sInternational Journal of Innovative Research and Scientific Studies, 8(3) 2025, pages: 1563-1572

IJIRSS International Journal of Innovative Research & Scientific Studies

ISSN: 2617-6548

URL: www.ijirss.com

Designing a Hypothetical Learning Trajectory with Open-Ended Tasks: An Approach to Teaching Angle Measurement

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Abstract

This study aims to examine the effectiveness of integrating the Hypothetical Learning Trajectory (HLT) with an open-ended approach in teaching angle measurement to pre-service elementary school teachers, addressing the passive learning behavior often observed in PGSD (Primary School Teacher Education) students. Using a design research methodology, this study focused on the first phase preparing for the experiment. It involved 30 third-semester PGSD students in the Basic Mathematics Concepts 2 course. The process included a literature review, student needs analysis, and the development of an initial learning trajectory. Data were collected through pretests, posttests, classroom observations, and student reflections. The findings revealed a notable improvement in students' conceptual understanding, with average test scores increasing from 56.4 (pretest) to 84.7 (posttest). Students also showed enhanced critical thinking, creativity, and confidence in problem-solving. However, some challenges remained, particularly related to disparities in student abilities and initial resistance to the open-ended learning model. The integration of HLT and open-ended approaches effectively fosters deeper mathematical understanding and active engagement among PGSD students, helping transform passive learning behavior into dynamic participation. The results support the implementation of HLT and open-ended learning in teacher education programs to cultivate essential 21st-century skills. Educators are encouraged to receive training in HLT development and explore digital tools to enhance learning experiences and accommodate diverse student needs.

Keywords: Active Learning, Hypothetical Learning Trajectory, Open-Ended Approach, Pre-Service Teachers.

DOI: 10.53894/ijirss.v8i3.6837
Funding: This study received no specific financial support.
History: Received: 18 April 2025 / **Revised:** 20 April 2025 / **Accepted:** 26 April 2025 / **Published:** 7 May 2025

Competing Interests: The authors declare that they have no competing interests.
Authors' Contributions: All authors contributed equally to the conception and design of the study. All authors have read and agreed to the published version of the manuscript.
Publisher: Innovative Research Publishing

1. Introduction

Students enrolled in the Elementary School Teacher Education Study Program (PGSD) play a crucial role in influencing the quality of education at the elementary level. As individuals who are prepared to educate the younger generation, they must have a good mastery of teaching materials as well as skills in delivering material to students effectively and engagingly. However, in the Basic Concepts of Mathematics 2 course at the PGSD Faculty of Teacher Training and Education [1], it was found that students prefer to receive explanations directly from lecturers rather than actively participate during the learning process. This shows that students tend to be passive in learning and less involved in activities that can improve their understanding of mathematical concepts. As a result of this lack of active involvement, their understanding is often temporary and easily forgotten in a short period [2-7]. The lack of active involvement of students in learning causes them to ask questions frequently, be reluctant to answer questions, and be less accustomed to thinking independently. This is a challenge in learning because one of the main goals of education is to develop critical and creative thinking skills [8]. If students only rely on explanations from lecturers without conducting independent exploration, their understanding will become shallow and unsustainable [9]. This passive attitude also has an impact on weak understanding of concepts, because students are not used to constructing their knowledge through meaningful learning experiences. In the context of teacher education, the lack of critical and creative thinking skills is very dangerous because teachers who are unable to think deeply and flexibly will have difficulty in developing innovative learning strategies that are responsive to the needs of students [10, 11].

To overcome this challenge, a learning strategy is needed that can promote students' active engagement in the learning process. One strategy that can be applied is through Hypothetical Learning Trajectory (HLT), which is applied with an open-ended approach [12, 13]. HLT is a concept in design research that aims to design a learning flow by developing students' thinking in understanding mathematical concepts. HLT consists of three main components, namely learning objectives, learning activity plans, and alleged development of student understanding [14, 15]. In the context of learning at PGSD, HLT serves as a guideline for lecturers in designing more effective teaching strategies, and at the level of the development of students' thinking [16]. By understanding how students think and learn, lecturers can develop learning activities that are more dynamic, interactive, and relevant to their needs [17].

The open-ended approach is a learning strategy that emphasizes providing open-ended problems that have a variety of possible answers or solutions. This method aims to provide space for students to explore various ways of solving problems, thereby encouraging them to think more creatively and critically [18, 19]. This approach provides flexibility in thinking and solving problems, so that students are not only fixated on one way of solving problems taught by the lecturer. With the variety in solutions that can be found, students will be encouraged to think more broadly and look at a problem from various perspectives. According to McGregor [20] and Ismail et al. [21], the open-ended approach can be used as a pedagogic strategy that can develop students' creative thinking. In addition, [10] emphasizes that basic and imaginative considering abilities can be improved through significant learning that combines open-ended questions and problem-solving. One of the main benefits of an open-ended approach in learning is the expanded conceptual understanding. When students are given a problem that does not have a definitive answer, they are forced to understand the concepts being studied to find various possible solutions. In this way, their understanding becomes more in-depth and not just memorizing certain formulas or procedures. In addition, this approach also encourages the development of student creativity. In solving open problems [22], they have the opportunity to think outside the box and try different approaches. This can train them to be more innovative in thinking and develop more effective solutions. In addition, Additionally, an open-ended approach might boost students' confidence in their ability to solve problems. By being given the freedom to explore various solutions, they will feel more confident in their thinking abilities. Another benefit is strengthening collaboration skills through group discussions. When students work in groups to solve an open-ended problem, they will learn how to discuss, share ideas, and constructively defend their arguments [21, 23]. In this case, an open-ended strategy was used to teach angle measurement using HLT. Contextual difficulties are one of the phases that make up the learning process, including group discussions, presentation of results, and reflection and analysis of various solutions found. In the early stages, students still have difficulty understanding the concept of clockwise movement about the size of the angle [24]. They are used to predetermined solutions and find it difficult to find various possible solutions on their own. However, as the discussion and exploration in the group progressed, they began to understand the relationship between the position of the hands and the angles formed. With interaction between students, they can share mutual understanding and help each other find the right concept. At the end of the lesson, students can not only determine the size of the angle more accurately, but also develop various ways to solve the given problem [25, 26].

The results of this study show that the application of HLT with an open-ended approach can have a positive impact on the mathematical understanding of PGSD students. Students who were initially passive became more active in discussing and expressing their opinions. They also began to get used to thinking more independently and not only relying on the lecturer's explanations. With the freedom to solve problems, they feel more challenged to find creative and innovative solutions. Therefore, there are several recommendations for future learning, namely, strengthening lecturer training in compiling HLTs so that they can design a more effective learning flow, and at the level of student development. In addition, the use of varied learning media is also needed to support the learning process to make it more interesting and interactive [27, 28]. A continuous evaluation of the effectiveness of this approach in the long term must also be carried out to see the extent to which this approach has an impact on student understanding. In addition, integration with digital technologies such as learning management systems (LMS) can also be considered to enrich students' learning experiences and provide wider access to various learning resources [10, 29, 30]. Students can become more engaged in their education and acquire the necessary abilities for their future careers as instructors if the proper approach is taken. Teachers who possess both critical and creative thinking abilities and a thorough comprehension of mathematical ideas will be more equipped to handle issues in the classroom. To obtain more optimal and durable learning outcomes, this strategy needs to be further developed in the future. To find out how this strategy can be modified to fit different student profiles and used in other PGSD courses, more research is required [31].

2. Literature Review

2.1. Hypothetical Learning Trajectory (HLT)

Hypothetical Learning Trajectory (HLT) is one of the main components of the design research approach, which aims to design and develop learning systematically based on an understanding of how students learn [12, 29]. Students can become more engaged in their education and acquire the necessary abilities for their future careers as instructors if the proper approach is taken. Teachers who possess both critical and creative thinking abilities and a thorough comprehension of mathematical ideas will be more equipped to handle issues in the classroom. To obtain more optimal and durable learning outcomes, this strategy needs to be further developed in the future. To find out how this strategy can be modified to fit different student profiles and used in other PGSD courses, more research is required [14, 32]. This approach also accommodates the principles of Realistic Mathematics Education (RME), where learning begins in a meaningful context and develops towards formal understanding [33, 34]. In addition, HLT supports dynamic, adaptive, and needs-based learning, which is critical in elementary school teacher education [35].

2.2. Open-Ended Approach in Mathematics Learning

An open-ended approach in mathematics learning is a strategy that provides students with the opportunity to solve problems that have more than one correct answer or multiple ways of solving. The goal is to encourage students to think creatively, critically, and flexibly in solving mathematical problems [23]. Unlike conventional questions that have one correct answer, open-ended questions direct students to explore various possibilities, compare strategies, and reflect on their thinking. According to Klavir and Hershkovitz [18], this approach is effective for improving mathematical reasoning skills because it allows students to build knowledge based on their own experience and cooperation with peers. In practice, students are encouraged to propose various solutions, discuss alternative solutions, and defend their arguments, so that a meaningful and contextual learning process occurs. McGregor [20] stated that the use of open-ended questions can develop high-level thinking skills such as divergent and reflective thinking. In addition, this approach also increases students' confidence because they feel given space to think freely and express ideas independently [10].

2.3. Common Difficulties in Angular Measurement Learning

Angle measurement is one of the important topics in basic math learning, but it is often a source of difficulty for students. One of the main difficulties faced by students is the misconception of the position of the clock, especially in the context of questions that involve the simultaneous movement of short and long needles [25]. Students tend to assume that the shorthand remains exactly at the hour mark, when in fact its position changes proportionally over time. This shows that there are weaknesses in conceptual understanding that cannot enough to be conveyed only through procedures or formulas. Students often fail to integrate visual representations with the actual concept of angles, especially when dealing with a time that is not round (e.g., 03.30 or 04.15) [26]. This common mistake indicates the need for a learning approach that links concepts to concrete experiences and accurate visual representations. Further, students' understanding of angles is often limited to geometric definitions as "space between two lines", without relating them to rotation or changes in relative position in dynamic contexts [26]. This hinders students' ability to apply angular concepts in real-life situations, such as reading clocks or analyzing rotational movements.

2.4. The Role of PGSD Students in Basic Mathematics Education

Elementary School Teacher Education (PGSD) students have a strategic role in improving the quality of mathematics learning at the elementary education level. As prospective teachers, they are not only required to master the material but must also have pedagogical skills that can arouse students' interest in learning and critical thinking [1]. However, various studies show that PGSD students still tend to be passive in the mathematics learning process, relying more on lecturers' explanations than exploring concepts independently [2, 3]. This passive attitude can negatively impact an in-depth understanding of mathematical concepts, as well as hinder the development of creative and reflective thinking skills that are essential for the teaching profession [10]. If students are not used to constructively building understanding, they will tend to replicate traditional learning models with minimal student participation. An activity-based learning approach and active engagement, such as open-ended tasks or problem-based projects, can help change passive learning patterns to be more Dynamic. Therefore, the PGSD study program needs to design a learning experience that is not only oriented towards mastery of the material but also develops high-level thinking competencies, communication, and collaboration.

2.5. Design-Based Learning (Design Research)

The design research approach in education aims to develop effective learning solutions through iterative cycles of design, implementation, and reflection [12, 36, 37]. In the context of mathematics education, design research allows the development of learning based on the real needs of students, by integrating theory and practice directly in the field [33, 38]. This makes design research a relevant approach to produce contextual and sustainable learning innovations.

2.6. Development of Critical, Creative, and Collaborative Thinking Skills

Critical, creative, and collaborative thinking skills are essential competencies of the 21st century that need to be developed through challenging and open learning strategies [10]. The open-ended approach is considered effective in encouraging students to explore various solutions, consider alternatives, and discuss productively with peers [20]. These kinds of activities reinforce the habits of reflective thinking and teamwork in the context of meaningful learning.

2.7. Integration of Technology in Mathematics Learning

The integration of technology in mathematics learning can expand students' access to interactive and in-depth learning resources. The use of Learning Management Systems (LMS), digital simulations, and interactive visual media has been proven to increase students' engagement and understanding of mathematical concepts [30]. Technology also enables personalization of learning and facilitates reflection on learning through online discussion forums and project-based assessments.

3. Methods

This study uses the design research method, which is a research approach that aims to develop and test the effectiveness of learning design iteratively through the cycle of design, implementation, and reflection [12]. In this approach, the research process focuses on theoretical testing and developing learning strategies that can effectively improve students' understanding and skills. The essence of design research is a cyclic process consisting of thought experiments and learning experiments [12, 14]. These two experiments are run repeatedly and complement each other in the process of designing better learning. In the long-term process, these two activities can be seen as cumulative cyclic processes as shown in Figure 1.

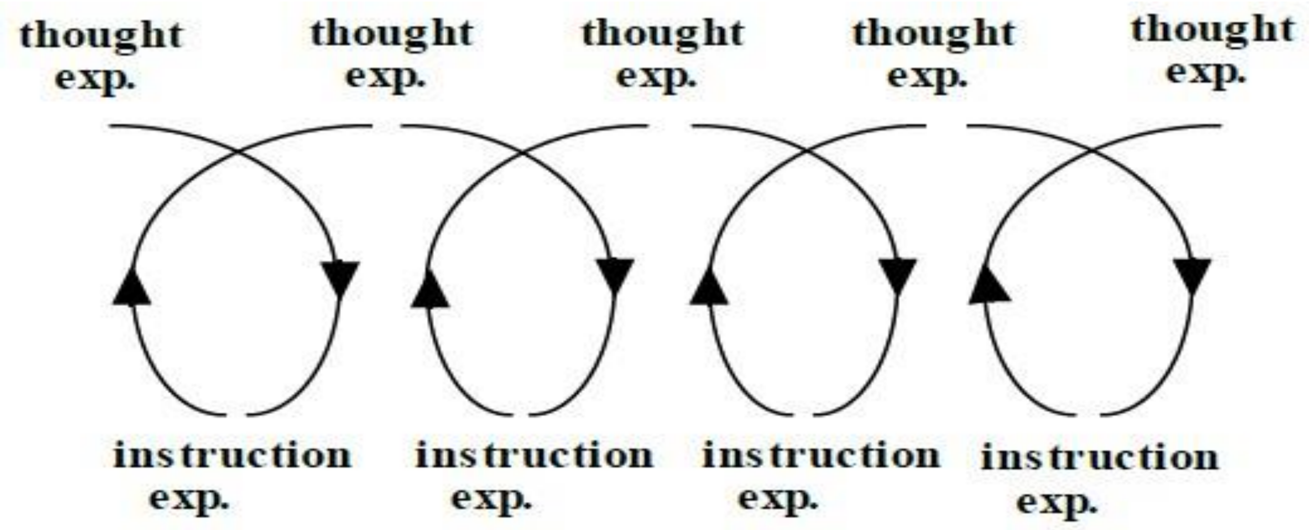

Figure 1.
Cyclical Thought Process & Classroom Experiment.
Source: Gravemeijer [33].

As stated by Gravemeijer and Cobb [12], design research consists of three main stages, namely preparing for the experiment, conducting the experiment, and retrospective analysis. However, in this study, the main focus is on the first stage, which is preparing for the experiment. This stage includes various processes that aim to develop an initial design of learning activities based on literature review and student needs analysis. This research was conducted on students of the Elementary School Teacher Education Study Program (PGSD) who are taking the Basic Concepts of Mathematics 2 course at the Faculty of Teacher Training and Education. The research subjects were 30 third-semester students who were selected purposively based on certain characteristics, namely passive tendencies in learning and difficulties in understanding mathematical concepts in Depth [2-5, 39]. The creation of learning designs through the Hypothetical Learning Trajectory (HLT) approach is the main emphasis of this study, combined with open-ended methods in angle measurement learning. In the preparation stage for the experiment, the first step is a literature review to identify previous theories and findings that are relevant to the development of learning design. The literature review covers various aspects, such as the basic concepts of design research, the principles of HLT, and how well the open-ended method works for learning mathematics. This study serves as the foundation for creating educational activities that should enhance students' conceptual understanding.

After the literature review is carried out, the next step is to analyze the needs of students. This analysis aims to understand how students learn the concept of angle measurement, the difficulties they face, and the extent of their involvement in mathematics learning. Data in this stage was collected through interviews with the lecturers in charge of the course and initial observations of the learning process that took place. This observation was carried out to determine how well students engaged with the course material and how much they contributed in class. According to the findings of the interviews and observations, students typically learn more passively, ask fewer questions, and depend more on lecturers' direct explanations of concepts than on their independent investigation. These findings are in line with previous research that showed that PGSD students often experience difficulties in critical and creative thinking in mathematics learning [10, 30].

The next step in the preparation stage for the experiment is the design of HLT, which is the basis for developing learning activities. HLT consists of three main components, namely learning objectives, learning activity design, and alleged development of student understanding [14]. In the context of this study, the learning objectives were formulated based on the concept of angle measurement, with a focus on understanding the relationship between the position of the clockwork and the magnitude of the angle formed. The design of learning activities is then developed by considering the open-ended principle, which is to provide open problems that allow students to explore various ways of solving. The activities designed include contextual problem solving, group discussions, presentation of results, and reflection on various solutions found. At this design stage, the problems given are designed so that students can relate the concept of angle measurement to everyday situations. One example of the problem used is how to determine the magnitude of the angle formed by the movement of the clockwise in various situations. This issue was chosen because it is relevant to students' daily lives and allows them to build a deeper conceptual grasp. Students are supposed to investigate different approaches to problem-solving through group discussions and contrast their answers with those of their peers.

Once the HLT design has been developed, the next stage is an initial trial on a small scale to evaluate how well the learning design works before it is widely applied in the teaching experiment stage. This trial was carried out in a lecture session with a small group of students, where the lecturer applied the designed activities and observed how students interacted with the learning material. During the trial, observations were made on the students' involvement in the discussion, the strategies they used in solving problems, and the extent to which they could develop a better conceptual understanding. In addition, a short interview was conducted after the learning session to get feedback from students regarding their experience in participating in HLT-based learning with an open-ended approach. The data obtained from observations and interviews were analyzed descriptively to evaluate whether the learning design met the needs of students. If obstacles or weaknesses are found in the design, revisions are made before the teaching experiment stage is carried out. The analysis in this stage focuses on how students understand the concepts being taught, how they participate in group discussions, and whether the planned exercises adequately foster the growth of their capacity for critical and creative thought [18, 19, 40].

The results of this preparatory stage for the experiment become the basis for the next stage, namely the teaching experiment, where the learning design that has been developed will be applied in a larger class. By using a design research approach, this research allows the development of more effective learning strategies based on the real needs of students. In addition, the implementation of HLT with an open-ended approach is expected to contribute to improving the quality of mathematics learning in PGSD, so that students not only understand concepts more deeply but also can develop thinking skills that are essential in the teaching profession [10, 19]. Thus, the preparation stage for the experiment in this study provides a strong foundation for the development of learning designs based on student needs and supported by relevant learning theories. Literature review, student needs analysis, HLT development, and initial trials are important steps in ensuring that the learning design applied is truly effective in improving the understanding of mathematics concepts of PGSD students. In the future, this research can be further developed by exploring how the teaching experiment and retrospective analysis stages can be used to further improve and refine the designed learning design.

4. Development

Learning Design Process: An open-ended method for measuring angles. The development of students' purported learning trajectories and the design of learning activities are crucial aspects to monitor and examine during the early design stage. The learning path and student learning trajectory for the topic of angle measurement are examined before creating learning activities. Additionally, the context, learning activities, and purported student learning trajectory utilized in angular measurement learning will be transformed into a student learning trajectory.

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4.1 Preliminary Design

The following image illustrates the Realistic Mathematics Education (RME) approach in teaching the concept of angles. Starting from real situations such as clockwise or angular curves (situational level), to the use of visual aids such as protractors and angular measurements (referential/model level), to achieving an abstract understanding of the types of angles, angular sizes, and the concept of angles as part of geometry (formal level). This approach helps students build conceptual understanding in a meaningful and contextual way.

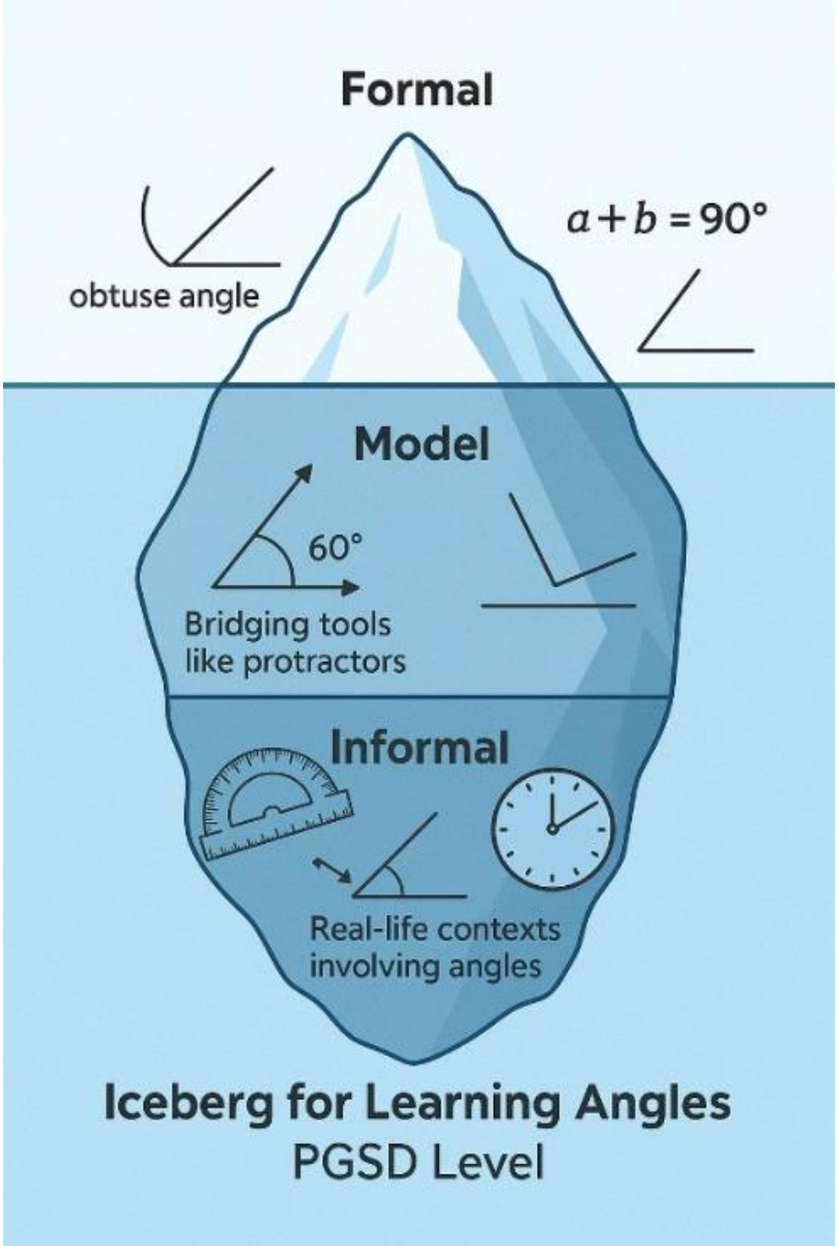

Figure 2.
The Iceberg Structure of Angular Concept Learning for PGSD Students: From Contextual Activities to Conceptual Understanding.

Learning began with lecturers distributing student activity sheets (MFIs) that had been designed using an open-ended approach. This MFI begins with contextual problems, which are then followed by a pretest to measure students' initial understanding of angle measurement. In the pretest, students are asked to determine the angle size formed at certain times, such as 02.00, 03.30, 04.15, and 05.18. The results of the pretest show that many students have difficulty calculating the angle correctly, mainly because they do not take into account the movement of the short needle. For example, in a question that asks about the angle formed at 3:30 a.m., many students answer 90°, because they already understand that the long needle is at 6, but they still think that the short needle remains at the number 3. The short needle should be between the numbers 3 and 4. Similar errors also occur in other questions, showing that students do not understand the concept that short needles also move in proportion to time. In the first activity, students start working in small groups and are asked to choose for themselves a time that is a multiple of one hour, in addition to the one given in the pretest. They then drew the clock according to the time they chose, positioned the long hands and short hands correctly, and calculated the magnitude of the angle formed. In the group discussion, students explain their answers to each other and conclude the results obtained. After that, one of the group members was chosen to present their work. At this stage, students begin to show a better understanding of the position of the needle and calculate the angle correctly.

In the second activity, students were asked to choose a time that was 30 minutes from their unit of time. However, at this stage, there are still some students who make mistakes in positioning the short needle because they still direct the short needle precisely at the hour number, without considering that the needle also moves when the long needle moves. They have also not been able to determine the amount of angle taken by the short needle within 30 minutes. With the direction of the lecturer, who does not directly provide an answer, students can finally find the concept that the movement of a short needle every minute is 0.5°. After the discussion, each group concluded its answers, and then several selected groups presented their work. In the presentation, students were able to explain well how they determined the size of the angle and the method they used. However, most students still struggle to find various other methods to get the same results. For example, if a group chooses a time of 4:00 a.m. and finds the angle formed is 120°, they use only one calculation method, which is 4 × 30°. There are several other alternatives, such as 180° – (2 × 30°) or 360° ÷ 3. This shows that their thinking is still not fully flexible in implementing an open-ended approach.

Entering the third activity, students are asked to choose a time that is 15 minutes or 45 minutes from the unit of time. Thanks to the experience from previous activities, students began to be more skilled in determining the position of short needles and calculating the magnitude of the angles. In the fourth activity, students were given a more complex challenge, namely determining the angle at a time that is not a multiple of five minutes. At this stage, students already understand the concept of short needle movement, but still have difficulty in determining the angle that the long needle takes in one minute. With the guidance of the lecturer, who encouraged discussion, the students finally discovered that the movement of the long needle every minute was 6°. In the presentation session for the third and fourth activities, the students who performed were able to explain well the angular calculations they did, and the other groups began to be able to provide alternative calculation methods. Their ability to apply the open-ended approach has increased compared to the previous presentation session. Despite this, they still have difficulty in finding a wider variety of methods. The entire learning process shows the gradual development of student understanding. At first, they have difficulty considering the movement of the short needle, but through discussion and direction from the lecturer, they can discover important concepts that support their understanding. Meanwhile, the application of the open-ended approach is starting to show positive results, with more and more students being able to think flexibly and find various alternative calculation methods. This shows that an open-ended approach can help improve students' conceptual understanding of angle measurement and train them to think more creatively in solving mathematical problems.

4.2. Results of Learning Implementation with an Open-Ended Approach

PGSD students' comprehension of the Basic Concepts of Mathematics 2 course is positively impacted by the use of an open-ended approach in angular measurement instruction. In general, the results of this study reflect a significant improvement in the aspects of conceptual understanding, critical thinking skills, and students' collaborative skills. Quantitative data obtained from the pretest and posttest showed an increase in the average score of students from 56.4 in the pretest to 84.7 in the posttest [2, 6, 25, 26, 41]. This improvement shows the effectiveness of learning strategies designed based on Hypothetical Learning Trajectory (HLT) with an open-ended approach [42, 43]. Most students who initially had difficulty in determining the magnitude of the angle due to a lack of understanding of the movement of the short-hand of the clock, after following a series of learning activities, were able to correct their own mistakes and provide various alternative solutions. Observation data also shows that active student involvement increases along with the implementation of learning. If at first only about 30% of students were actively discussing in groups, by the end of the learning session, the participation rate increased to 85%. This improvement is supported by the open-ended method, where students are allowed to explore various problem-solving strategies without being limited to one correct answer [20, 44].

4.3. Qualitative Analysis of the Learning Process

Based on the results of observation and reflection on student activities, several important findings were found that reflect how the open-ended approach can change student learning patterns. Here are some of the key aspects analyzed:

4.3.1. Better Conceptual Understanding

At the beginning of the lesson, students had difficulty understanding the concept of angle measurement formed between short hands and long clocks. The most common mistake is not taking into account the movement of the short needle in tandem with the movement of the long needle. However, through group discussions and reflection on their own mistakes, students' understanding gradually improves [14, 45]. For example, in the question at 3:30 a.m., the majority of students in the pretest answered that the angle formed was 90°, assuming that the short needle was still at 3. However, after discussing in groups, they realized that the short needle should have shifted closer to the number 4. Through this reflection, they then understood that the angle formed was 105°.

4.3.2. Improvement of Critical and Creative Thinking Skills

The open-ended approach allows students to explore different ways of determining the size of the angle. For example, when asked to determine the angle at 4:00 a.m., some students use a straightforward approach by multiplying 30° per hour (4 x 30° = 120°). Meanwhile, another group found an alternative solution using the concept of complementary angles, which is to calculate the angle from 180° and then reduce it by the angle of 60° formed by the movement of the long needle (180° - 60° = 120°). The ability of students to come up with a variety of solution methods shows that they not only memorize concepts but also understand them deeply and can apply them in a variety of contexts [18]. This is in line with the findings of McGregor [20], who stated that open-ended question-based learning can improve students' creative thinking skills.

4.4. Increased Confidence and Collaboration

Open-ended learning also encourages students to be more confident in expressing their opinions. Classroom observations showed that students who were initially reluctant to speak began to actively present arguments and respond to the opinions of their peers. This happens because of the open discussion atmosphere, where each answer is considered valid as long as it has a logical basis of thought [10, 27]. In the presentation session, students also began to be more confident in explaining their answers to classmates [46]. If at the beginning of the learning, only 25% of students are willing to answer questions or participate in class discussions, this figure increases to 75% at the end of the learning session. Thus, this approach not only improves mathematical understanding but also develops communication and cooperation skills.

4.4.1. Challenges in Implementing Open-Ended Approaches

Although the results are positive, there are some challenges in implementing this approach. One of the main challenges is the difference in student understanding levels in one class. Some students who understand concepts faster tend to dominate group discussions, while slower students are sometimes passive. To overcome this, lecturers play a role in directing the discussion by providing triggering questions for all students to participate. In addition, some students are still familiar with conventional learning methods based on single answers, so they feel hesitant when asked to find alternative solutions. However, with practice and habituation, students begin to accept that in an open-ended approach, the exploration of various problem-solving strategies is something to be expected [6, 12, 26, 47].

4.5. Implications of Research Results

The findings of this study have significant ramifications for university-level mathematics teaching, particularly for aspiring teachers. It has been demonstrated that the HLT-based open-ended approach can enhance students' capacity for critical thought, creativity, and teamwork. Consequently, several suggestions are made for additional implementation:

- Strengthening Lecturer Training in Developing HLT: For this approach to be applied more widely, lecturers need to receive training in designing HLT that is at the level of student understanding.
- Integration with Digital Technology: The use of technology, such as a learning management system (LMS), can help students reflect on their understanding through online discussions and interactive exercises.
- Continuous Evaluation: To ensure the effectiveness of this approach, it is necessary to conduct periodic evaluations of student understanding through assignments and project-based assessments.

5. Conclusions

Using an open-ended method in conjunction with the Hypothetical Learning Trajectory (HLT) in angular measurement learning has been shown to significantly improve PGSD students' conceptual understanding and critical thinking abilities. Through this study, it was found that this method not only helps students understand the basic concepts of angle measurement in more depth but also improves their critical thinking skills, creativity, and confidence in solving problems. The open-ended approach allows students to explore various possible solutions in determining the magnitude of the angle formed between the long and short hands on the clock. With flexibility in problem-solving, students are more encouraged to think analytically and not just fixate on one standard way of solving.

The results showed a significant increase in pretest and posttest scores, reflecting the development of students' understanding after participating in a series of open-ended learning activities. If at first many students have difficulty in determining the size of the angle because they do not consider the movement of the short needle, after the learning process, they begin to understand that every movement of the long needle also affects the position of the short needle. This shows that students who previously only relied on procedural memorization began to develop a better conceptual understanding. In addition, through group discussions facilitated in this approach, students have the opportunity to compare different methods of solving and understand concepts from a broader perspective.

In addition to the improvement in conceptual understanding, this study also noted an increase in students' critical and creative thinking skills. By being given the freedom to seek and explore various alternative problem-solving strategies, students are more active in developing more flexible strategies. Not only are they able to answer questions more diversely, but they can also provide logical arguments for their answers. The ability to evaluate different solutions and compare them with other strategies suggests that students are getting used to thinking more reflectively and analytically. These findings are in line with previous research that stated that open-ended learning can improve higher-level thinking skills because students are encouraged to look at a problem from different perspectives [20].

Another notable impact of the open-ended HLT approach is the increased confidence of students in expressing opinions and participating in discussions. Initially passive, students became more active in asking questions and presenting ideas after engaging in this learning model. This shift reflects not only deeper material comprehension but also enhanced communication skills, which are critical for future educators in teaching mathematical concepts to young learners. However, the study also revealed challenges. A significant issue was the variation in students' levels of understanding. High-achieving students tended to dominate discussions, while those with lower comprehension were less confident. This can be addressed through active lecturer facilitation and the use of guiding questions to ensure inclusive participation. Technology-based tools, such as Learning Management Systems (LMS), may also support differentiated learning by offering flexible access to materials.

Another challenge was students' initial resistance to shifting from traditional, single-answer learning methods to a more open-ended paradigm. Many were hesitant to propose alternative solutions, fearing their answers differed from their peers. Gradual exposure and consistent practice helped students adapt to the idea that multiple valid solutions can exist in mathematics. These findings have important implications for teacher education. The open-ended HLT approach not only enhances conceptual understanding but also cultivates critical, creative, and collaborative thinking skills essential for effective teaching. Its application should be expanded beyond mathematics fundamentals to other problem-solving courses. Supporting this, lecturers need training in designing open-ended activities tailored to student readiness and promoting active discussion. Future research should investigate the approach's application in various subjects and explore the potential of digital technologies, including interactive media and AI tools, to optimize learning. Overall, this study confirms that open-ended HLT is a powerful strategy to foster mathematical thinking and professional competence in pre-service teachers.

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