



The development of didactical tetrahedron design on learning integers to improve algebraic thinking skills

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Abstract

This study aims to develop a didactical tetrahedron design for teaching integers by integrating the components of teacher, student, learning materials, and technology to enhance students' algebraic thinking skills. The study adopts a Design-Based Research approach grounded in Realistic Mathematics Education (RME). The research was conducted in two phases involving the development of a hypothetical didactical tetrahedron design, which was later refined through classroom teaching experiments. Both qualitative and quantitative instruments are used. The statistical analysis of the questionnaires and the narrative analysis of the qualitative instruments have been triangulated. Based on the analysis of students' difficulties, a conjectured didactical tetrahedron design was developed. Then the developed design was implemented in the teaching experiment. The implementation of the didactical tetrahedron design significantly improved students' algebraic thinking abilities. Students in the experimental group showed higher N-Gain scores and better conceptual understanding than those in the control group. The didactical tetrahedron design is effective in addressing common difficulties students face when learning integers and promotes the development of algebraic thinking. This design provides educators with a structured and technology-integrated instructional model that supports meaningful learning and enhances students' algebraic thinking skills.

Keywords: Algebraic thinking, Didactical tetrahedron design, Integers, Mathematics learning.

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1. Introduction

Human life in the 21st century has undergone significant changes compared to previous centuries, marked by rapid advancements in science and technology. These advancements have impacted various fields, including education [1]. One of the most important subjects taught in every school is mathematics. Mathematics covers structured topics such as algebra, geometry, trigonometry, calculus, probability and statistics, as well as discrete mathematics [2]. Integers are a crucial subject that students need to grasp in order to build a strong foundation for mastering algebra [3].

Algebraic thinking is an important skill that students need to master [4-7]. With algebraic thinking, students can better understand and solve problems encountered in everyday life [8]. Algebraic thinking is crucial in mathematics education because it forms the foundation for understanding more advanced mathematical concepts and problem-solving strategies. Specifically, when focusing on integers, algebraic thinking enables students to grasp fundamental operations, relationships, and properties that are essential for solving a wide range of mathematical problems. Algebraic thinking provides the groundwork for learning higher-level mathematical concepts. Understanding integers and their properties is crucial for grasping abstract algebraic concepts such as equations and functions [9]. This foundational knowledge ensures that students can handle more complex mathematical ideas as they progress.

Mastery of integers through algebraic thinking enhances problem-solving skills. Algebraic thinking helps students identify patterns, generalize mathematical relationships, and apply systematic approaches to solve problems involving integers [10]. This ability is critical for tackling a wide range of mathematical challenges. Working with integers and understanding their operations helps students develop logical reasoning and analytical skills. These skills are essential for constructing and evaluating mathematical arguments and for making informed decisions in various contexts [11]. Early algebraic thinking with integers supports future learning in mathematics. Students who develop strong algebraic skills with integers are better prepared for learning more advanced algebraic concepts and mathematical reasoning in subsequent educational stages. Previous studies show that students face many obstacles and difficulties in learning about integers [12-15]. Effective learning approaches and the use of appropriate teaching media are necessary to address problems in number learning and improve students' algebraic thinking skills.

Teachers are expected to design mathematics instruction that meets the needs of students. The teaching-learning trajectory forms the basis of instruction and programs that are designed consistently, logically, and understandably. Local instruction theory is used in the learning process involving teaching-learning trajectories. It is applied to the learning process related to a specific topic, supported by appropriate resources, Gravemeijer [16]. Gravemeijer [16] uses the analogy of learning as a journey. Local instruction theory provides a "travel plan" or "itinerary," where the teacher creates a plan for an actual journey with the students. In this theory, the teacher's insights are used to select instructional activities and then to design hypotheses for the learning trajectory for the students. The quality of the learning trajectory can be enhanced through the use of local instruction theory. In general philosophy, mathematics education is based on the didactic triangle of students, teachers, and mathematical content. However, technological advances add a new dimension. Technology becomes a new dimension (the fourth dimension), forming a didactical tetrahedron in the appropriate educational context.

One serious obstacle in learning about numbers, according to Fahlevi [17], is students' low sensitivity to numbers. Students tend to think about numerical concepts only in the context of answering questions. Numbers and related concepts are not clearly understood. A non-realistic approach to learning about numbers can hinder one's ability to grasp numerical concepts.

There is a fundamental issue that hinders the improvement of algebraic thinking skills, especially concerning number concepts. Students often lack a good number sense. This problem can be addressed by providing students with a learning design based on local instruction theory, considering the didactical tetrahedron's relationships among students, teachers, mathematical content, and technology.

Meaningful understanding of number concepts will develop if students work with problem situations that arise in various contexts close to their everyday lives or are meaningful to them. Additionally, using appropriate media can support successful learning. One strategy for improving students' algebraic thinking skills is the application of a realistic approach, known as Realistic Mathematics Education (RME), integrated with appropriate technology in instruction [18-21]. Algebraic thinking involves interpreting data or events in mathematical language to explain and predict phenomena [22]. Students can learn from problems relevant to their daily lives to achieve a suitable understanding. The use of appropriate media and technology can also support successful learning with RME and be effective in developing students' algebraic thinking skills [23-26].

2. Method

This study aims to develop the didactical tetrahedron design on learning integers. To achieve the goals of this research, the method used is design research, specifically educational design research. Educational design research is a type of research conducted to design and test a solution to a problem that has been thoroughly examined and understood. This approach is highly effective in theory building because it emphasizes valid and empirical studies. When done well, both the process and results of the research will contribute valuable insights to educational practice [27]. Not only aiming to refine practice or instructional design, design research also pays full attention to theoretical issues. As explained by Collins et al. [28], design research has a dual purpose: to refine both theory and practice. The focus of this research is to design and develop a didactical tetrahedron for teaching numbers with a realistic approach to enhance students' algebraic thinking skills.

The research is conducted in two phases. The first phase results in a hypothetical didactical tetrahedron design, while the second phase results in an empirical didactical tetrahedron design. Each phase follows the three steps of design research proposed by Gravemeijer [16]. These steps are preliminary design, teaching experiment, and retrospective analysis. The preliminary design begins with identifying and analyzing students' learning obstacles or difficulties, setting the learning objectives to be achieved, and preparing anticipations for the teaching experiment. During the teaching experiment, the preliminary design is implemented. At this stage, the researcher investigates whether students' mental activities align with the previously prepared anticipations. Information obtained from the teaching experiment can be used to modify the sequence of instructional activities and to formulate new hypotheses about students' expected mental activities. Through retrospective analysis, local instruction theory is reconstructed based on the information gathered in the previous steps.

The research activities are conducted in two phases. The first phase involves the development of a hypothetical didactical tetrahedron design. This hypothetical didactical tetrahedron design will be revised and refined to produce an empirical didactical tetrahedron design in the second phase.

In the first phase, a didactical tetrahedron design for teaching numbers with a realistic approach is developed, consisting of a hypothetical learning trajectory, local instruction theory, and a description of the didactical tetrahedron teaching materials. This phase begins with a literature review related to number concepts by analyzing textbooks, educational videos, and research results related to number teaching. Preliminary research is conducted to identify students' learning obstacles and difficulties, initial understanding, and students' algebraic thinking skills. This exploratory study produces findings that are used to develop a hypothetical didactical tetrahedron design, including the hypothetical learning trajectory, local instruction theory, and a description of the didactical tetrahedron teaching materials for number concepts. The design is validated by experts and tested with a limited group of 10 students. The hypothetical didactical tetrahedron design is then implemented in the first teaching experiment. This activity aims to gather information on the results of the implemented design. The findings from the first phase are used to improve the hypothetical didactical tetrahedron design, which will then be implemented in the second teaching experiment. These findings serve as the basis for retrospective analysis. All data collected, including documentation of learning activities, assignments, test results, and student interviews, are analyzed and evaluated to revise the didactical tetrahedron design for implementation in the next phase.

The initial design in the second phase is developed based on the previously revised hypothetical didactical tetrahedron design. Teaching experiments and retrospective analysis are repeated in the second phase until an empirical didactical tetrahedron design is produced.

3. Results and Discussion

3.1. Identification of Students' Difficulties in Solving Algebraic Thinking Ability Problems

Based on the preliminary study conducted by administering algebraic thinking ability tests to 28 seventh-grade junior high school students on the topic of integers, the researcher identified several difficulties that students encountered in solving these problems. Some of these difficulties are as follows. a. Students have difficulty using relational patterns to analyze situations. b. Students struggle to translate between different representations. c. Students face challenges in creating and using symbols, visual or spatial notation, and words or sentences to solve mathematical problems. d. Students encounter difficulties in modeling and solving problems.

Next, a review of the literature has also been conducted on the difficulties students face in learning about integers and their role in algebra, including the following noteworthy issues [29-33].

- Understanding Integer Concept: Students often struggle with the concept of negative numbers, particularly understanding that a negative number represents a value less than zero and interpreting the absolute value as the distance from zero on the number line.
- Visualization of integers on a number line can be difficult. Students may have trouble placing negative numbers correctly and understanding their relative positions to positive numbers.
- Applying the rules for adding and subtracting integers can be confusing, especially when dealing with multiple signs (e.g., positive and negative numbers). Students might struggle with remembering and applying the rules correctly.
- Understanding the rules for multiplying and dividing integers, such as the rule that the product or quotient of two integers with the same sign is positive and with different signs is negative, can be challenging.
- Students may find it challenging to apply integer operations within algebraic expressions and equations. For instance, combining like terms or solving equations with integer coefficients can be problematic.
- Grasping the concept of additive inverses (e.g., -a is the additive inverse of a) and their role in solving equations can be difficult for students.
- Translating real-world scenarios into mathematical problems involving integers can be particularly difficult. Students may struggle with setting up the correct equations or interpreting the results in the context of the problem.
- Understanding the context of a problem and how integers apply within that context (e.g., temperatures below zero, debts, elevations) can be challenging for students, impacting their ability to solve problems accurately.

3.2. Development of Didactical Tetrahedron Design

The design of the didactical tetrahedron is hypothesized to be developed based on the results of preliminary analyses conducted. During the design development process, the relationships between the components of the didactical tetrahedron are observed. During this process, teacher activities are involved in planning integer instruction. Considering the characteristics and needs of the students, the teacher selects appropriate technology and teaching approaches for function instruction. This interaction can be referred to as e-teaching [34].

This hypothesized didactical tetrahedron design consists of several learning activities. Table 1 shows the learning activities in the didactical tetrahedron, Hypothetical Learning Trajectory description.

Table 1.

Fhe didactical tetrahedro	n (Hypothetical Learning	g Trajectory) description

Learning	Learning Activities	Student Response	Anticipation
Objectives		Predictions	· · · · · ·
To activate	Teachers use the Quizizz application	• Initial Understanding:	The teacher provides
students' prior	to provide interactive quizzes related	Students may understand	additional
knowledge and	to initial knowledge of numbers.	some types of numbers but	explanations using an
introduce the	Example questions:	do not yet understand their	interactive number line
concept of	• "What type of number starts from	relationships or applications	in GeoGebra.
numbers	1 and does not have the number 0?"	in concrete forms.	The teacher uses short
through	"What number is -3?"	• Common Mistakes:	animated videos from
technology	• Teachers provide digital	Students tend to confuse	YouTube to illustrate
media.	illustrations using GeoGebra to show	natural and whole numbers	the types of numbers.
	the location of numbers on the number	and understand negative	• •
	line.	numbers.	
Students	Digital Simulation:	Intermediate Understanding:	The teacher provides
understand the	Students use GeoGebra to visualize the	Students understand the	additional tutorials
relationship	relationship between types of numbers	types of numbers and can	using
between types	on a number line.	represent them visually, but	GeoGebra to resolve
of numbers and	The teacher gives questions such as:	may make mistakes when	confusion.
can represent	"Show where the numbers 2 and 3 are	ordering negative numbers.	
numbers using	located on this digital number line."		
technological	Students are given time to try to place		
tools.	the numbers on the line.		
	The teacher monitors student progress		
	through the results of the simulation.		
	Group Discussion:		
	Students work in small groups to solve		
	tasks using GeoGebra. Example task:		
	Classifying given numbers into		
	number categories.		
Students are	Digital Project:	Advanced Understanding:	The teacher provides
able to apply	Students use the GeoGebra application	Students are able to	personal feedback
number	to solve number-based problems, such	generalize number	using GeoGebra.
concepts to	as: "Calculate the total goods of the	concepts and apply them	-
solve real	merchant using integers". Students are	independently, but may still	
problems	asked to create a diagram that shows	require guidance on	
through	the type of number from the data	problems of high	
technology	given.	complexity.	
applications.	Contextual Problem Solving:		
	The teacher gives story-based		
	problems that utilize GeoGebra.		
	Students simulate the answer using		
	GeoGebra.		
	Presentation of Results:		
	Students present the results of their		
	work using GeoGebra, visualizing the		
	types of numbers and steps to solve the		
	problem.		

3.3. Implementation of the Didactical Tetrahedron Design

The implementation of the design in the Teaching Experiment aims to test the developed didactic tetrahedron design for integers. The developed learning design was implemented in a 7th-grade class consisting of 58 students at SMP 139 Jakarta. The analysis results from the teaching experiment are expected to provide feedback for improving the quality of the didactic tetrahedron design.

3.3.1. Preliminary Activities

The lesson begins with the teacher conducting preliminary activities to prepare the students for the day's learning. Initially, students participate in a short prayer session, followed by the teacher taking attendance and ensuring that all students have the necessary materials for the lesson. The teacher then introduces the topic of integers and outlines the scope of the material, objectives, competencies, benefits, learning steps, and the assessment methods that will be used. The lesson progresses with an interactive quiz conducted through Quizizz, engaging students with digital illustrations from GeoGebra to help them visualize integers on a number line. The teacher then encourages students to think about how integers are used

in daily life, sparking a brief discussion where students offer their predictions about the practical applications of integers. To further solidify the lesson's relevance, the teacher connects the mathematical content to real-life scenarios. These activities are important for setting a positive learning environment, but their effectiveness depends on how well the teacher can engage students, adapt to their varying levels of readiness, and communicate the material clearly.

3.3.2. Core Activities

In the core activities, the teacher divides the students into small groups, each consisting of 4-5 members. This division allows for peer collaboration and promotes active interaction. The students begin by observing and listening attentively as the teacher explains key concepts related to integers and number lines. They are also shown an audiovisual presentation that further illustrates these concepts, using GeoGebra to visualize the location of integers on a number line. After receiving worksheets with related questions, students are prompted to read the instructions and ask any clarifying questions. The teacher fosters group discussions, encouraging students to share their questions and responses, while also inviting other groups to offer their insights. With the teacher's guidance, the students collaborate to solve problems related to integers and number lines, including tasks like identifying integers, explaining their properties, and determining their positions on a number line. The use of GeoGebra allows students to visualize the answers, making abstract concepts more tangible. Throughout the lesson, the teacher circulates among the groups, providing targeted support and addressing any difficulties students encounter. The teacher emphasizes accuracy, encourages discussion about different problem-solving methods, and directs the students to analyze their solutions thoroughly. As students continue working, they are asked to compile their findings into a report, summarizing their discussion and solutions. Group representatives present their results, and other students, along with the teacher, provide feedback and engage in further analysis. This reflective process encourages students to synthesize their learning and strengthens their understanding of the material. The teacher acknowledges and appreciates the students' participation, reinforcing a positive learning environment.

3.3.3. Closing Activities

The lesson concludes with closing activities that reinforce the material learned. The teacher assigns a project using GeoGebra to further challenge students in solving problems related to integers and number line presentations. Following this, a post-test is administered through Quizizz to assess students' understanding of the topic. Students are then briefed on what to expect in the next lesson. To encourage continued learning, the teacher directs students to find additional references related to integers, whether through library books or online resources. This final step provides an opportunity for students to deepen their understanding and prepare for future lessons. The overall structure of the lesson fosters engagement, critical thinking, collaboration, and reflection while using technology effectively to enhance students' understanding of mathematical concepts.

3.4. Analysis of Students' Algebraic Thinking Skills

After the implementation of the didactical tetrahedron design, students from two classes were given a test on algebraic thinking skills related to integer material. Each

class was designated as either a control class or an experimental class, with both the control and experimental classes consisting of 29 students each. The data analysis results indicate that the experimental class and the control class differed significantly. The results of the algebraic thinking skills test are summarized in Figures 1 and 2.

Descriptives								
	Class		Statistic	Std. Error				
NGain_Score	Experiment	Mean	.7446	.00674				
		95% Confidence Interval for	Lower Bound	.7308				
		Mean	.7584					
		5% Trimmed Mean		.7443				
		Median		.7451				
		Variance	.001					
		Std. Deviation	Std. Deviation					
		Minimum	.69					
		Maximum	.81					
		Range	.12					
		Interquartile Range	Interquartile Range					
		Skewness	.056	.434				
		Kurtosis	-1.238	.845				
	Control	Mean	.4894	.00562				
		95% Confidence Interval for	Lower Bound	.4779				
		Mean	Upper Bound	.5009				
	5% Trimmed Mean			.4892				
		Median	.4906					
		Variance	.001					
		Std. Deviation	.03026					
		Minimum	.44					
		Maximum	.54					
		Range	.11					
		Interquartile Range	.05					
		Skewness	.080	.434				
		Kurtosis	-1.025	.845				

Figure 1.

The results of the N-Gain test for the experimental class and the control class.

Based on the results of the N-Gain test, the experiment class showed an average improvement score of 0.7446, with a maximum value of 0.81 and a minimum value of 0.69. According to the N-Gain test table, this improvement can be categorized as high for the experimental class. In the control class, as shown in Figure 1, the average improvement score was 0.4894, with a maximum value of 0.54 and a minimum value of 0.44. According to the N-Gain test table, this improvement is categorized as low for the control class.

Independent Samples Test										
Levene's Test for Equality of Variances			t-test for Equality of Means							
									95% Confidenc	e Interval of the
								Std. Error	or Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Difference	Lower	Upper
NGain_Score	Equal variances assumed	2.070	.156	29.074	56	.000	.25515	.00878	.23757	.27273
	Equal variances not assumed			29.074	54.240	.000	.25515	.00878	.23756	.27275

Figure 2.

Results of the t-test for the control and experimental classes.

Based on the t-test results, the significance value (2-tailed) is less than 0.05, specifically 0.000. According to the hypothesis, we reject H0. Therefore, it can be concluded that the improvement in students' algebraic thinking skills is better in the class using the didactical tetrahedron design compared to the class not using the didactical tetrahedron design.

4. Conclusion

Based on the results and discussion presented previously, several conclusions can be drawn as follows: (1) based on the results of the preliminary research, students' difficulties in solving integer problems related to algebraic thinking indicators have been identified;

(2) The didactical tetrahedron design for learning integers was developed with attention to algebraic thinking indicators; (3) the validated and limited trial of the didactical tetrahedron design has been implemented in a teaching experiment; (4) based on the analysis of experimental data, the use of the didactical tetrahedron design for learning integers has proven effective in enhancing algebraic thinking skills.

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