowledge but an independent source for obtaining new knowledge [26].

Using augmented reality technology, you can visualize mathematical abstractions, display a generated object on the smartphone screen and create a three-dimensional geometric shapes, curves or planes. At the same time, the students' planar drawings turn into interactive 3D objects. The interaction of virtual objects visualizes an action that is almost impossible to carry out on a piece of paper. For example, to build a section of a shape with a plane.

The majority of publications by foreign experts devoted to the study of the influence of digital technologies on the development of students' thinking only confirmed their amateurish observations [27, 28]. Their research usually notes that representatives of the online generation read and remember less and less preferring to consume ready-made information in passing and combining this process with other activities.

The primary focus of many authors of manuals and textbooks introducing different digital geometry systems into education has largely revolved around delineating the functions and capabilities of these systems. As illustrative examples, they often choose problems designed for traditional computational skill development. However, certain teachers and proponents advocating for the integration of interactive dynamic systems into mathematics instruction have taken a more advanced approach. In their publications and developments, one can discover compelling examples of add-on function implementation.

We believe that a mathematics teacher should have the skills to work in various technological environments, including AR and VR technologies in accordance with the requirements of modern digital society.

5. Conclusion

In the course of this research, the hypothesis put forward was fully confirmed and the tasks set were solved which allowed us to formulate the following conclusions:

- The analysis of scientific and methodological literature and the generalization of pedagogical experience indicate that the problem of the development of spatial thinking among students is currently relevant.
- The introduction of the principles of selection of the content of the educational material of the geometry course, its structuring and presentation, the implementation of which will ensure the development of spatial thinking in students in the process of forming geometric concepts is justified.
- The didactic requirements for the creation of a set of tasks aimed at the development of students' spatial thinking are defined as the mandatory presence of tasks involving the possibility of choosing the location of given shapes.
- A pedagogical digital software tool for self-diagnosis of the level of spatial thinking development has been developed.

- The conducted pedagogical experiment showed the effectiveness of the developed methodology for the use of digital technologies for the development of spatial thinking.

It can be concluded that there are various ways to solve classical graphic problems using digital technologies and many of them are described in the pedagogical literature. This area of pedagogical activity is developing with the improvement of digital technologies and contributes to progressive changes in teaching graphic disciplines.

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