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A model of developing mathematics teachers to enhance the use of TPACK, PBL, CBL, and CLIL integrated learning

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Abstract

The purposes of this study were to design a model for developing learning management skills in TPACK, PBL, CBL, and CLIL integrated learning, and to implement this model with mathematics teachers. The research was conducted in two phases: the first phase involved a focus group discussion among nine scholars to develop the model and design activities that integrate these principles into mathematics instruction. The second phase focused on implementing the model with 20 volunteer mathematics teachers from northeastern Thailand. The instruments included a problem analysis questionnaire, self-assessment forms, teaching skills evaluation rubrics, and a satisfaction questionnaire. Data were analyzed using mean scores, standard deviations, percentages, and a one-sample t-test, with an 80% criterion set for success. The results indicated that the model effectively enhanced teachers' learning management skills, increased their confidence, and led to high levels of satisfaction. This study contributes a practical and effective approach for integrating TPACK, PBL, CBL, and CLIL into mathematics education, offering a valuable framework for teacher development programs.

Keywords: Learning activity package (LAP), Skill, research instrument development skills, Preservice Teachers.

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1. Introduction

The teaching profession is a career that demands adaptation and flexibility as the world orbits around changing trends like 21st century learning, technology disruption, artificial intelligence, etc. Technology has transformed the needs in education [1, 2]. At the moment, students are now expected to compete in a world where digital literacy, technological fluency, and the ability to adapt to fast-changing environments are as critical as developing their knowledge of school subjects like mathematics and science [3]. This has placed a new level of responsibility on mathematics educational scholars as it could be considered their burden to equip students with not only fundamental knowledge but also the skills needed to thrive in a technology-driven world [4].

As a result, the role of mathematics teachers has expanded in this modern era as they might need to prepare students to meet broader educational goals. To be specific, teachers' jobs vary from helping students acquire essential problem-solving skills, fostering critical thinking and creativity, to analytical reasoning—capabilities vital in our data-driven, technology-rich society [4, 5]. It is also their responsibility to continuously update their teaching practices to align with the evolving educational objectives of the 21st century. Opportunities and challenges emerging from this situation are demanding for mathematics educators, who must stay informed about advancements in pedagogy, technology integration, and innovative learning strategies.

However, the Thai educational system has been criticized for its lack of preparedness, particularly in rural areas where resources are often limited [6]. Many schools rely on passive teaching methods and operate in classrooms with minimal technological support, which is particularly concerning for mathematics education [7, 8]. Considering the rising expectations of learning in the modern era, active engagement, critical thinking, and authentic learning situations could be crucial components for effective mathematics instruction. These demanding challenges indicate the need for improvements in mathematics education in Thailand, especially for teachers who need to find instructional methods that could serve the needs of modern learners.

To address these challenges, integrating various instructional principles can help meet the needs of modern education. A combination of frameworks such as Technological Pedagogical Content Knowledge (TPACK), project-based learning (PBL), community-based learning (CBL), and content and language integrated learning (CLIL) can be applied to enhance the teaching and learning experience. In particular, TPACK emphasizes the meaningful integration of technology in learning [9-11]; PBL promotes critical problem-solving skills [12, 13]; CBL connects learning to real-life [14, 15] contextual content; and CLIL supports the simultaneous development of language skills and subject knowledge [16, 17]. Equipping mathematics teachers with the tools to effectively manage their classrooms using these integrated principles can significantly improve educational outcomes for students, particularly in the Thai context.

This study aims to develop mathematics teachers' skills in learning management within the Thai educational framework by integrating TPACK, PBL, CBL, and CLIL. This research seeks to offer a comprehensive model that enhances both teacher and student competencies, preparing them to meet the demands of modern education.

2. Literature Review

2.1. Technological Pedagogical Content Knowledge: TPACK

To be conclusive, the framework of Technological Pedagogical Content Knowledge (TPACK) could be a holistic method for incorporating technology into education by merging three essential areas of knowledge [9-11]. In detail, the combination of Content Knowledge (CK) - mastery of the subject matter, Pedagogical Knowledge (PK) - effective teaching methods for that subject, and Technological Knowledge (TK) - familiarity with the technological tools that can be utilized, are the core principles of the first instructional method used in the current study [9]. In practice, Hill and Uribe-Florez [18] highlight the importance of TPACK in enabling secondary school mathematics teachers to effectively implement technology in their classrooms. They found that a well-rounded TPACK foundation significantly improves the integration of technology in math education. Similarly, Marbán and Sintema [19] observed that pre-service teachers with a strong TPACK background and positive attitudes towards ICT integration are more successful in applying technology in mathematics teaching. Additionally, Promwongsai and Poonputta [20] demonstrated that incorporating TPACK with specific teaching strategies, like TGT (Teams-Games-Tournaments), can significantly enhance students' achievement in mathematical topics such as histograms. Furthermore, Rakes et al. [11] explored the relationship between TPACK and effective teaching practices in mathematics. They emphasized that a solid TPACK framework enables teachers to select and use technology in ways that align with best teaching practices, ultimately leading to better student outcomes in mathematics. This body of research underscores the critical role of TPACK in modernizing and enhancing mathematics instruction through technology.

Therefore, it could be assumed that TPACK can empower teachers to design engaging, technology-enhanced learning experiences in math classes. The example of TPACK implementation is that a teacher might apply the TPACK framework by using dynamic geometry software to explore geometric concepts interactively. Instead of traditional paper-based problems, students could manipulate shapes and observe the effects of transformations in real time. Additionally, the teacher might incorporate an online graphing tool where students can visualize and analyze functions, enhancing their understanding of algebraic concepts. Through these activities, students could develop mathematical understanding and build technological skills and collaborative skills, which should lead to more interactive and engaging learning processes.

2.2. Project-based Learning: PBL

Scholars Krajcik and Shin [12], Boss and Larmer [13] and Guo et al. [21] have provided definitions for Project-Based Learning (PBL), which can be summarized as a method that focuses on providing opportunities for students to actively engage in solving real-world problems through projects, shifting the focus from passive content consumption to active learning. In mathematics education, PBL encourages students to inquire, research, and collaborate to find solutions to complex mathematical problems [22]. Additionally, PBL is an active, student-centered form of instruction characterized by students' learning outcomes, collaboration, affective attitudes, and thinking skills [23, 24]. PBL enhances student engagement, motivation, and learning outcomes by facilitating active involvement and collaboration [21,25,27] demonstrating that PBL significantly improves interpersonal communication, teamwork skills, cognitive outcomes, skills, and affective or behavioral aspects [21, 23, 27]. Students can engage with both content and problem-solving scenarios, leading to opportunities to apply collaborative skills. This can make learning more meaningful and applicable to authentic situations.

2.3. Community-Based Learning: CBL

In community integrated learning, community-Based Learning (CBL) is applied in the current study as it is considered an instructional approach that integrates academic learning with real-world community issues and experiences [14]. It is often used as a strategy to teach both content and practical skills simultaneously [15]. In mathematics education, CBL can be an effective method that helps students acquire mathematical knowledge while engaging with meaningful and relevant material related to their communities. According to Aini et al. [28] CBL has been shown to enhance students' mathematical communication skills by incorporating community-based projects within the learning cycle, making mathematical concepts more accessible and engaging. Phan and Ngo [29] highlighted that implementing a multidisciplinary, community-based approach in educational programs allows students to apply their mathematical knowledge in real-world contexts, fostering a deeper understanding and greater retention of mathematical principles. Supporting this, Zizka et al. [30] found that CBL is a powerful method for promoting sustainability and authentic engagement in STEM education, as it connects academic learning to real-world community challenges, which in turn motivates students to apply their mathematical skills in meaningful ways. In practice, a mathematics teacher could implement CBL by engaging students in real-world problems, such as analyzing community data on environmental issues. This task would involve collecting data, interpreting the results, and discussing findings—all within the context of learning mathematics. This approach not only enhances students' mathematical skills but also builds their connection to the community, making the learning experience more authentic and meaningful.

2.4. Content Language Integrated Learning: CLIL

Lastly, a CLIL mathematics classroom could allow teachers to instruct academic content through a foreign language, which could promote both linguistic skills and the content of the core subject. Scholars Scholars Coyle et al. [16], Ruiz-Cecilia et al. [17] and Wunberg et al. [31] have defined Content and Language Integrated Learning (CLIL) as the combination of teaching subject content with language learning. The dual-focused approach allows students to engage with authentic language use while learning mathematical concepts, making the process of language acquisition more practical and contextually relevant. Recent studies have highlighted the benefits of CLIL in mathematics education. For instance Ruiz-Cecilia et al. [17] conducted a systematic literature review and found that CLIL enhances mathematical learning by providing a context where students simultaneously develop their language skills. This approach not only aids in understanding complex mathematical concepts but also helps students gain fluency in the language of instruction. Thai et al. [32] discovered that the method significantly improved students' mathematical competencies alongside their language proficiency. The study showed that students who were taught mathematics through CLIL performed better in both mathematics and language assessments. Likewise, Wunberg et al. [31] found that CLIL positively influences academic self-concepts in both English and mathematics. Their research suggests that learning mathematics through a foreign language can enhance students' confidence in their mathematical abilities when CLIL is effectively implemented. In practice, a CLIL mathematics lesson could involve students learning about statistical analysis while being taught in the target language. Students would engage with relevant vocabulary, analyze data sets, and present their findings in the foreign language. This method not only enhances their mathematical understanding but also develops their language proficiency, blending content mastery with language skills development. The dual-focused approach allows students to engage with authentic language use while learning mathematical concepts, making the process of language acquisition more practical and contextually relevant. Recent studies have highlighted the benefits of CLIL in mathematics education.

Consequently, the principles and the proven benefits of TPACK, PBL, CBL, and CLIL could lead us to assume that integrating these methods has the potential to present an opportunity to enhance mathematics teachers' instructional capabilities. With the knowledge of these frameworks, teachers can develop dynamic learning management plans that engage students on multiple levels. For example, mathematics teachers can design activities where students use technology to analyze community data and solve real-world problems through project-based tasks, while simultaneously applying mathematical concepts as well as presenting their data in English. This blending of technological, content, and problem-solving approaches could encourage creativity and real-world application in the classroom.

Moreover, previous studies have demonstrated the effectiveness of TPACK, PBL, CBL, and CLIL in mathematics education, focusing on both student development and teacher training. However, previous investigations encouraged the integration of methods. According to previous studies, individual principles or combinations of two principles have been explored, but to our knowledge, no study has yet combined all four principles—TPACK, PBL, CBL, and CLIL—despite their significant potential when integrated. Testing these integrated frameworks on teachers would significantly contribute to the field. Therefore, this study aims to develop mathematics teachers' learning management skills in the Thai context by examining needs, creating a development model, and implementing it to enhance their use of TPACK, PBL, CBL, and CLIL.

3. Methodology

3.1. Research Design

The study was a research and development (R&D) design comprising two integral phases. The first phase focused on model development, involving a focus group discussion among scholars to create a model aimed at enhancing mathematics teachers' knowledge of TPACK, PBL, CBL, and CLIL. This phase also included the development of activities designed to help teachers integrate these principles into their mathematics instruction. The second phase involved the implementation of the developed model. This phase included a survey to identify challenges in mathematics teaching, self-evaluations conducted by participants before and after engaging with the model, assessments of teaching skills, and a satisfaction survey to evaluate the overall effectiveness of the model.

3.2. Phase 1 Developing A Model for Developing Mathematics Teachers to Enhance the Use of TPACK, PBL, CBL, and CLIL Integrated Learning

In the first phase of the study, a model for developing mathematics teachers to enhance the use of TPACK, PBL, CBL, and CLIL integrated learning was developed. This model development phase involved the active participation of nine expert scholars specializing in TPACK, PBL, CBL, and CLIL education management and pedagogy. This coincides with the notion of connoisseurship, wherein Prasith-Rathsint and Sookasame [33] propose that the optimal number of specialists is between 8 to 10, not to exceed 15. To assess the model's effectiveness, it was subjected to evaluation using [34] the Standard-Based Assessment Framework, which comprises two critical dimensions: propriety and feasibility. Subsequently, data were analyzed using the mean ($\bar{x}$), standard deviation (SD), median (Mdn), and interquartile range (IR). A consensus among the expert panel was deemed robust if the interquartile range (IR) was less than 1.50 and the median score fell within the range of 2.51 and above [35].

3.3. Phase 2 The Implementation of the Model

3.3.1. Participants

The group included 20 volunteer mathematics teachers from northeastern Thailand. All participants were treated in accordance with ethical guidelines, ensuring their privacy and confidentiality were maintained throughout the study.

3.3.2. Instruments

1) Problem analysis questionnaire

The questionnaire was developed to assess the challenges faced in mathematics teaching within the specific contextual area where the model would be implemented. Its purpose was to ensure that teacher participants were aware of the problems in their teaching context before engaging with the model. The questionnaire consisted of 10 items rated on a 5-point Likert scale. The item-objective congruence (IOC) ranged from 0.5 to 1.0, with a Cronbach's alpha of 0.82.

2) Mathematics teachers' self-assessment form

The form was designed to allow teachers to self-assess their teaching skills in relation to the integration of TPACK, PBL, CBL, and CLIL after participating in the model. It utilized a 5-point rating scale and focused on three main aspects: knowledge and skills in integrating TPACK, PBL, CBL, and CLIL in mathematics classes (11 items), the use of technology-based learning media (10 items), and authentic evaluation and assessment practices (10 items). In total, the form comprised 31 items. The item-objective congruence (IOC) is 0.5-1.0, with Cronbach's alpha of 0.84.

3) Mathematics teachers' teaching skills evaluation rubric

As the model includes positive feedback, inspectors assigned to evaluate teachers in their schools used a rubric to assess their teaching after the model's implementation. The rubric assessed three key aspects: knowledge and skills in integrating TPACK, PBL, CBL, and CLIL in mathematics classes (11 items), the use of technology-based learning media (10 items), and authentic evaluation and assessment practices (9 items). The rubric was rated on a scale of 1 to 3, with maximum scores of 33, 30, and 27 for each aspect, respectively. The item-objective congruence (IOC) ranged from 0.5 to 1.0, and the rubric demonstrated high reliability, with a Cronbach's alpha of 0.91.

4) Satisfaction questionnaire

The satisfaction questionnaire was designed to assess teachers' experiences with the model across four key aspects: usefulness (7 items), feasibility (5 items), appropriateness (5 items), and accuracy (5 items). The questionnaire used a 5-point Likert scale. The item-objective congruence (IOC) ranged from 0.5 to 1.0, with Cronbach's alpha of 0.87.

3.4. Data Collection and Data Analysis

In the second phase, the model was implemented with teacher participants, who first completed a mathematics teaching problem questionnaire. They then engaged in various activities within the model, including coaching, positive feedback sessions, participation in a professional learning community, and self-assessment, where they completed a self-assessment form. Finally, inspectors evaluated the teacher participants' skills and reported the findings back to the researchers, while the participants also completed a satisfaction questionnaire. The data were analyzed using mean scores, standard deviation, percentage, and a one-sample t-test, with the determining criterion set at 80% of the full score.

4. Results

4.1. Phase 1 Developing A Model for Developing Mathematics Teachers to Enhance the Use of TPACK, PBL, CBL, and CLIL Integrated Learning

4.1.1. Model Development

The model for developing mathematics teachers to enhance the use of TPACK, PBL, CBL, and CLIL integrated learning has processes of the mode can be illustrated in the figure 1 below.

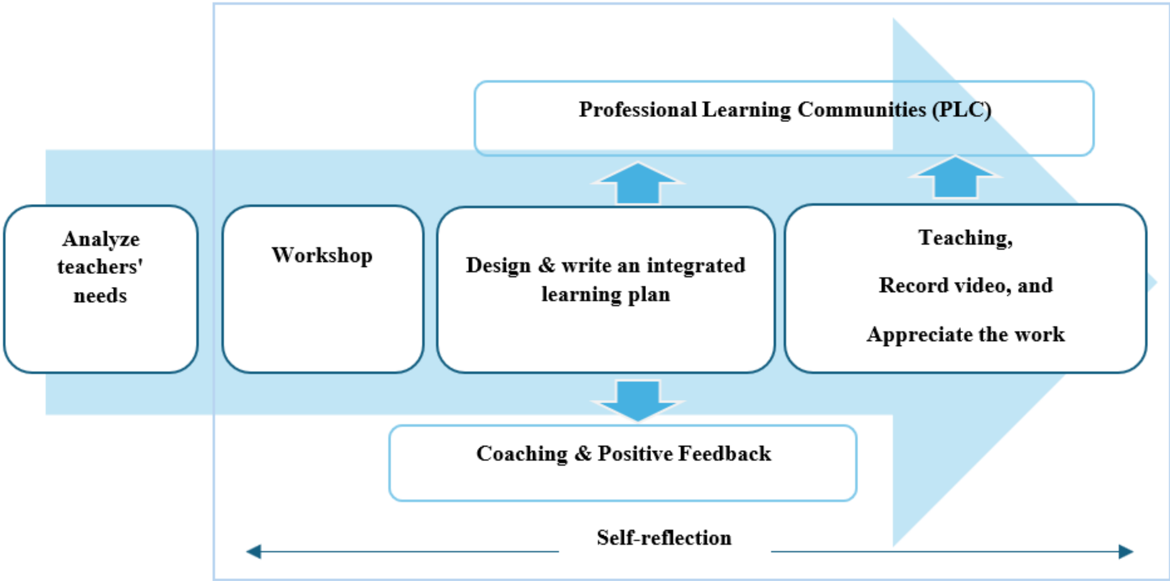

Figure 1.
Model development.

From Figure 1 can be explained as follows:

Step 1 Analyze teachers' needs: This research studies the problems and needs of teachers to gain fundamental knowledge for their development, acknowledging the various environments and contexts in which they operate. For instance, certain educators may want to incorporate technology into the classroom or receive training in specific disciplines [36]. If the development aligns with the needs, it will engage instructors' attention and foster collaboration [37].

Step 2 Workshop: At this stage, training is provided on integrated learning management, TPACK, CLIL, PBL, and CBL, including applicable methodologies utilized in instruction. The workshop serves as a mechanism to augment educators' competencies, understanding, and methodologies, facilitating their acquisition of knowledge that aligns with the goals of professional development [38-40].

Step 3: Design and Write an Integrated Learning Plan: The teacher has designed three math subject plans that take 2-3 hours each. The coaching sessions are designed to be hands-on, allowing teachers to engage directly with the concepts, practice new strategies, and receive immediate support. Positive feedback involves participants in the active design of learning management plans. Teachers created plans that integrate the TPACK, PBL, CBL, and CLIL principles into their mathematics lessons.

After the design phase, they receive constructive feedback from peers and experts, helping them refine their plans and enhance their teaching strategies. This iterative process of design and feedback ensures that the learning management plans are both practical and effective. Consistent with past research [40, 41].

Step 4: Teaching, recording video, and appreciating the work. The teacher conducts the teaching and records the video, then shows the video to the teacher's friends. Consistent with past research [40-42].

The supplementary activities conducted in Steps 3–4 used PLC: The Professional Learning Community (PLC) component brings together various stakeholders, including teachers, administrators, and other educational professionals, in regular meetings. These gatherings focus on collaboratively addressing contextual challenges faced in mathematics education.

The supplementary activities conducted in Steps 2–4 used self-reflection. Finally, in the self-reflection process, teachers assess their strengths and areas for improvement, fostering continuous professional growth. This is to help teachers understand the impact of the TPACK, PBL, CBL, and CLIL integration on their teaching and student outcomes.

4.1.2. Model Evaluation

The evaluation of the development model for mathematics teachers, which integrates TPACK, PBL, CBL, and CLIL, revealed that the model was rated as highly appropriate, with an average score of 4.94, and highly feasible, with an average score of 4.62. The median scores for individual items ranged from 4.00 to 5.00, and the interquartile range (IQR) ranged from 0.00 to 1.00, reflecting positive feedback from the experts regarding the model's effectiveness and applicability (Table 1).

Table 1.
Model evaluation

Aspects of evaluation	x̄	SD	Level	Q_3	Q_1	Mdn	IR
1. Appropriateness of the model	4.94	0.25	Very high				
1.1 Analyzing teachers' needs	5.00	0.00	Very high	5	5	5.00	0.00
1.2 Workshop activities	5.00	0.00	Very high	5	5	5.00	0.00
1.3 Design and write an integrated learning plan using TPACK, PBL CBL and CLIL	5.00	0.00	Very high	5	5	5.00	0.00
1.4 Teaching practice, record video, and appreciate the work	5.00	0.00	Very high	5	5	5.00	0.00
1.5 Self-reflection	4.78	0.44	Very high	5	5	5.00	0.00
1.6 The processes of Coaching and Positive Feedback	5.00	0.00	Very high	5	5	5.00	0.00
1.7 Professional learning community and self-reflection	4.78	0.44	Very high	5	5	5.00	0.00
2. Feasibility	4.62	0.61	Very high				
2.1 This development model can be effectively applied in actual classroom teaching.	4.78	0.44	Very high	5	5	5.00	0.00
2.2 This development model can be adapted to any content.	3.67	0.50	High	4	3	4.00	1.00
2.3 This development model can be applied across all grade levels and subjects.	5.00	0.00	Very high	5	5	5.00	0.00
2.4 The development model are worth the time invested.	4.78	0.44	Very high	5	5	5.00	0.00
2.5 The development model are cost-effective.	4.89	0.33	Very high	5	5	5.00	0.00

4.2. Phase 2 The Implementation of the Model

4.2.1. Participants' Self-Evaluation Regarding Learning Management Skills of TPACK, PBL, CBL, and CLIL

The participants' self-evaluation of their learning management skills in integrating TPACK, PBL, CBL, and CLIL into mathematics instruction indicated a very high level of competence across all assessed areas ($\bar{x}$ = 4.62, S.D = 0.33). Among the specific skills assessed, the use of technology-based learning media received the highest mean score of 4.70 (S.D = 0.34). The integration of TPACK, PBL, CBL, and CLIL in mathematics classes was also rated highly, with a mean score of 4.62 (S.D = 0.35). Additionally, the ability to conduct authentic evaluation and assessment practices was rated with a mean score of 4.52 (S.D = 0.46). These findings reflect the participants' strong self-perceived capabilities in applying these educational frameworks within their teaching practices (Table 2).

Table 2.
Participants' self-evaluation regarding learning management skills of TPACK, PBL, CBL, and CLIL

Learning management skills	x̄	SD	Skill level
1. Knowledge and skills in integrating TPACK, PBL, CBL, and CLIL in mathematics classes	4.62	0.35	Very high
2. The use of technology-based learning media	4.70	0.34	Very high
3. Authentic evaluation and assessment practices	4.52	0.46	Very high
Overall	4.62	0.33	Very high

4.2.2. Participants' learning management skills after the implementation of the model (Coach as assessor)

The participants' learning management skills were also evaluated by the coach using a rubric, with scores compared against an 80% benchmark of the full mark. The results showed that mathematics teachers had overall learning management skills ($\bar{x}$ = 79.00, SD = 8.90) that were 80% (the set criteria, $\bar{x}$ = 72) higher than the average grade, statistically significant at the level of .05. (t = 2.71, p = 0.01). More specifically, the participants demonstrated the ability to develop their learning management skills in TPACK, PBL, CBL, and CLIL to the expected level. These findings suggest that the model was effective in enhancing participants' skills as intended (Table 3).

Table 3.
Participants' learning management skills after the implementation of the model (Coach as assessor).

Learning management skills	Full mark	Criteria (80%)	x̄	SD	Mean difference	t	p
1. Knowledge and skills in integrating TPACK, PBL, CBL, and CLIL in mathematics classes	33	26.40	28.60	3.93	2.200	2.506	0.01*
2. The use of technology-based learning media	30	24.00	27.20	3.30	3.20	4.33	0.00*
3. Authentic evaluation and assessment practices	27	21	23.20	2.73	1.60	2.62	0.01*
Overall	90	72	79.00	8.90	5.40	2.71	0.01*

Note: *p-value < 0.05.

4.3. Participants' Satisfaction with the Model

The results of the study indicate that the participants had a positive experience with the model. Overall, their satisfaction with the model was rated at a very high level ($\bar{x}$ = 4.66, S.D = 0.52). This high level of satisfaction was consistent across all assessed aspects, including usefulness ($\bar{x}$ = 4.56, S.D = 0.60), feasibility ($\bar{x}$ = 4.67, S.D = 0.31), appropriateness ($\bar{x}$ = 4.82, S.D = 0.22), and correctness ($\bar{x}$ = 4.65, S.D = 0.35). These findings suggest that the processes within the model effectively guided participants through a comprehensive and supportive learning experience, enabling them to develop their skills and knowledge in a meaningful way (Table 4).

Table 4.
Participants' satisfaction with the model

Aspects of evaluation	$\bar{x}$	SD	Level of satisfaction
Usefulness	4.56	0.60	Very high
Feasibility	4.67	0.31	Very high
Appropriateness	4.82	0.22	Very high
Correctness	4.65	0.35	Very high
Overall	4.66	0.52	Very high

5. Discussion

The findings of this study demonstrate that the implemented model had a positive impact on the participants' learning management skills in various aspects. Firstly, it allowed teachers to identify and understand the challenges within their classrooms, which is a critical step in effective teaching. Secondly, the model increased their confidence in managing their classrooms, as evidenced by their self-reported improvements. Furthermore, the teachers significantly enhanced their learning management skills throughout the process, and they expressed high levels of satisfaction with their experience in the project.

The positive outcomes observed can be attributed to the comprehensive and systematic processes involved in the model. The combination of expert coaching, constructive feedback, and support from a professional learning community (PLC) was instrumental in helping the teachers gain both knowledge and confidence. These elements of the model facilitated the continuous development of their learning management skills, as the longitudinal nature of the process provided sustained guidance and support [43]. The core activities of the model, namely coaching, positive feedback, and a strong learning community, could be considered crucial factors that lead to the growth in teacher participants' learning management skills regarding TPACK, PBL, CBL, and CLIL.

Additionally, the model emphasizes the authentic use of learning management design in classroom settings. Therefore, not only can teachers practice their learning management skills of TPACK, PBL, CBL, and CLIL, but students also gain benefits as they are engaged in active and engaging mathematics classes. To simplify, as the teachers develop their skills and become more confident, their ability to foster student growth improves, leading to a cycle of success where both teachers and students thrive [44, 45]. This satisfaction among teachers is likely due to the tangible improvements they observe in their students, which reinforces the effectiveness of the model.

The results of this study are consistent with previous research by Rakes et al. [11], Ruiz-Cecilia et al. [17], Hill and Uribe-Florez [18], Marbán and Sintema [19], Almulla [23], Guo et al. [21], Aini et al. [28], Phan and Ngo [29], Zizka et al. [30], Wunberg et al. [31] and Thai et al. [32] who also found that integrating TPACK, PBL, CBL, and CLIL frameworks positively influences mathematics education and the professional development of mathematics teachers.

6. Conclusion

In conclusion, the study aimed to develop a model for mathematics teachers' learning management skills of TPACK, PBL, CBL, and CLIL through the process of focus group discussion. The model was implemented with mathematics teachers in the Thai educational context. The results show that the model led to expected outcomes in terms of teachers' self-beliefs, skills development, and satisfaction with learning. This model is recommended for implementation in teacher training programs, and the processes of coaching, positive feedback, and professional learning communities are encouraged as they are the key activities leading to the outcomes of the study. Moreover, further studies on mathematics teacher education could pay attention to the integration of active learning instructional methods as teachers' skills are more in demand in the changing era of the world.

However, as the study was conducted with a relatively small sample size of 20 teachers, it could be considered a limitation of the study. This is because teacher development is challenging in practice, as teachers have their job responsibilities, and finding time to participate in such programs can be difficult. Future studies should consider ways to involve a greater number of professional teachers in similar programs. Additionally, the study lacked a comparative control group, which may affect the generalizability of the findings. Further research could explore the long-term effects of this model on teacher development and student outcomes, providing a broader picture of the model's effectiveness.

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