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The efficiency of defense-augmented production function for G20 countries: An analysis of the relationship with Country Fragile Index

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Abstract

This study investigates the efficiency of defense-augmented production in G20 countries and its relationship with the Fragile Index, assessing the impact of defense spending on production efficiency. This study utilizes Stochastic Frontier Analysis (SFA) and panel autoregressive distributed lag (ARDL) models, analyzing a panel dataset covering 2006 to 2021. Key variables include GDP, labor, capital, defense spending, and the Fragile Index for G20 countries. The study demonstrates a positive correlation between production inefficiency and country fragility. Countries with higher levels of fragility exhibit lower production efficiency. Additionally, while capital and labor inputs enhance production efficiency, defense spending is linked to inefficiencies. The findings emphasize the need for policymakers and defense sector stakeholders in G20 nations to address fragility-related factors in order to enhance overall production efficiency in defense-related industries. This study offers new insights into the relationship between defense spending, production efficiency, and national fragility. It adds to the literature on defense economics and production efficiency, providing a comprehensive framework for assessing the impact of fragility on economic performance in G20 countries.

Keywords: Efficiency, Fragile index, Production function, Stochastic frontier analysis.

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1. Introduction

Defense spending is essential for ensuring a country's security and military capabilities. It encompasses various public goods and services, including tanks, aircraft, maintenance, training, etc. Consequently, countries dedicate a significant portion of their resources to defense spending. The G20 comprises diverse nations with differing defense spending, labor, and capital levels.

The relationship between a country's level of fragility and its defense-driven production efficiency is a crucial concern for policymakers and defense industry stakeholders. Fragile countries are more susceptible to security threats and may struggle to maintain a strong defense industry. While initiatives to reduce a country's fragility generally positively affect production efficiency, certain potential risks may hinder production efficiency.

This paper aims to investigate the efficiency of defense-driven production in G20 countries and examine its relationship with each country's fragility index. We utilize a dataset that includes information on output, defense spending, labor, capital, and the fragility index for every G20 nation. Our analysis focuses on the production efficiency of defense-driven output and explores the connection between production efficiency and the fragility index of each country. By examining this relationship, we aim to highlight the differences in defense-driven production among G20 nations and the impact of a country's level of fragility on production efficiency. The implications of our findings may encourage policymakers and stakeholders in the defense industry within G20 countries to reform their strategies.

In conclusion, this paper enhances our understanding of the relationship between defense-driven production efficiency and country fragility in G20 nations. By analyzing the data, we aim to provide insights to inform policy decisions related to defense spending, domestic defense industry development, and international security.

2. Theoretical Framework

Defense is regarded as a pure public good because it benefits all members of society, regardless of whether individuals contribute to its provision or enjoy its advantages. Pure public goods are characterized as goods or services that are non-excludable and non-rivalrous, meaning they cannot be easily denied to those who do not pay for them, and consumption by one person does not diminish the availability of the good for others. Defense, encompassing military protection and national security, falls into this category. Additionally, national defense is crucial for maintaining stability, protecting citizens, and promoting peace and security, which makes it an essential public good.

A Nobel Prize-winning economist, Stiglitz [1], discusses the limitations of markets in achieving efficient outcomes. He refers to this economic phenomenon as "market failure." The concept of market failure suggests that markets, when left to operate freely, may not allocate resources efficiently to produce sufficient goods or services. This creates a situation where individuals lack adequate incentives to pay for public goods, resulting in market failure. In such cases, government intervention may be essential to rectify market failures and ensure the provision of public goods. Such intervention can involve funding public goods through direct government spending, regulation, or other policy measures to correct market failures and encourage efficient resource allocation.

In defense economics, it is difficult to identify a single person who was the pioneer in writing about defense economics, as the concept has likely been around and discussed for centuries. Adam Smith wrote extensively on the relationship between government spending and economic growth. He emphasized balancing military spending with other economic priorities and stressed that the government has only three duties. The first is to protect society from the violence and invasion of different countries [2]. However, one of the earliest and most influential thinkers in this field was Jean-Baptiste Say, the French economist who explored the link between military spending and economic growth in the early 19th century. He argued that military expenditures can positively and negatively affect the economy. On the one hand, he acknowledged that military spending could stimulate economic growth by creating jobs and increasing demand for goods and services [3].

On the other hand, he argued that military spending could deplete resources that could be utilized for more productive purposes and that war and other military conflicts could disrupt trade and economic activity. He also underscored the importance of balancing military spending with other economic priorities, asserting that government spending on defense should be confined to what is strictly necessary for national defense. These ideas helped to lay the groundwork for later discussions on the economics of defense and the trade-offs involved in military spending.

Another economist who made significant contributions to the field of economics, including defense economics, is Hayek [4]. His work emphasized the importance of markets, competition, and individual liberty in achieving economic efficiency and growth. He argued that government intervention in the economy, including defense spending, should be limited to sustain these conditions. Furthermore, he examined the dangers of central planning, particularly regarding defense spending, and how it can lead to the erosion of individual liberty and the rise of totalitarianism. These ideas have had a lasting impact on modern economic thought and policy, including defense economics.

Similarly, Mises [5] argued that without market prices and a profit-and-loss system, a socialist economy could not allocate resources efficiently or make rational decisions about production. This issue was referred to as the calculation problem in socialist economies. He also asserted that government intervention in the economy would result in market inefficiencies and distortions. In his view, the free-market system was the most effective mechanism for coordinating economic activity and ensuring the efficient allocation of resources, which would lead to increased economic prosperity and a more vigorous national defense. By allowing individuals to pursue their self-interest and make decisions based on market signals, Mises believed the economy would naturally adjust to changing conditions and generate the resources necessary to defend the nation.

Friedman [6] promoted a minimalist view of government intervention in the economy, arguing that a limited role for the state was necessary for national defense and other essential public goods. Furthermore, big government spending could burden the economy and lead to inflation and other economic problems. However, reducing government spending, including military spending, would allow more resources to be allocated to the private sector, leading to economic growth and increased prosperity. Moreover, resources used for military spending could be better used for other purposes, such as education and health care.

Buchanan [7] introduced the inefficiency of government spending on defense. He argued that defense spending and procurement decisions are subject to the same biases and inefficiencies as any other type of government spending. The military-industrial complex can lead to the production of weapons that are not suited for the actual defense needs of a country but are instead driven by the interests of the military-industrial complex. He emphasized the importance of considering the political and institutional factors that shape defense spending decisions. He also encouraged critically examining the incentives and biases that can lead to inefficient and wasteful defense spending.

The importance of considering trade-offs and opportunity costs in defense spending decisions was introduced by Samuelson [8]. He emphasized the importance of considering the trade-offs involved in defense spending decisions. He argued that military spending should be evaluated in the context of other important priorities, such as economic growth and development, education, and health care. He also emphasized the importance of considering the opportunity cost of defense spending, that is, the alternative uses of the resources used for defense. The economists mentioned above and others have made significant contributions to our understanding of the relationship between economics and national defense, and their ideas continue to be relevant in discussions about government spending and resource allocation.

The theoretical framework that links country fragility to defense expenditures is often grounded in the notion that fragility can drive defense spending and that excessive defense spending can contribute to fragility. The "security dilemma" concept is frequently employed to illustrate this relationship. The security dilemma posits that as one country escalates its defense spending, other nations may interpret this as a threat and respond by boosting their defense budgets. This can precipitate an arms race, in which countries become caught in a cycle of escalating defense expenditures that ultimately undermine stability and foster fragility.

Moreover, another theoretical framework that connects country fragility to defense expenses is the concept of "rent-seeking behavior." This framework suggests that elites in fragile countries may use defense spending to extract rents or resources from the state. That can lead to excessive defense spending that does not effectively improve security or stability but serves the small elite's interests [9].

In addition, the "military-industrial complex" is often used to explain the relationship between defense spending and fragility. This framework suggests that the defense industry can become powerful and influential and may lobby for increased defense spending even when not necessary for security reasons. This can contribute to excessive defense spending that undermines stability and contributes to fragility [10].

In conclusion, the theoretical frameworks connecting country fragility to defense expenses highlight the complex relationship between defense spending and fragility. These frameworks indicate that excessive defense spending can lead to fragility and that fragile countries may be more prone to rent-seeking behavior or be influenced by powerful military-industrial complexes.

The theoretical framework linking country fragility to production efficiency is often based on the notion that fragility weakens the economic and institutional factors crucial for efficient production. Fragile countries may face challenges such as weak governance, corruption, political instability, conflict, and insufficient infrastructure, which can hinder firms' ability to operate effectively. Additionally, fragility can create uncertainties and risks, discouraging investment in productive activities [11, 12].

One theoretical framework often used to explore the relationship between country fragility and production efficiency is "institutional quality." This framework suggests that strong and effective institutions, including the rule of law, property rights, and regulatory frameworks, are necessary for efficient production. Countries with solid institutional quality may need help creating a business environment conducive to efficient production.

Another theoretical framework often used to explore the relationship between country fragility and production efficiency is political economy. This framework suggests that the distribution of power and resources within a country can significantly impact production efficiency. In fragile countries, elites may use their power to extract rents or resources from productive activities, undermining efficiency and discouraging investment [13].

In conclusion, the theoretical frameworks that connect country fragility to production efficiency emphasize the importance of institutional quality and political economy factors. These frameworks suggest that fragile countries face significant challenges in creating a business environment conducive to efficient production and that addressing issues related to governance, corruption, and political instability is necessary to improve production efficiency.

3. Literature Review

Many economists think that government defense spending crowds out productive expenditures and investment, which is detrimental to economic growth. Others, however, have shown that defense spending accelerates economic growth. As a result, several studies have been conducted utilizing various samples, periods, and theoretical and empirical methodologies to evaluate the influence of defense spending on economic performance. However, the findings are frequently debatable, and the defense-growth nexus is still crucial for in-depth analysis.

Hou and Chen [14] examined the impact of defense spending on economic growth for 35 developing countries between 1975 and 2009 using the Augmented Slow Growth Model. He finds that defense negatively affects economic growth using the Generalized Method of Moments (GMM).

According to Brumm [15], increased national defense spending encourages economic growth by enhancing the protection of property rights. Defense expenditures as a percentage of GDP have also been shown to have a statistically significant beneficial effect on the rate of per capita GDP growth.

Grobar and Porter [16] concluded that increased defense spending slows economic development by reducing saving rates. Furthermore, the positive effects of defense spending on economic growth in developing countries are generally outweighed by its adverse effects.

In a two-sector economic model with a civilian and defense sector, technological change occurs in both industries, with defense acting as an externality in the civilian sector. Mueller and Atesoglu [17] found that a shift in defense spending has a positive and considerable impact on the economy's growth rate.

In a meta-analysis, Alptekin and Levine [18] analyzed 32 empirical studies to assess the link between defense expenditures and economic development. The authors concluded that while not all LDCs support the hypothesis of a negative relationship between military spending and growth, a positive effect of military expenditures on economic growth is supported for rich nations.

4. Data and Methodology

This study utilized data from the World Bank's database concerning GDP, labor, and capital. Military expenditure information was obtained from SIPRI Military Expenditure, an independent international institute based in Stockholm. Data on the Fragility Index is sourced from The Fund for Peace (FFP), a non-profit organization located in Washington, D.C. All collected data includes insights about G20 nations from 2006 to 2021. The following countries are represented in the G20: Argentina, Australia, Brazil, Canada, China, France, Germany, Greece, Italy, Japan, the Republic of South Korea, Mexico, Russia, Saudi Arabia, South Africa, Turkey, the United Kingdom, the United States, and the European Union. To prevent duplicate data, it is important to note that the European Union was excluded.

The study adopted two stages to estimate the effects of fragility and efficiency on military expenses. In the first stage, we estimated efficiency through stochastic frontier analysis. In the second stage, we employed the panel autoregressive distributed lag (ARDL) model to examine the impacts of fragility and efficiency on military expenditures. The following sections provide further details about the methodologies used.

4.1. Efficiency Estimate

The study aims to estimate the Production efficiency of G20 countries using Stochastic Frontier Analysis (SFA). It then estimates the yearly technical efficiency associated with the G20 countries. Next, it examines the relationship between countries' fragility, production efficiency, and military expenditures.

The study used the Stochastic Frontier Analysis (SFA) method developed first by Meeusen and van Den Broeck [19] and Aigner et al. [20] to estimate country production efficiency. The parameters of this method are calculated using the maximum likelihood method [21]. The method suggests that the distance between actual production and the maximum possible one represents a reasonable estimate of a country's technical inefficiency.

This study, with guidance from Battese and Coelli [22], used Y_{it} to denote the actual production of the i^{th} country at period t. Then production can be expressed as

$$Y_{it} = f\left(x_{jit};\ \beta\right).\exp(\varepsilon_{it}) \qquad (1)$$

Where X represents the vector of inputs, the subscript "i" stands for the countries, "t" for the time period, and "j" is the input type. B represents the parameter vector. The error is represented by ε_{it}, and it's composed of two error components, the v and u, where v_{it} stands for normally distributed random error with a mean of zero and a σ_v^2 variance and u_{it} is normally distributed, non-negative, and zero-truncated variables that represent inefficiency.

$$\varepsilon_{it} = v_{it} - u_{it} \qquad 2)$$

We additionally employed a logarithmic production function. As a result, technical efficiency is described as

$$TE_{it} = E[exp(-u_{it})\backslash\varepsilon_{it}] \qquad (3)$$

$$TE_{it} = \left\{\frac{1-\Phi[\eta_{it}\sigma_i^* - (\mu_i^*/\sigma_i^*)]}{1-\Phi(-\mu_i^*/\sigma_i^*)}\right\} exp\left\{-\eta_{it}\mu_i^* + \frac{1}{2}\eta_{it}^2\sigma_i^{*2}\right\} \qquad 4)$$

Where $\Phi(\cdot)$ present the standard normal distribution function and η_{it} represent the time-varying model parameter to be estimated.

$$\mu_i^* = \frac{\mu\sigma_v^2 - \eta_i'\varepsilon_i\sigma^2}{\sigma_v^2 + \eta_i'\eta_i\sigma^2}\ , \quad \sigma_i^* = \frac{\sigma_v^2\sigma^2}{\sigma_v^2 + \eta_i'\eta_i\sigma^2} \qquad (5)$$

In this type of literature, the Translog production function is the one that is most commonly employed. Therefore, the estimates from this study can be compared to those from earlier investigations. In addition, this study assumes that gross domestic product (GDP) is an output function of capital, labor, and time.

$$GDP_{it} = f(K_{it}, L_{it},\ T_{it}) \qquad .(6)$$

Where GDP_{it} is the Actual Nominal Gross Domestic Product for the country i at time t. and is intended to measure the total output produced by a country measured in millions of dollars. K_{it} is capital for country i at time t, measured by the Gross Capital Formation measured in millions of dollars. L_{it} is actual number of workers in country i at time t, measured in a million persons.

To estimate production efficiency, first, we specify the functional form f open paren x sub i. t, beta, close paren.$f(x_{it}, \beta)$The most common functional form used in the literature is the translog function. The Translog Production Frontier specification for this study is as follows:

$$\ln GDP_{it} = \beta_0 + \beta_1 \ln K_{it} + \beta_2 \ln L_{it} + \beta_3(T_{it}) + 0.5\,[\,+\beta_4\,(T_{it}^2) + \beta_5\,(\ln K_{it})^2 + \beta_6\,(\ln L_{it})^2] + \beta_7\,(\ln K_{it})(ln\ L_{it}) + \beta_8\,(T_{it})(\ln K_{it}) + \beta_9(T_{it})(\ln L_{it}) + v_{it} - u_{it} (7)$$

4.2. Panel ARDL Model

The ARDL model was first introduced by Pesaran et al. [23]. The main advantage of this model was that it found the relationship between a dependent variable and a set of independent variables with unknown stationarity levels. This model is applicable when variables are I(0), I(1), or both. The model used in this study can be expressed as follows:

$$Y_{it} = \phi_i + \sum_{j=1}^{k} \psi_j X_{it} + \epsilon_{it} \qquad (8)$$

Where Y_{it} is the defense expenditure (DS_{it}) of the country (i) in the year (t); X_{it} is a vector of regressors including the country's efficiency and country-level of frugality. The two vectors ϕ and ψ represent country-specific fixed effects and the regression parameters that estimate the impact of j independent variables; correspondingly, script epsilon is the error term.

From Equation 8, the ARDL model can be derived as follows:

$$Y_{it} = \mu_i + \sum_{j=1}^{m} \gamma_{it} Y_{it-j} + \sum_{j=1}^{n} \rho_{it} X_{it-j} + \epsilon_{it} \qquad (9)$$

Where μ_i expresses the country constant, the Bayesian information criterion (BIC) is used to obtain optimal lag lengths m and n. The "Conditional Error Correction
Model" can be presented as follows:

$$\Delta Y_{it} = \varphi_i + \sum_{j=1}^{p-1} \alpha_{ij} Y_{i,t-j} + \sum_{j=1}^{q-1} \beta_{it} X_{i,t-j} + \vartheta_i\big(Y_{i,t-1} - \delta_i' X_{i,t-1}\big) + \epsilon_{it} \qquad (9)$$

Where $\delta_i\ measures\ the\ long\ run\ parameters, \alpha, and\ \beta\ are\ the\ short\ run\ co$ and $\vartheta_i$is the error-correction term (ECT) that measures the speed of adjustment toward equilibrium.

The pooled mean group (PMG) model estimates the model's parameters. At the same time, the used dependent variable is represented by the term $DS_{it,}$ which reflects the amount of money allocated for military purposes in country i at time t, including defense expenditures and equipment measured in Billions of dollars. The independent variable used in this model is efficiency Effit, calculated from the previous section using Stochastic frontier analysis. The other independent variable is Gross Domestic Product (GDP), which reflects the total value of all goods and services produced within a country yearly, measured in billions of dollars. The Fragile States Index (FSI) is an index that was designed by the Fund for Peace (FFP). It is a set of valuable instruments and methods created to measure vulnerability and determine how it could influence field efforts. The methodology depends on open-source data, employs qualitative and quantitative indicators, and generates quantifiable outcomes. Twelve conflict risk indicators are utilized to assess a state at any time. The indications offer a moment that may be compared to other moments in a time series to see if things are improving or worse, which indicates its vulnerability to political, economic, and social instability. The FSI is used as an independent variable that describes why efficiency differs from one country to another during the period of the study.

Descriptive statistics for all study variables across each country are presented in Table 1. This table offers statistical information on GDP, capital, labor, and defense spending. It also includes various statistical measures for each country in the corresponding columns, such as the mean, maximum, minimum, and standard deviation:

5. The Results

We began by describing the data, as illustrated in Table 1. This table shows that the United States has the world's highest GDP and defense expenditure, while China has a substantial labor force and capital accumulation. Argentina displays relatively lower GDP and defense expenditures compared to other G20 countries. This may be linked to the Argentinian economy's challenges, which include prolonged periods of economic volatility and high inflation.

Australia and Saudi Arabia have relatively lower employed labor forces than other G20 countries. Despite being a developed country, Australia has a smaller population than many other G20 nations, contributing to a relatively lower employed labor force in absolute numbers. On the other hand, Saudi Arabia has a significant emigrant population, and the percentage of employed Saudi nationals in the labor force has historically been lower than that of some other G20 countries.

Canada is regarded as the least fragile G20 country, while Indonesia is viewed as the most fragile. This distinction arises mainly from Canada's political stability, well-functioning democratic system, strong institutions, and high standard of living. The country features a diversified economy, abundant natural resources, and a robust social welfare system. Its effective governance, low levels of corruption, and social cohesion significantly contribute to its stability. In contrast, despite being a diverse and populous nation, Indonesia faces challenges related to governance, social disparities, and natural disasters. Although it is an emerging economy with substantial potential, it also struggles with issues such as corruption, infrastructure development, poverty, and regional inequalities. These challenges may create perceptions of increased fragility in certain areas.

This descriptive analysis does not explicitly imply a direct relationship between Military expenditures, efficiency, and fragility. Fragility is a complex concept that encompasses various dimensions, such as political stability, social cohesion, economic resilience, and the effectiveness of governance.

While countries with higher GDPs and efficient production processes may generally possess stronger economic foundations, this does not necessarily indicate their degree of fragility. Fragility can be shaped by political instability, social unrest, conflict, economic disparities, natural disasters, and more.

It's important to recognize that each country's fragility assessment is distinctive and complex. It necessitates a thorough analysis of factors that are specific to that country's political, social, and economic context.

Therefore, while the descriptive analysis provides insights into certain economic aspects of the G20 countries mentioned, it does not directly imply a relationship between GDP or GDP production efficiency and fragility. To evaluate such relationships, a more nuanced analysis and examination of specific indicators and contexts would be required.

Table 1.
Descriptive statistics for G20 countries for the period between 2006-2021.

country		**GDP (M$)**	**Capital (M$)**	**Labor (M)**	**Defense (M$)**	**Fragility**
Argentina	Mean	464,624.50	80,134.63	17.45	3,748.30	45.83
	Maximum	643,628.70	117,221.30	18.89	5,482.62	50.10
	Minimum	232,557.30	43,442.83	16.08	1,847.55	40.80
	Std. Dev.	117,044.60	20,977.80	0.79	1,171.60	2.62
Australia	Mean	1,269,262.00	327,553.70	11.64	24,109.62	23.94
	Maximum	1,576,380.00	439,544.50	13.10	31,753.72	29.20
	Minimum	747,906.60	205,849.10	10.14	14,239.78	19.70
	Std. Dev.	256,306.90	64,345.89	0.89	4,579.23	2.92
Brazil	Mean	1,912,239.00	360,967.90	87.63	26,776.28	67.02
	Maximum	2,616,202.00	571,019.20	92.09	36,936.21	75.80
	Minimum	1,107,640.00	197,342.40	82.69	16,404.87	61.40
	Std. Dev.	437,874.90	120,714.60	2.80	6,122.91	4.08
Canada	Mean	1,652,622.00	391,240.80	18.17	20,055.06	24.49
	Maximum	1,988,336.00	472,033.60	19.58	26,449.16	27.90
	Minimum	1,319,265.00	301,860.70	16.90	14,809.89	18.70
	Std. Dev.	181,830.80	51,565.07	0.81	2,879.97	2.94
China	Mean	9,588,575.00	4,214,236.00	749.56	165,053.60	77.39
	Maximum	17,734,063.00	7,596,483.00	760.48	293,351.90	84.60
	Minimum	2,752,132.00	1,098,385.00	738.99	51,453.37	68.90
	Std. Dev.	4,410,367.00	1,902,633.00	7.84	73,446.97	4.92
France	Mean	2,693,637.00	625,754.00	27.54	51,433.64	33.58
	Maximum	2,957,880.00	740,259.40	28.62	56,647.00	35.30
	Minimum	2,320,536.00	539,239.50	26.73	45,647.47	30.50
	Std. Dev.	177,992.10	56,257.61	0.43	3,374.91	1.30
Germany	Mean	3,650,400.00	764,153.50	40.40	44,463.07	31.01
	Maximum	4,259,935.00	992,152.40	43.04	56,017.03	39.70
	Minimum	2,994,704.00	616,031.90	37.42	35,883.66	23.20
	Std. Dev.	306,459.80	101,150.10	1.78	5,141.13	5.27
India	Mean	2,021,766.00	683,650.20	439.27	51,559.02	76.41
	Maximum	3,176,295.00	990,504.70	463.00	76,598.03	79.60
	Minimum	940,259.90	366,687.30	424.19	23,951.93	70.40
	Std. Dev.	659,761.50	161,794.10	11.15	15,865.74	2.92
Indonesia	Mean	839,398.20	274,765.90	117.44	6,370.40	77.64
	Maximum	1,186,093.00	378,033.50	132.02	9,386.96	89.20
	Minimum	364,570.50	92,601.70	98.13	2,611.88	67.60
	Std. Dev.	250,515.50	94,771.72	10.55	2,245.75 y	6.45
Italy	Mean	2,086,026.00	399,590.80	22.79	29,699.32	43.00
	Maximum	2,408,655.00	524,569.40	23.39	36,839.99	45.80
	Minimum	1,836,638.00	314,198.50	22.22	22,180.85	35.10
	Std. Dev.	157,080.20	61,028.83	0.35	3,733.92	3.11
Japan	Mean	5,154,567.00	1,276,004.00	64.53	49,406.42	33.59
	Maximum	6,272,363.00	1,505,676.00	67.49	60,762.21	43.50
	Minimum	4,444,931.00	1,118,582.00	62.66	40,530.05	28.00
	Std. Dev.	526,263.30	98,665.73	1.65	6,012.29	3.91
	Mean	1,385,579.00	436,618.10	25.97	35,117.01	37.23

Korea, Rep.	Maximum	1,810,956.00	581,135.40	27.73	50,226.95	41.60
	Minimum	943,941.90	277,518.80	23.81	24,575.66	32.00
	Std. Dev.	267,404.00	83,288.58	1.43	7,859.91	2.99
Mexico	Mean	1,145,709.00	257,958.40	48.10	5,651.70	72.33
	Maximum	1,315,351.00	287,687.10	53.88	8,680.83	76.10
	Minimum	900,045.40	206,010.60	42.37	3,035.13	67.20
	Std. Dev.	116,136.30	26,520.46	3.87	1,437.70	2.36
Russian Federation	Mean	1,633,565.00	376,206.20	70.53	64,141.01	78.41
	Maximum	2,292,473.00	542,218.20	71.56	88,352.90	87.10
	Minimum	989,930.50	209,584.10	68.81	34,517.78	72.60
	Std. Dev.	371,717.00	97,777.17	0.87	14,036.87	3.47
Saudi Arabia	Mean	645,317.10	180,583.30	11.65	57,747.01	73.38
	Maximum	833,541.20	234,153.90	15.03	87,185.87	77.50
	Minimum	376,900.10	83,729.00	8.09	29,580.51	68.80
	Std. Dev.	147,760.60	42,476.71	2.24	16,845.12	3.03
South Africa	Mean	373,496.50	65,185.50	15.37	3,685.48	67.06
	Maximum	458,201.50	86,385.82	16.49	4,594.15	72.90
	Minimum	303,860.90	41,865.02	13.86	3,139.31	55.70
	Std. Dev.	46,977.01	11,016.67	0.82	445.43	4.81
Turkey	Mean	795,119.80	228,687.50	25.67	17,099.93	77.22
	Maximum	957,783.00	284,000.00	29.80	20,436.92	82.20
	Minimum	557,057.80	148,000.00	20.72	13,036.51	74.10
	Std. Dev.	106,550.60	38,993.54	3.17	1,794.86	2.56
United Kingdom	Mean	2,797,292.00	485,834.70	31.27	62,997.18	34.71
	Maximum	3,131,378.00	573,137.30	33.40	73,448.03	41.50
	Minimum	2,417,638.00	362,104.70	29.64	51,633.54	32.40
	Std. Dev.	203,264.50	58,744.51	1.46	6,344.06	2.33
United States	Mean	17,593,896.00	3,644,801.00	150.07	685,590.90	35.77
	Maximum	23,315,081.00	4,920,488.00	159.88	800,672.20	44.60
	Minimum	13,815,587.00	2,572,572.00	142.58	558,335.00	32.80
	Std. Dev.	2,913,992.00	680,229.00	5.34	67,038.56	2.85
All	Mean	3,037,005.00	793,364.50	103.9	73,931.84	53.16
	Maximum	23,315,081.00	7,596,483.00	760.47	800,672.20	89.20
	Minimum	232,557.30	41,865.02	8.09	1,847.55	18.70
	Std. Dev.	4,195,200.00	1,203,761.00	179.71	150,507.50	20.96

The Stochastic Frontier analysis method estimates the model (7). The square and product terms and most of the coefficient estimates for production inputs in Table 2 are all statistically significant. These nonlinear components wouldn't matter if the specification were a straightforward Cobb-Douglas production function. The estimated parameter γ represents the variance of the error term's inefficiency component divided by the overall error variance. It is roughly 0.98 and significant, indicating that the inefficiency component accounts for most of the variation in total error. As a result, we conclude that utilizing the stochastic frontier model is suitable.

Table 2.
Stochastic production frontier results for G20 countries during the period 2006-2021.

Variables	Coefficient	Standard-Error	T-Ratio
intercept	0.0329	4.8975	0.0067
Ln (K)	-0.5032	0.3116	-1.6146
Ln (L)	2.8246***	0.4578	6.1703
T	-0.1153***	0.0399	-2.8906
0.5*T^2	-0.0012***	0.0005	-2.5630
0.5*Ln (K)*Ln (K)	0.0607***	0.0149	4.0828
0.5*Ln (L)* Ln (L)	-0.1415***	0.0271	-5.2194
Ln (K)* Ln (L)	-0.0158	0.0133	-1.1877
T* Ln (K)	0.0019	0.0015	1.2823
T* Ln (L)	0.0041*	0.0022	1.8927
sigma-squared	0.0375**	0.0148	2.5267
gamma	0.8951***	0.0441	20.2912
mu	0.3599***	0.1097	3.2819
eta	0.0307***	0.0056	5.4964

Note: *** coefficients have 1% significance level, ** coefficients have a 5% significance level, and * coefficients have a 10% significance level

The results shown in Table 2 indicate that the effect of capital on production exhibits a nonlinear U-shaped relationship. There is a negative but insignificant relationship at low levels of capital, so we cannot conclude that there is a negative relationship between capital and output. However, since we have a positive and significant coefficient for the squared log of capital, at high levels of capital, as capital increases, output also increases, leading to increasing returns to scale.

Similarly, labor has a nonlinear relationship with output. Still, this relationship depicts an inverted U-shape, as the coefficient of log(l) is positive while the coefficient of squared log(L) is negative. This suggests that initially, increasing labor leads to greater output. However, after a certain point, the rate of output increase declines as labor continues to rise.

The coefficient of the interaction term between log(K) and log(L) was negative but not significant, which may indicate that the diminishing marginal product of one input becomes insignificant as the level of the other input increases.

Counterintuitively, the coefficients for time and time squared exhibit significant negative signs. Since time may serve as a proxy for technological progress, this could suggest that the productivity associated with technological advancements is declining at an increasing rate. This outcome might result from improper implementation or negative externalities linked to new technologies.

The coefficients of the interaction terms between time and log(K) and between time and log(L) were positive. This suggests that as technology advances, the productivity of labor and capital also increases. However, since the coefficient of the first interaction term was insignificant, we cannot confirm its effect.

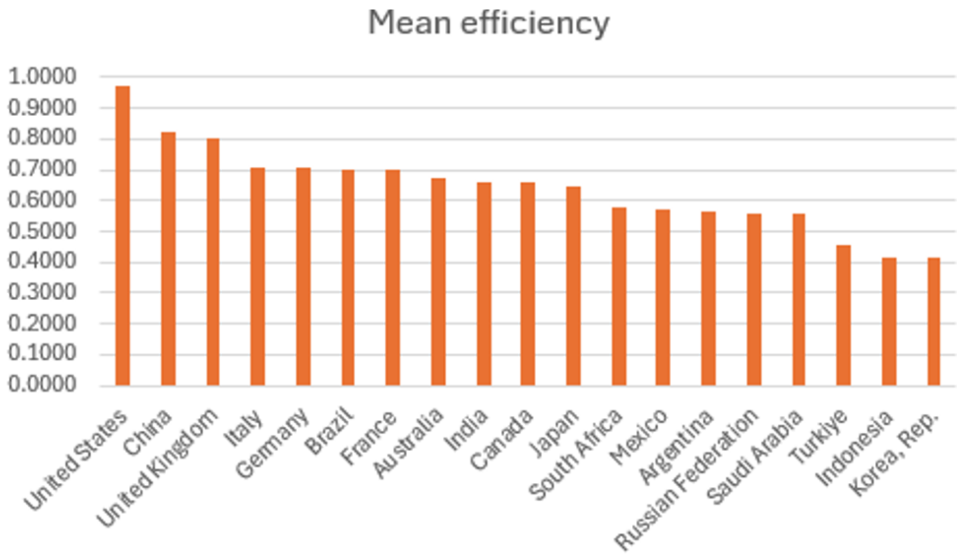

Figure 1.
The mean efficiency for G20 countries during 2006-2021.

Figure 1 displays the mean estimated efficiency derived from the previously mentioned stochastic frontier model. The figure indicates that the United States, followed by China and the United Kingdom, exhibited the highest mean efficiency throughout the study period. Conversely, the Republic of Korea, Indonesia, and Turkey recorded the lowest mean efficiency in succession.

After estimating the annual efficiency of each G20 country during the study period, the research progressed to evaluating the feasibility of using a panel ARDL bounds testing approach. Therefore, we began by conducting the panel unit root test to assess data stationarity. The results of this test are shown in Table 3.

Table 3.
Panel unit root tes.

Variable	Levin, Lin & Chu Test				Im, Pesaran, and Shin W Test			
	Level		First difference		Level		First difference	
	Statistic	Prob	Statistic	Prob	Statistic	Prob	Statistic	Prob
Eff	-65.0657	0.0000	-	-	-68.7133	0.0000	-	-
FRGALITY	-3.1322	0.0009	-1.2887	0.0988	-0.8304	0.2032	-1.6536	0.0491
DEF	3.8046	0.9999	-6.0367	0.0000	5.5004	1.0000	-1.9885	0.0234

Test results indicate that countries' yearly production efficiency is stationary at a certain level, while their fragility and military expenditures are stationary at the first difference.

The study then employed Kao test statistics to examine the long-run cointegration relationship among our variables. The results of the test are presented in Table 4.

Table 4.
Kao test statistics.

Null Hypothesis	ADF T-statistics	P-value	Result
No cointegration	2.3926	0.0084	Cointegrations exist

The test confirms the presence of a long-run cointegration relationship among our variables at the 10% significance level. Then, we tested for the presence of collinearity among independent variables. We found that the pairwise correlation between the two independent variables was equal to -0.3593, which rejects the presence of a collinearity problem among our independent variables.

The diagnostic test above supports using the PMG model to estimate the effects of efficiency and fragility on military expenditure. The model's results are shown in Table 5.

Table 5.
Panel ARDL – PMG estimator.

	Variable	Coefficient	Std. Error	t-Statistic	Prob.
Long run Coefficients	EFF	-2594.1388***	478.1077	-5.4258	0.0000
	FRGALITY	6.0966***	0.5091	11.9756	0.0000
short run Coefficients	COINTEQ	-0.5494***	0.1490	-3.6880	0.0003
	D(DEFB(-1))	0.2141**	0.0911	2.3492	0.0196
	D(EFF)	-987041.0167	699435.0963	-1.4112	0.1594
	D(EFF(-1))	881757.2674	680120.4930	1.2965	0.1960
	EFF*FRGALITY	-5.3147***	1.4832	-3.5832	0.0004
	C	1611.8578	417.7260	3.8586	0.0001

The Panel ARDL (PMG) model presented in Table 5 shows both short-run and long-run coefficients that illustrate the impact of countries' efficiency and fragility on military expenditures. Additionally, the model estimates the error correction term, which is negative and significant, indicating the presence of convergence over the long term and confirming that equilibrium will be restored after 1.8 years.

The study reveals a significant positive relationship between fragility and military expense over the long term. Specifically, a 1% increase in fragility is associated with a $6.1 billion rise in military expenditure. Conversely, the study identifies only a negative relationship between efficiency and military expense over the long term, indicating that a 1% increase in efficiency results in a reduction of military expenses by $2,594 billion.

Furthermore, the study shows that increasing any independent variable will minimize the impact of the other independent variable in the short term. It also indicates that past military expenditures significantly positively influence current military expenditures in the short term.

The findings of this study align with several key theoretical insights discussed earlier in the literature. Buchanan [7] highlighted the inefficiencies inherent in government defense spending, primarily due to the influence of the military-industrial complex. This correlates with the study's results, which indicate that higher levels of country fragility significantly increase military expenditures while simultaneously reducing production efficiency. Stiglitz's [1] concept of market failure further supports this, as inefficient defense spending in fragile countries can reflect poor resource allocation stemming from institutional weaknesses. Similarly, Say [24] pointed out the dual nature of military expenditures, noting that while necessary, excessive defense spending can divert resources from more productive sectors. The non-linear relationships observed in this study between capital, labor, and output illustrate this trade-off, where defense spending initially boosts output but ultimately leads to diminishing returns, as Samuelson [8] also suggested in his discussions on opportunity costs. These connections reinforce the study’s conclusion that reducing fragility and improving institutional quality are essential for enhancing production efficiency in defense-related industries.

6. Conclusions

This study analyzes the relationship between defense-augmented production efficiency and country fragility among G20 nations from 2006 to 2021. The findings confirm that higher levels of country fragility correlate with lower production efficiency, highlighting the adverse effect of fragility on economic productivity in the defense sector.

The analysis of capital and labor inputs reveals a non-linear relationship with output, suggesting that optimal levels of these inputs can enhance production efficiency. Specifically, the study finds that capital investments yield returns only after surpassing a certain threshold, while labor input exhibits diminishing returns beyond an optimal level. Counterintuitively, the coefficients for technological progress indicate that productivity is declining at an increasing rate, possibly due to improper implementation or negative externalities associated with new technologies. These insights emphasize the necessity for strategic planning in capital investments and labor utilization to achieve optimal production efficiency.

Furthermore, the long-term analysis indicates that increasing fragility significantly drives up military expenditures, while enhancements in production efficiency can substantially reduce these costs. Over the long term, a 1% rise in fragility is linked to a $6.1 billion increase in military expenditures, whereas a 1% improvement in efficiency corresponds to a $2.594 billion decrease in these costs. The study also suggests that in the short term, rising any independent variable lessens the impact of the other on military expenses, with previous military expenditures positively influencing current spending.

These findings highlight the need to reduce country fragility to improve production efficiency in defense-related industries. To optimize defense spending, policymakers should establish strong institutional frameworks, encourage transparency and accountability, and strategically invest in capital and labor.

Future research could build on the current study by examining the factors contributing to the declining productivity of technological advancements in defense sectors. Investigating the reasons behind improper implementation or negative externalities related to new technologies could provide valuable insights into best practices for technology adoption and its positive impacts on production efficiency.

Additionally, conducting in-depth case studies on specific G20 countries would enhance the understanding of the unique challenges and strategies affecting defense production efficiency and fragility. These case studies could provide detailed insights into the policies and practices that have been either successful or ineffective, enabling a more nuanced comprehension of the complex dynamics at play.

Including non-G20 countries in the analysis would enable a comparative study of the effects of fragility and efficiency on global defense spending. This comparison might uncover whether the findings from G20 nations apply in other contexts and could highlight global trends or specific regional challenges that affect defense spending efficiency.

Another critical area of research is exploring the impact of external shocks, such as economic crises or geopolitical tensions, on the relationship between the efficiency of defense spending and national fragility. Understanding how these shocks influence defense-related production could assist policymakers in developing more resilient strategies for maintaining efficiency during turbulent times.

Lastly, conducting longitudinal studies to evaluate the long-term effects of policy changes aimed at reducing fragility and enhancing production efficiency in defense-related industries would yield critical insights. These studies could monitor the progress of specific interventions over time, providing evidence of their effectiveness and assisting in refining future policy approaches.

Future research can explore these areas to better understand the complex dynamics among defense spending, production efficiency, and national fragility. This will ultimately assist policymakers in making informed decisions to strengthen national security and economic stability.

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