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Al-Farabi trigonometry problems in modern school as a tool for the development of students' computational thinking

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Abstract

This study aims to develop and theoretically justify a methodology for teaching trigonometry within the secondary school algebra course, enhanced with tasks from Al-Farabi's trigonometric legacy. This approach is based on interdisciplinary projects with elements of micro-learning, and it evaluates the impact of this methodology on developing schoolchildren's computational thinking. The methodological basis of the study is a quantitative approach using the experimental method. To evaluate the effectiveness of the proposed methodology, a pedagogical experiment was conducted involving 103 9th-grade schoolchildren from schools in Almaty (Kazakhstan), divided into experimental and control groups. The homogeneity of the sample was verified using the criterion χ^2 test based on the results of measuring the cognitive ability levels according to Amthauer [1] intelligence structure tests. Criterion χ^2 was also used to verify the validity of differences in the characteristics of computational thinking of these groups after the completion of experimental training. The obtained empirical value of the test was 13.92, which exceeds the critical value of 5.99, indicating statistically significant differences between the groups with a 95% confidence level. The observed effect is attributed to the application of the experimental teaching methodology. The proposed methodology can be used in teaching trigonometry to increase its effectiveness.

Keywords: Al-Farabi's trigonometric legacy, Computational thinking, Problem solving, Digitalization of education, Interdisciplinary projects, Micro-learning technology, The principle of historicism in education.

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1. Introduction

In the modern educational process, it is important to take into account the ways how today's generation of schoolchildren perceives information, characterized by fragmented attention and difficulties with concentration. Micro-learning through interactive resources, which involves delivering content in small, easily digestible portions, appears to be the best solution to this challenge. It enhances learning efficiency, especially in complex areas of mathematics such as trigonometry, which often causes difficulties in comprehension and lack of interest among schoolchildren. Traditional teaching methods frequently fail to foster sustained motivation in this context.

This approach facilitates better understanding and memorization of the material and allows instruction to be tailored to schoolchildren's individual abilities. The use of historical context in teaching trigonometry significantly enhances the educational impact by enabling schoolchildren to trace the evolution of mathematical ideas and realize the practical necessity of their emergence. This contributes to a deeper understanding of the subject and arouses interest in trigonometry as a science with a rich background.

Trigonometry has undergone a long path of development. It originated in ancient times as one of the branches of astronomy, serving as its computing tool, and had a purely geometric nature, primarily representing the "calculation of chords." The most comprehensive trigonometric knowledge of that period is presented in the famous «Almagest» by the ancient Greek astronomer Ptolemy [2, 3]. As a distinct branch of mathematics, the history of trigonometry began in the Middle Ages. Indian medieval astronomers made significant contributions to it. It was during this time that scientists replaced chords with sines. This discovery made it possible to introduce functions related to the study of the sides and angles of a right triangle. It was then that trigonometry began to separate from astronomy, becoming a branch of mathematics.

The Indian astronomical treatises, along with Ptolemy's Almagest, served as a starting point for the development of trigonometric concepts and methods in the countries of the Near and Middle East. Ptolemy's Almagest played a particularly important role in this. At the beginning of the 9th century, it was translated from Greek into Arabic and later commented on and reworked by many scholars of the medieval East. One of his first commentators was al-Farabi (870–950), the greatest scientist, thinker, and encyclopedist of the early Middle Ages, a native of Kazakhstan.

Al-Farabi improves the trigonometric apparatus of Ptolemy to facilitate the understanding of difficult mathematical calculations available in the work of the ancient Greek astronomer. And in the trigonometric chapters of his «Book of Appendices to the Almagest» [4] he presents a fairly developed trigonometry, created by him in connection with the application of mathematical methods to solve various problems of mathematical astronomy and geography. It was translated relatively recently from a photocopy of the only Arabic manuscript kept in the British Museum in London and studied by the famous Kazakh scientist in the field of the history of mathematics and pedagogy of the Islamic East [4] and is reflected in the works [4, 5] «Mathematical heritage of al-Farabi», «Mathematical treatises», «Commentaries on the «Almagest»» of Ptolemy, highly appreciated by foreign Farabist scientists [6]. Previously, this work had not been translated into any language, its existence was unknown.

The main focus of this work is on the construction of a sine table, which was critically important for performing necessary calculations and solving many practical problems in astronomy and geodesy. Building such a table is impossible without determining the value of the sine of one degree. Mathematicians of the medieval East placed great importance on finding methods for calculating this value. One of the first to succeed in this endeavor in the medieval East was Al-Farabi. The solution he presented holds great potential for developing schoolchildren's computational thinking [7, 8]. This problem is initially formulated as a computational challenge, which can be effectively addressed only with the help of a computer and other information-processing tools.

The scientist breaks it down into two rather large tasks:

- Determining the sine of one degree and
- Building a table of other values of this function based on it.

The process of solving them includes the following sequence of actions:

- Analyzing the problem statement, breaking it down into small tasks that are easier to analyze and solve individually (task decomposition);
- Focusing attention on solving each of them individually and ignoring minor details in order to increase the effectiveness of their solutions (abstraction);
- Identification of patterns in solving a problem;
- Construction of an algorithm for solving a problem in the form of step-by-step instructions and its implementation using various computing tools;
- Analysis and evaluation of the results obtained, generalization of the algorithm, and transfer of the process of solving this problem to the process of solving a wide range of tasks.

It is this sequence of thought processes that is characteristic of computational thinking [8].

Al-Farabi's trigonometric legacy possesses additional didactic potential and is worthy of study, both in modern schools and in higher pedagogical education for the preparation of teachers of mathematics and computer science. Its inclusion in the content of trigonometry training will allow schoolchildren, along with the development of computational thinking, to realize the practical significance and importance of this section of mathematics, and to increase the level of their mathematical education.

Historical facts and information have long been actively used in teaching mathematics, and their effectiveness has been confirmed by numerous studies. Today, with the growing emphasis on the humanitarian aspect of mathematical education, the problem of integrating elements of history into mathematical education remains relevant. Contemporary scholars

emphasize the need to consider the genesis of mathematical ideas and methods in teaching mathematics, suggesting various ways to integrate historical information into lessons and extracurricular activities to improve the quality of mathematical education [9, 10]. Various forms and methods of presenting information from the history of mathematics are given in Nikotina [11]. This is a historical conversation, an excursion into history, a historical essay, historical tasks, etc. The works of Mazurova [12] and Abramova [13] confirm their positive influence on the subject knowledge and motivation of schoolchildrens.

According to Friedman [14] it is advisable to begin the study of each new topic in mathematics with a brief historical digression demonstrating the genesis of the concepts considered in it, the practical prerequisites for their appearance and evolution. This approach helps schoolchildren better understand why certain concepts and theories are studied in the mathematics course. Schoolchildren should be able to see the interrelationship of mathematics with other sciences, many of which develop based on the achievements of mathematics and encourage mathematicians to develop new methods and solutions, which, in turn, contribute to the development and enrichment of mathematics itself. At the same time, the inclusion of elements of history in teaching mathematics should be carried out in an organic connection with the content of the studied material. This also applies to the introduction of al-Farabi's trigonometric legacy into the learning process.

A brief historical overview of the emergence of al-Farabi's trigonometric heritage, along with the integration of the scientist's unique approach to solving problems into the content of teaching the trigonometry section in the school algebra course, can enrich the teaching of trigonometry by strengthening its practical relevance. This not only expands the system of subject knowledge of schoolchildren but also ensures the development of their computational thinking. In school education, the development of computational thinking is identified in official documents as one of the main learning outcomes in many countries of the world, including in the Republic of Kazakhstan, [15, 16].

One of the possible and most effective ways to introduce al-Farabi's trigonometry into the modern school system may be to offer schoolchildren the task of constructing a sine matrix, as the main task in the legacy of the trigonometry scientist, combining all his tasks into a single whole, as an interdisciplinary (algebra and computer science) project assignment for the entire period of studying the section. The use of the micro-learning technology mentioned above will enhance the educational effect.

The innovative educational technology of micro-education has been widely discussed by the global pedagogical community in recent years. A number of studies confirm the convenience and effectiveness of its use in teaching [17, 18] as well as its beneficial effect on memorizing information [19].

Microlearning is effectively used in the study of not only theoretical but also practice-oriented material [20, 21] which makes it possible to use it in teaching schoolchildren trigonometry with content enriched with historical elements. This approach enhances both emotional (interest in learning, motivation) and cognitive (understanding, memory, thinking, reasoning) involvement in the learning process. The positive impact of the level of involvement in the education of university schoolchildren in "smart classrooms" using "inverted learning" technology on the formation of computational thinking skills was noted in the work of [22].

When schoolchildren master trigonometry, enriched with al-Farabi's planar problems within the framework of an interdisciplinary project using microlearning technology in the context of developing schoolchildren's computational thinking skills, it is quite possible, since the enriched content of trigonometry training can be divided into short fragments and presented in the form of mini-blocks aimed at solving a specific task that forms a specific computational thinking skill.

However, an analysis of numerous studies in the field of computational thinking [23, 24] shows that the didactic potential of al-Farabi's trigonometric legacy in the context of the development of computational thinking has not yet been the subject of a separate study. Moreover, despite a significant amount of research on the use of historical elements in teaching mathematics, interdisciplinary projects, and micro-learning in the field of education, the possibility of introducing al-Farabi's trigonometric legacy into modern educational practice and applying interdisciplinary projects with micro-learning elements in his teaching to develop schoolchildren's computational thinking remains unexplored.

4. Research Methodology

In the course of the conducted study:

- The scientific works of Kazakhstan and foreign scientists in the field of computational thinking, one of the key cognitive skills of a digital society person, have been studied Wing [7], Aho [25], Khenner [8], Barr et al. [23], Zhao and Shute [26], Kong et al. [27], Angeli et al. [28], Haseski et al. [29], Harangus and Katai [30], Xie et al. [31] and Mukasheva and Paevskaya [24]. The various interpretations of the term «computational thinking», its main components, opportunities, and ways of their development among schoolchildren at different levels of the educational system are studied.
- The mathematical treatises of al-Farabi and the works of Kubessov [4] on the mathematical legacy of al-Farabi has been studied, on the basis of which it has been revealed that al-Farabi's trigonometry on a plane and related issues of constructing tables of trigonometric functions contain significant potential for the development of computational thinking.
- The scientific and methodological works of scientists devoted to the technology of micro-education, its features, advantages, and positive experiences of application in the educational process are studied. Based on the analysis, the possibilities of using this technology in the study of trigonometry as part of a school algebra course have been explored.
- A set of general scientific methods (analysis, synthesis, generalization, comparison) was used to comprehensively study the state of the problem under consideration, identify the degree of its study, and determine the totality of pedagogical conditions for its solution. The possibilities of the interactive GeoGebra environment in teaching al-Farabi

trigonometry on a plane are studied, as well as the possibilities of using an interdisciplinary project approach to ensure the effective development of schoolchildren's computational thinking skills.

- In order to determine the effectiveness of the proposed methodology for teaching trigonometry, enriched with the tasks of al-Farabi, a set of empirical methods (pedagogical experiment, observation, testing) and statistical methods for processing and analyzing the obtained quantitative data were used. 103 pupils of the 9th grade of schools in Almaty (Kazakhstan), divided into experimental and control groups, took part in the conducted pedagogical experiment. The uniformity of the sample was checked using criterion χ^2 based on the results of measuring the level of cognitive abilities of schoolchildren on Amthauer [1] intelligence structure tests.

Criterion χ^2 was also used to verify the validity of differences in the characteristics of computational thinking of these groups after completion of experimental training. The empirical value of criterion 13.92 exceeded the critical value of 5.99, which indicates statistically significant differences between the groups with a 95% confidence level. The tests used to measure the components of computational thinking are based on B. Bloom's taxonomy. Their reliability and validity are confirmed using the Cronbach's alpha coefficient. Direct and indirect observation of schoolchildren's work in the learning process was also used, and the products of their activities were studied.

5. Results and Discussion

In the school education system of the Republic of Kazakhstan, schoolchildren receive the first information about trigonometric functions in geometry lessons in the 8th grade. The systematic study of trigonometric functions begins in the 9th grade in the algebra course. An analysis of the standard curriculum of the course [16] shows that almost all tasks from the trigonometric heritage of al-Farabi perfectly fit into the modern curriculum and can be studied in parallel with the basic material.

Despite the possibility of integrating al-Farabi's trigonometric heritage and other historical information into the curriculum, and even the presence of brief historical references in algebra textbooks [32] teaching trigonometry is often reduced to a formal presentation without due attention to the historical context, which negatively affects schoolchildren's understanding of mathematics as a developing science. The famous psychologist and methodologist [14] notes that for this reason alone, «many schoolchildren lack correct ideas about mathematics as a science, they do not know the basic facts of the history of its origin and development, its current state and problems» [11].

Al-Farabi's trigonometric legacy is a treasure trove of useful information on plane trigonometry. Familiarization with it, first of all, will allow us to see the development of trigonometry, its basic concepts, expand schoolchildren's understanding of both trigonometric problems in general and possible ways to solve them, and promote a more informed understanding of trigonometric formulas, which will undoubtedly have an impact on improving schoolchildren's knowledge in the field of trigonometry.

Al-Farabi's key achievement in the field of plane trigonometry is solving the problem of constructing a table of sines and other trigonometric functions, which he divided into two consecutive stages.

Finding the sine of one degree, as one of the important stages of its solution, is a rather complicated and cumbersome task. For the first time in the medieval East, al-Farabi was able to solve it. When solving it, he proceeded from the method of calculating the chord of one degree of Ptolemy. Two things are new to him:

- First, he replaces the chord of the arc with the sine of half of the same arc,
- Secondly, he uses more advanced arithmetic of sexagesimal fractions than the Greeks and more improved methods of approximate calculation.

It is not possible to solve this problem without dividing it into separate subtasks that are simpler from the point of view of the solution. To this end, al-Farabi identifies groups of subtasks in his work that are necessary to achieve this result. This:

- Tasks for finding the values of the 5° , 60° , 36° and 18° sine needed in further work;
- Tasks for proving the formulas of the sine of the sum and difference of angles, as well as the sine of the half argument, familiarity with which allows you to realize their practical value;
- The task of finding the numerical value of the sine of one degree based on the results of solving the previous two groups of problems.
- Building a table of its other values based on the found value of the sine of one degree is no less difficult task. It includes:
- Identification of patterns in solving the problem and development of an algorithm for constructing a sine table;
- Implementation of the developed algorithm and generalization of the obtained results.

Similarly, for the effective implementation of an interdisciplinary project to build a sine table, the entire content of the trigonometry section can be divided into small blocks, supplemented by the tasks described above by al-Farabi, in which each of the microblocks is focused on developing a specific computational thinking skill.

At the beginning of his trigonometric chapters [3], al-Farabi defines the sine «as half of the chord of a doubled arc», i.e. if $D = chd2\alpha$ (Fig.1), then BC is the sine line of the $AB = \alpha$, and $\sin \alpha = \frac{1}{2} \cdot chd2\alpha$ (1)

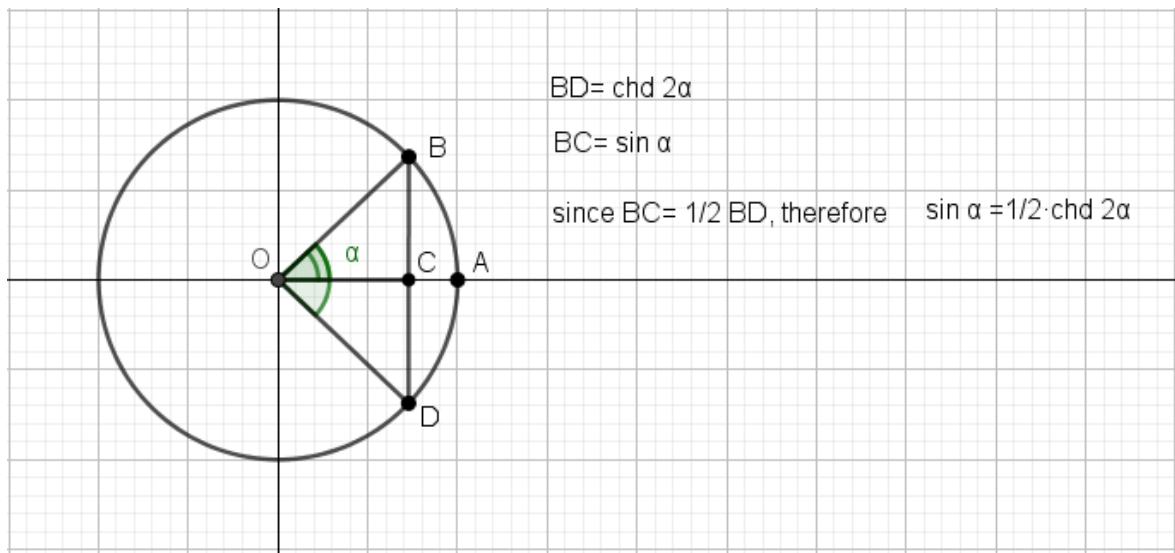


Figure 1.
Determination of the sine by al-Farabi.

This is one of the first definitions of the sine known to us when commenting on Ptolemy. It can be considered in parallel with its traditional definition.

Later in his works, al-Farabi everywhere replaced the chord of the arc with the sine of the angle. Although such a replacement in itself does not seem so significant, the transition from the chord to the half-chord favored the widespread introduction into astronomy of various trigonometric functions related to the sides and angles of a right triangle in a circle.

Further, they explain other lines in a trigonometric circle of Ptolemaic radius (60 units): «ABC is a circle, E is its center, AC is the diameter. Draw EB at a right angle from point E. We define the arc AG, draw the line AG, lower GD perpendicular to AC and GH perpendicular to BE, connect G and C. Then the line AG is the chord of the arc AG, GC is the chord of its complement, GD is the sine of the arc AG, GH is its cosine equal to the line DE, AD is the arrow of the arc AG; BH is the arrow of the arc GB, the arc GB is the complement of the arc AG to a quarter circle, the arc GBC is the complement of AC to half a circle» (see Figure 2) [4].

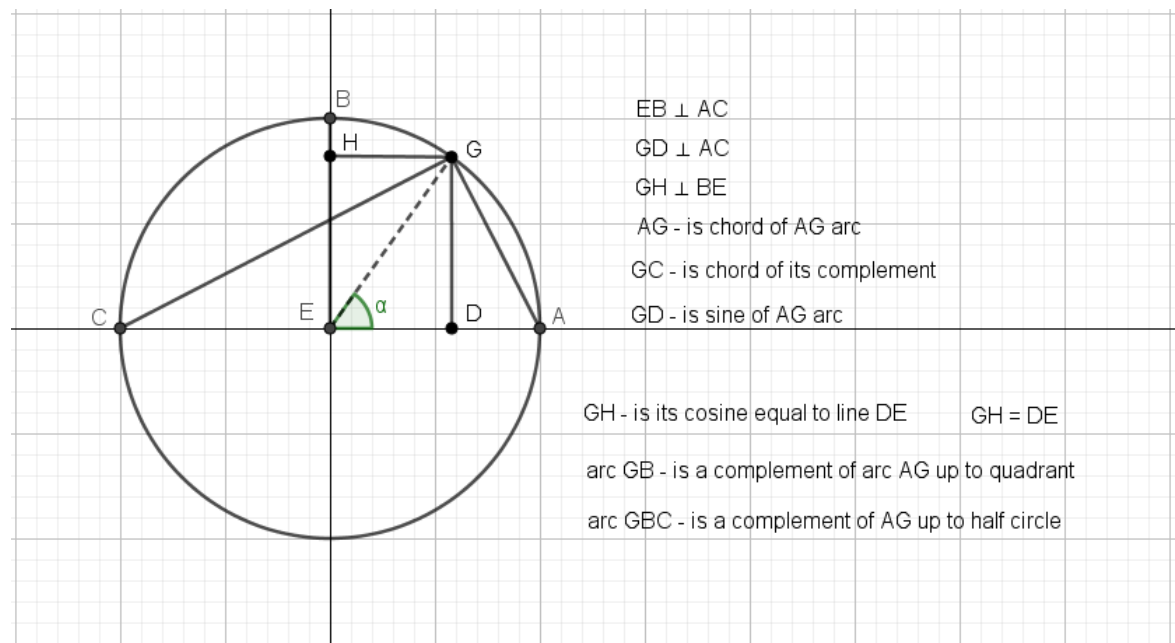


Figure 2.
Basic trigonometric lines according to al-Farabi.

And according to the well-known chord of the arc α , a method for finding the chord of its complement is described: «Let ABC be a circle, AC be its diameter. Let the arc AB be given, and draw the lines AB and BC (Fig.3). We will consider the chord AB to be well-known. Then the chord BC is also known» [4].

It is easy to show that $AB = \text{chd}90^\circ = \sqrt{2}AE$.

And, if you enter the notation $AE=R$ and use formula (1), you can get

$$\sin 45^\circ = \frac{\text{chd}90^\circ}{2} = \frac{R\sqrt{2}}{2}.$$

The proof of this statement and all the other tasks in this series is feasible for schoolchildrens.

The second group of tasks in the scientist's work consists of proving basic trigonometric formulas. Having the values of the chords of two angles, al-Farabi proves theorems that make it possible to calculate the chords of the difference and the sum of these angles, as well as the chords of the half-angle. This group of tasks will allow us to further find the required values of chords and their corresponding sines of other angles.

Here is one of them – «On finding the magnitude of the chord of the difference between two arcs, the chords of which are known»:

«Let ABCD be a semicircle, its diameter AD and its chords AB and AC are known. Connect B and C. Then, BC is also known», - he writes (see Figure 5).

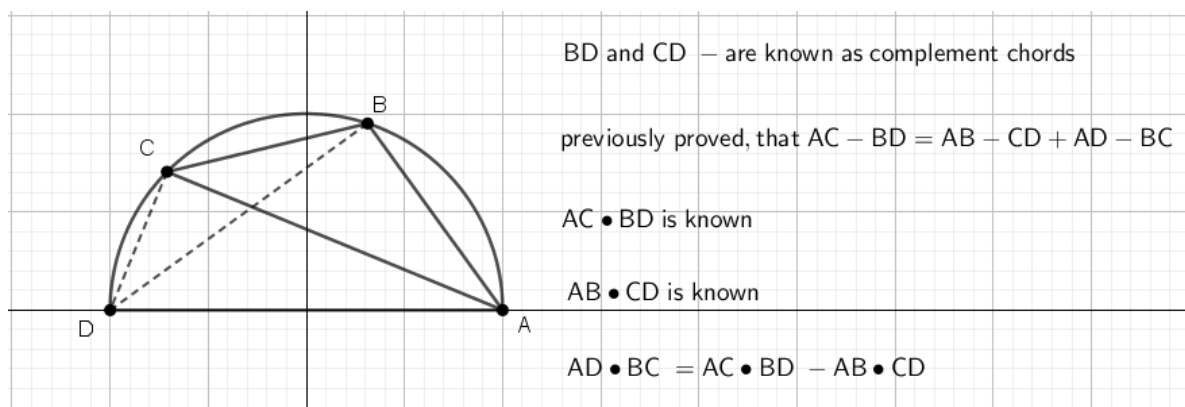


Figure 5.
Illustration of the chord of angle difference theorem.

Indeed, by the condition of the problem, the chords AB and AC are known. It is easy to determine and therefore the chords of their complements BD and CD are known.

By virtue of the well-known Ptolemy's theorem, the chord BC is easily located in the quadrilateral ABCD, inscribed in a circle:

$$BC = (AC \cdot BD - AB \cdot CD) / AD. \quad (3)$$

Introducing new designations

$$AC = \text{chd}\alpha; CD = \text{chd}(180^\circ - \alpha) - \text{the chord of its complement};$$

$$AB = \text{chd}\beta; BD = \text{chd}(180^\circ - \beta) - \text{the chord of its complement};$$

where α and β are the angles corresponding to the arcs AC and AB, respectively, and considering that BC is the chord of the angle difference between DAB and DAC:

$$BC = \text{chd}(\alpha - \beta); AD = \text{chd}180^\circ, \text{ we get:}$$

$$\text{chd}(\alpha - \beta) = \frac{1}{\text{chd}180^\circ} [\text{chd}\alpha \cdot \text{chd}(180^\circ - \beta) - \text{chd}\beta \cdot \text{chd}(180^\circ - \alpha)].$$

It is equivalent to the well-known formula of the sine of the difference of two angles

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \sin \beta \cdot \cos \alpha, \text{ which can be obtained by replacing the chord sinus with the aid ratio} \quad (1).$$

In exactly the same way, he proves the theorem on the chord of the sum of angles with given chords, as well as the formula for the chord of half an angle. The statements proved above are used further to compile new sums and differences: $\text{chd}180^\circ$, $\text{chd}120^\circ$, $\text{chd}108^\circ$, $\text{chd}72^\circ$, $\text{chd}60^\circ$, and 12° , 6° , 3° , $3/2^\circ$, $3/4^\circ$.

To build a sine table, in addition to the values of chords and sines found above, it is important to know the chord and sine of one degree. To determine them, al-Farabi followed Ptolemy, who found a very ingenious way out by applying one of Euclid's theorems that the ratio of segments cut off by a straight line drawn from the vertex of a triangle on the opposite side is less than the ratio of the angles themselves: if $\alpha > \beta$, to $\frac{\text{chd} \alpha}{\text{crd} \beta} < \frac{\alpha}{\beta}$. He provides its proof by analogy with Ptolemy

in his work.

Based on the proven statement, the inequalities are valid

$$\text{crd}1^\circ : \text{crd} \frac{3^\circ}{4} < 1 : \frac{3}{4}, \quad \text{crd}1^\circ < \frac{4}{3} \cdot \text{crd} \frac{3^\circ}{4} \approx 1^\circ 2' 49'' 52''' \quad \text{and}$$

$$\text{crd} \frac{3^\circ}{2} : \text{crd}1^\circ < \frac{3}{2} : 1, \quad \text{crd}1^\circ > \frac{2}{3} \cdot \text{crd} \frac{3^\circ}{2} \approx 1^\circ 2' 49'' 48'''.$$

$$\text{From which it follows, that } 1^\circ 2' 49'' 48''' < \text{crd}1^\circ < 1^\circ 2' 49'' 52'''.$$

For the approximate value of the chord of one degree, al-Farabi takes the arithmetic mean of the two found $\text{crd}1$ values $\text{rd}1^\circ \approx 1^\circ 2' 49'' 50'''$, where does \sin come from $\sin 1^\circ \approx 1^\circ 2' 49'' 43'''$ and $\cos 1^\circ \approx 59^\circ 59' 27'' 30'''$. In decimal terms, al-Farabi's approximation for $\sin 1^\circ$ is 0.017452389 instead of the correct 0.017452406.

And if the value of the chord of one degree, found by Ptolemy, is accurate to five decimal places, then al-Farabi's value has it exactly up to and including six decimal places. This achievement of al-Farabi in improving the accuracy of calculations of the sine of one degree was further developed by other mathematicians of the medieval East.

Furthermore, based on the found values of $crd1^\circ$ and $\sin 1^\circ$, a table of other values of the chord and sine is constructed. Recognizing the pattern, it is easy to find the value of $crd1^\circ \approx 1^p 2' 49'' 50'''$ by the value $crd179^\circ \approx 119^p 59' 43'' 33'''$, as chords of its complement. It is necessary to calculate $chd2^\circ$, for which he uses the formula of the chord of the sum of two arcs, the chords of which are known. Then it immediately calculates the value of $chd178^\circ$, as the chords of its complement, necessary to calculate $chd3^\circ$, etc. The cyclic repetition of these actions allows you to determine all the chords of arcs from one degree to 180° .

The value of the cosine of one degree is necessary for him to calculate the tangent and cotangent of one degree. They, in turn, are necessary for compiling tables of these functions.

Like Ptolemy [2] al-Farabi uses the sexagesimal system and takes the radius of a circle equal to 60 parts (units). In the same units, he calculates both chords and sines.

The sexagesimal system was widely used by Greek astronomers and more recently by Arabs, as well as ancient and medieval astronomers, primarily to represent fractions. That's why medieval scientists often called sexagesimal fractions «astronomical». These fractions were used to record astronomical coordinates—angles. This tradition has been preserved to this day. There are 60 minutes in one degree and 60 seconds in one minute. Note that al-Farabi uses the sexagesimal system when calculating fractions.

It is worth noting that the proofs of all the theorems presented in the scholar's work are impeccable from the point of view of mathematical logic. At the same time, they are presented in the form of a clear sequence of actions and are explained by drawings. Each of them can be considered as an independent task. A clear algorithm for their solution makes it possible to visualize them in the GeoGebra software environment [33-36]. Pre-designed screencasts of their solutions in this software environment are provided to schoolchildren as they explore each new problem. Such multimedia microcontent is absorbed more efficiently, while a large amount of data can lead to cognitive overload and disrupt the learning process [30]. It can be accessed anywhere and at any time using various mobile applications, instant messengers, and social networks, which is very convenient for the digital generation of schoolchildren, who are characterized by fragmentation of perception and difficulty maintaining concentration. The inclusion of Al-Farabi's tasks in the content of trigonometry within the algebra course, and their instruction using micro-learning technology, will enrich the content of the school algebra course and have a positive impact on the emergence and development of schoolchildren's interest in trigonometry. It will also ensure better assimilation of subject-historical knowledge and the development of computational thinking skills necessary for successful life in a modern digital society. This is supported by experimental data. Various approaches exist to assess the level of development of computational thinking [37]. In this study, the development of computational thinking skills occurs when schoolchildren master trigonometry in an algebra course, supplemented by al-Farabi tasks, as part of work on an interdisciplinary project assignment. To diagnose the level of its development, three key components of computational thinking are identified: meaningful, operational, and motivational. A three-level model with appropriate evaluation criteria has been developed for each of them [38]:

1. The content component reflects the knowledge gained as a result of mastering trigonometry in the algebra school curriculum, enriched with tasks from the legacy of al-Farabi. This is a set of concepts and formulas of trigonometry, as well as approaches to the development and solution of problems related to the construction of trigonometric tables. Methods of its diagnosis: evaluation of the completed project assignment and knowledge testing.

Table 1.

Levels and indicators for assessing the formation of the meaningful component of computational thinking.

The meaningful component of computational thinking	
Level	Indicators of formation
low	<ul style="list-style-type: none"> • Knowledge is not formed; facts and phenomena are described intuitively, without understanding the connection between them. It is difficult to apply them. • has difficulties in constructing trigonometric tables; • has difficulties in implementing algorithms, presenting data, debugging programs, and justifying and summarizing the results obtained.
medium	<ul style="list-style-type: none"> • demonstrates knowledge of the basic concepts and formulas of trigonometry, the ability to apply them, but has difficulty proving them. • understands the principles of building trigonometric tables; • uses a modular approach when writing programs; • knows program debugging techniques; • has difficulty substantiating and summarizing the results obtained
high	<ul style="list-style-type: none"> • has generalized and systematized knowledge of the section; • analyzes trigonometric formulas, knows how to prove them in different ways; • It correctly divides the task into subtasks and can reduce the solution to a problem that has already been solved before; • consciously chooses the tools for implementing the algorithm for constructing a table of trigonometric functions; • Competently debugs programs; is able to justify and generalize the result of the program

2. The operational component is determined by a set of mental skills (algorithmization, abstraction, decomposition, generalization, evaluation) necessary to solve the problem of constructing a table of trigonometric functions based on the approach used by al-Farabi, by dividing it into parts, as well as to select methods for solving them and interpreting the results obtained. This set of skills is formed when mastering the method of solving the problem of finding the sine of one degree and constructing a table of other sine values based on it, and is determined by the level of intellectual development of the schoolchildren, reflecting their ability to compare, classify, generalize, and evaluate. Therefore, Amthauer [1] intelligence structure tests (sentence addition (logical selection), word exclusion, analogy, generalization) [1, 39] can be used as diagnostic methods, as well as an analysis of the approaches used by schoolchildren in solving the problem of constructing a sine table using the al-Farabi algorithm and evaluating the results of its implementation.

The ability to solve a problem by dividing it into subtasks (modular program implementation), highlighting the main features in them, and abstracting from unimportant details to reduce the complexity of solving problems, as well as constructing and implementing an algorithm for solving it, is evaluated when submitting a project assignment.

Observation of schoolchildren during program debugging is highly informative. When schoolchildren test a program module by module, it demonstrates the manifestation of computational thinking skills. A side effect of unit testing is that the schoolchildren begin to use procedures and functions more often, breaking down the program into smaller logical parts. Good program code should be understandable, and this is achieved by highlighting clear logical structures in the form of procedures and functions, the meaning of which should be obvious even without comments.

Splitting a task into a set of small subtasks allows you to write code that can interact with the code of developed by others.

The levels and indicators for assessing the formation of the operational component of computational thinking are presented in Table 2:

Table 2.

Levels and indicators for assessing the formation of the operational component of computational thinking.

The operational component of computational thinking	
Level	Indicators of formation
low	<p>Schoolchild:</p> <ul style="list-style-type: none"> • It cannot divide tasks and algorithms into separate parts that can be understood, developed and analyzed separately; • It is not able to identify important features of objects and is distracted by unimportant features to reduce the complexity of solving problems; • He has difficulty developing algorithms for solving problems; he does not understand enough about the work of basic algorithmic structures. • He is unable to solve new problems based on those already solved or to adapt the developed algorithms to a class of similar problems; • Unable to assess the quality of the problem statement and the algorithm for its solution.
medium	<p>Schoolchild:</p> <ul style="list-style-type: none"> • It is able to divide tasks and algorithms into separate parts with minor inaccuracies, which can be understood, developed, and analyzed separately; • It is able to identify the main features of objects with minor inaccuracies and distract from unimportant details to reduce the complexity of solving problems; • He is able to build algorithms for solving typical problems, including the allocation of individual functions, but he makes minor mistakes when constructing algorithms for solving atypical problems; • When solving new problems based on those already solved, it causes inaccuracies; it has difficulty adapting the developed algorithm to a class of similar problems. • He is able to evaluate the quality of the problem statement and its solution algorithm with minor inaccuracies.
high	<p>The schoolchildren's actions are fully conscious and logically justified.</p> <p>Schoolchild:</p> <ul style="list-style-type: none"> • It is able to divide tasks and algorithms into separate parts that can be understood, developed, and analyzed separately; • It is able to identify important features of objects and distract from unimportant details to reduce the complexity of solving problems; • Is able to solve problems by clearly defining the sequence of steps; is able to predict the results of the algorithm, see possible problems that may arise during the implementation of the algorithm; is able to optimize the algorithm. • It is able to combine objects or phenomena according to their essential features and properties, solve new problems based on those already solved, and adapt algorithms to a class of similar problems; • He is able to evaluate the quality of the problem statement and the algorithm for its solution.

3. The motivational component characterizes interest and manifests itself in readiness for search activity, initiative, and independence in it.

Methods of diagnosis include the assessment of schoolchildren's activity in the learning process, their initiative, and independence in completing tasks, as well as observation.

The levels and indicators for assessing the formation of the motivational component of computational thinking are presented.

Table 3.

Levels and indicators for assessing the formation of the motivational component of computational thinking.

The motivational component of computational thinking	
level	Indicators of formation
low	<ul style="list-style-type: none"> • Weak interest in solving problems, passive attitude toward learning activities, lack of initiative and independence; • needs constant support when completing tasks; • has difficulties in theoretical training.
medium	<ul style="list-style-type: none"> • the manifestation of an episodic interest in solving problems; • low activity in the classroom, the need for constant supervision and guidance from the teacher; <p>Most often, the schoolchildren are attracted to the practical implementation of the method, rather than its theoretical justification.</p>
high	<ul style="list-style-type: none"> • steady and pronounced interest in solving problems, high activity and independence in educational activities, initiative and desire for self-development; • motivated for professional growth; • actively studying additional literature; • He can independently suggest new ways to solve the problem; • clearly expresses a desire to solve the problem in a more efficient way; • shows interest in the practical application of acquired knowledge in other fields.

The tables provide a minimally sufficient list of evaluation indicators, differentiated by levels for each individual component of computational thinking, based on which an integrated assessment of the level of development of computational thinking is formed.

The developed methodology for teaching trigonometry, enriched with tasks from al-Farabi's trigonometric legacy and based on an interdisciplinary project with elements of micro-learning, was tested in a real educational setting with 9th-grade secondary school students. It ensures deeper learning of subject knowledge by students and improves their academic performance in trigonometry and programming, contributing to the development of their computational thinking skills. The total number of students who participated in the pedagogical experiment was 103, including 51 students in the control group and 52 students in the experimental group.

To ensure the reliability of the experimental results at its initial stage, a study was conducted on the uniformity of the samples of the control and experimental groups in cognitive abilities. The results of testing schoolchildren's cognitive abilities using the Amthauer [1] method are taken as a basis [1].

The first four subtests of Amthauer [1] intelligence structure tests were used [39]. These tests characterize schoolchildren's ability to compare, classify, and generalize concepts, identify their essential features, and establish various relationships between concepts. The results of the analysis allow us to build an area of actual verbal development of schoolchildren and determine the requirements for educational information, compliance with which will make it more understandable. The total score, which is an assessment of mental development on the Amthauer [1] test, was calculated by summing up the scores received by the subject for completing each of the four subtests. The test results were evaluated by correlating them with group data based on the calculation of percentiles, which show the relative position of each schoolchild in the sample [40]. Despite the uneven units of measurement, percentile estimates are clear, universal in relation to various methods, and easy to calculate. The empirical distribution of test scores is divided into three quartiles (see Figure 6).

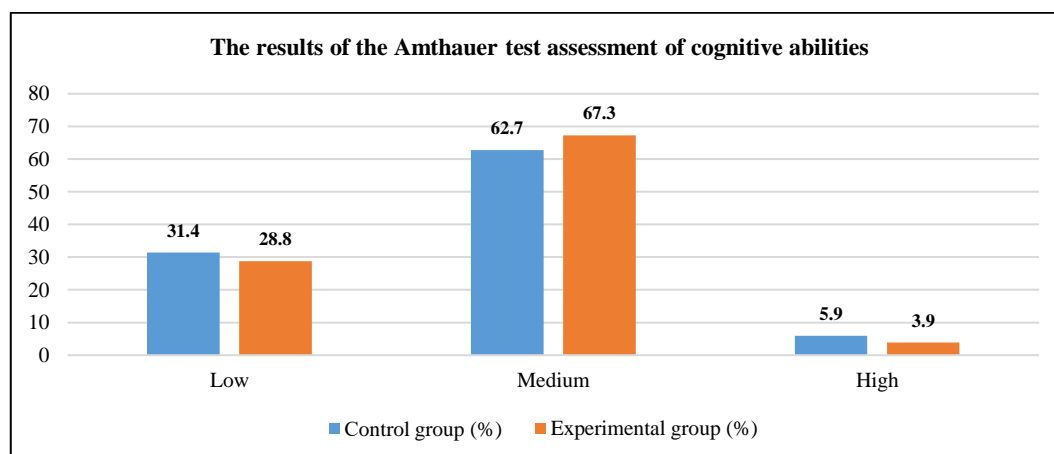


Figure 6.

The results of the assessment of cognitive abilities by the Amthauer test at the initial stage of the experiment.

It should be noted that quantile scales refer to scales of order, i.e., they provide information about which of the subjects has the most pronounced test property, but do not show how much or how many times stronger it is. To test the homogeneity of the control and experimental groups, Pearson's criterion χ^2 was used.

In this case, the empirical value of the criterion χ^2 for the number of degrees of freedom $k=2$ is 0,6832. It is less than the critical value of χ^2 , equal to 5,991 with a significance level of $p=0,05$, so we can conclude that there are no statistically significant differences in the characteristics of the control and experimental groups at the initial stage of the experiment. The groups are homogeneous and conditionally equivalent.

Introductory testing was conducted to assess the level of readiness of schoolchildren in the control and experimental groups to master trigonometry in the algebra course, supplemented with tasks from the trigonometric heritage of al-Farabi, and to implement an interdisciplinary project (algebra-computer science) on building a table of trigonometric functions. The results clearly demonstrate the low level of schoolchildren's training in programming and mathematics, both in the control and experimental groups (see Figures 7, 8).

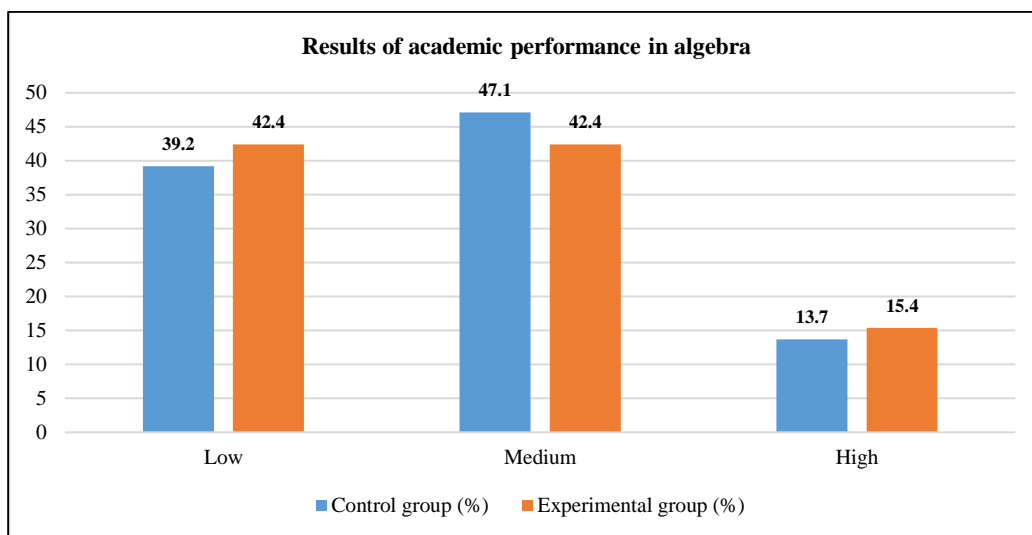


Figure 7.

The results of schoolchildren's performance in algebra at the initial stage of the experiment.

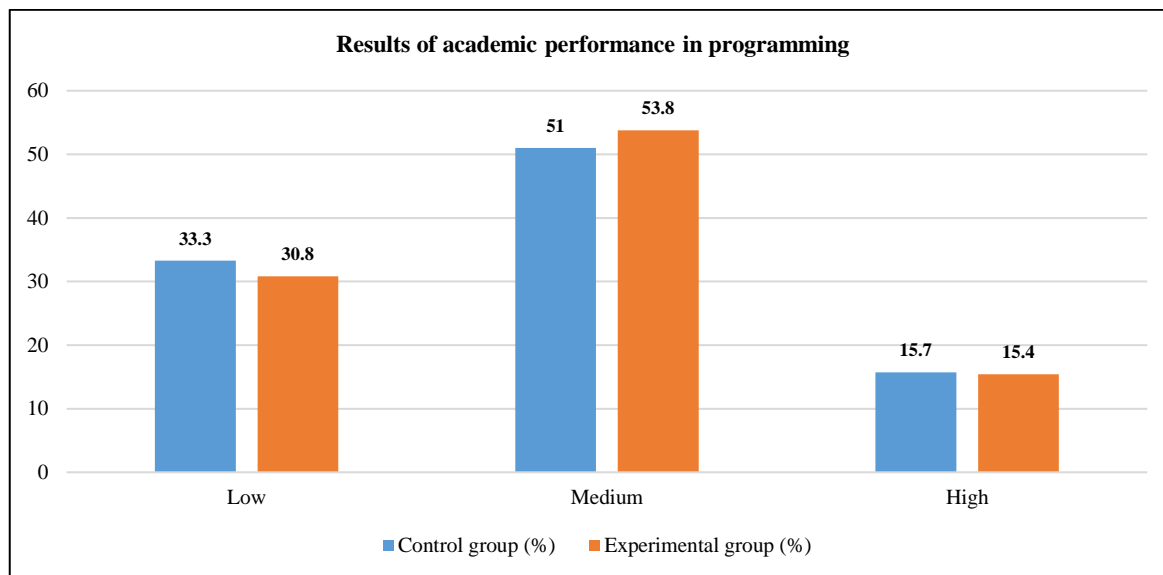


Figure 8.

The results of schoolchildren's academic performance in programming at the initial stage of the experiment.

The analysis of the data on academic achievement in algebra and programming showed that the empirical values of the criterion χ^2 at the number of degrees of freedom $k=2$ are equal to 0.475 for algebra and 0.175 for programming, respectively. They are less than the critical value of χ^2 , which is equal to 5.991 at the significance level $p=0.05$. This indicates that there are no statistically significant differences in the knowledge of students in the control and experimental groups at the initial stage of the experiment and confirms that these groups are homogeneous and conditionally equal.

At the formative stage of the pedagogical experiment, the effectiveness of the proposed methodology was tested. The schoolchildren of the control group were trained according to the traditional methodology, while the schoolchildren of the experimental group studied according to the developed methodology, which provided for the introduction of certain and reasonable pedagogical conditions that ensure better assimilation of educational material and the development of

schoolchildren's computational thinking skills. The first pedagogical condition is to supplement the content of teaching trigonometry in the compulsory algebra course with a system of tasks from the trigonometric heritage of al-Farabi and the use of micro-learning technology.

The second pedagogical condition in the framework of teaching trigonometry is the work of schoolchildren on an interdisciplinary project assignment to automate the process of constructing a table of sines and other trigonometric functions using the al-Farabi algorithm. The organization of schoolchildren's design and research work is mandatory within each section of the curriculum [16].

The assessment of the level of development of schoolchildren's computational thinking is based on the summation of the results for all its components.

The quantitative assessment of the content-related component of schoolchildren's computational thinking is based on the scores they achieved during knowledge testing and laboratory work. The test tasks are designed taking into account the taxonomy of B. Bloom, in which learning goals are determined by the levels of mental processes such as memorization, understanding, application, analysis, synthesis, and evaluation. The reliability of the test instrument is confirmed by the high Cronbach's alpha coefficient (0.82). The evaluation of laboratory work considers the completeness and quality of the problem solution, as well as the reasonableness of the conclusions.

The operational component of schoolchildren's computational thinking is evaluated in the same way as the assessment of their cognitive abilities at the beginning of the experiment using the first four subtests from Amthauer [1] intelligence structure tests, as well as an analysis of the approaches used by schoolchildren when writing laboratory works.

The motivational component is assessed by analyzing the schoolchildren's independent work in the learning process and completing a project assignment.

To quantify the level of development of schoolchildren's computational thinking, the level of development of each component of computational thinking is assigned a score (1 – low, 2 – medium, 3 – high). Based on these scores, the integrated weight of the level of development of computational thinking can be expressed for 27 possible cases. According to the obtained values, three levels of computational thinking development were identified in this study: low (3-4 points), medium (5-6 points), and high (7-9 points).

The results of the diagnosis of computational thinking of schoolchildren in the control and experimental groups at the end of the experiment are shown below (Figure 9).

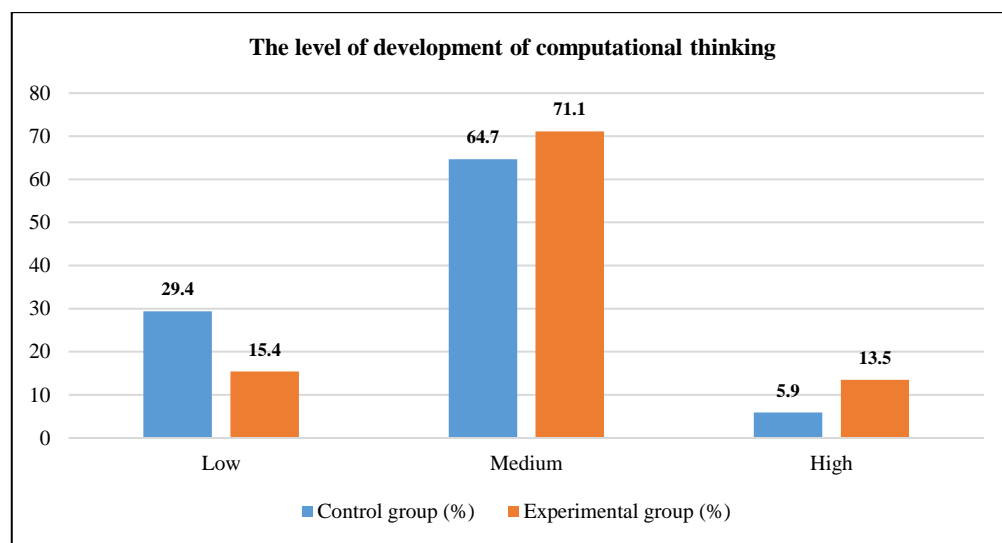


Figure 9.
Diagnostic results of computational thinking.

Based on the results of the statistical analysis, the significance of statistical differences in the total scores of the schoolchildren's computational thinking development level was determined, taking into account the selected confidence level $p=0.05$ (5%). The empirical value of the χ^2 criterion is 7.654, which is more than the critical value of 5.991. This indicates that the quality of training in the experimental group is higher than in the control group, with a significance level of 0.05.

The revealed difference allows us to assert that the application of the developed trigonometry teaching methodology, supplemented by al-Farabi's tasks, within the framework of an interdisciplinary project with elements of microlearning leads to statistically significant (at the level of 95% according to Pearson's criterion χ^2) differences in results and contributes to an increase in the level of development of computational thinking among schoolchildren.

The conducted study also allows us to conclude that the application of the developed methodology for developing schoolchildren's computational thinking in teaching trigonometry within the algebra course by including tasks from al-Farabi's trigonometric heritage in its content enhances their subject knowledge. Moreover, the higher the schoolchildren's level of computational thinking, the higher their academic performance in trigonometry. The increase in the average trigonometry score after the experiment is clearly shown in Figure 10.

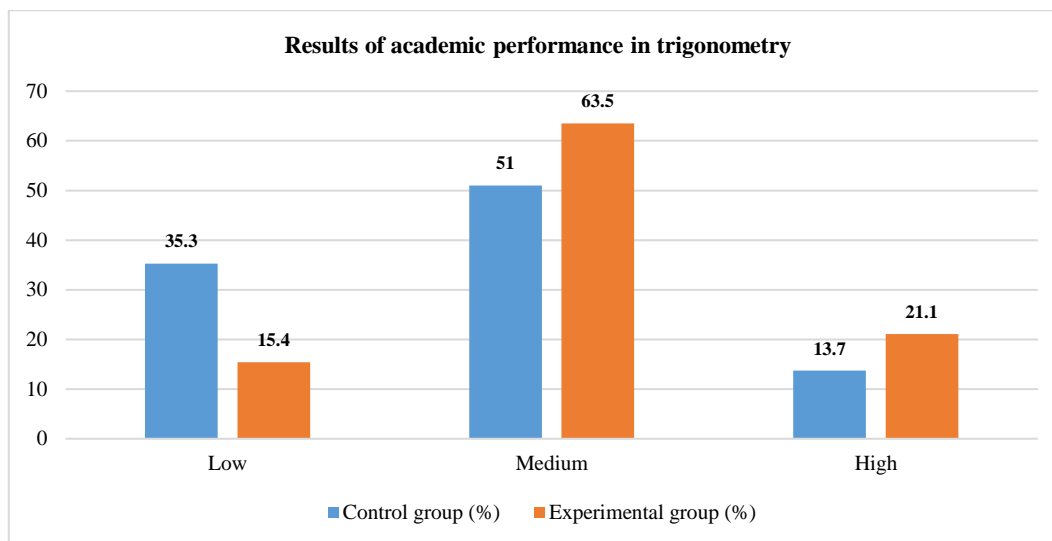


Figure 10.

The results of schoolchildren's progress in trigonometry in the algebra course.

The empirical value of Pearson's χ^2 criterion for diagnosing performance in trigonometry is 10,749, which is greater than the critical value of 5,991. This indicates that the academic performance of the experimental group is higher than that of the control group at the significance level of 0.05, and these results are not random, as well as the level of development of students' computational thinking after experimental training. Academic performance is not only influenced by cognitive ability but also by other factors such as motivation and teaching methodology.

A pedagogical experiment in the form of the implementation of trigonometry teaching methods enriched with tasks from the trigonometric heritage of al-Farabi in the real educational process of secondary schools within the framework of an interdisciplinary project approach using micro-learning technology in modern conditions of digitalization of education showed a significant increase in schoolchildren's academic performance and the level of development of their computational thinking compared with traditional teaching methods.

6. Conclusion

Al-Farabi's trigonometric legacy possesses enormous didactic potential and deserves to be integrated into the modern school education system. It allows for enriching the content of trigonometry teaching, increasing its applied focus, and expanding schoolchildren's understanding of trigonometric problems in general, and possible ways to solve them. Moreover, it promotes the development of schoolchildren's interest in the studied material and a deeper, conscious assimilation of it. The improvement in the quality of knowledge acquisition can be explained by the fact that in the trigonometric legacy of al-Farabi, the need to study every new concept, every new statement, every trigonometric formula is obvious, since without them it is impossible to build a table of sines and other trigonometric functions of great practical importance. Al-Farabi's trigonometry not only contains valuable knowledge but also has significant potential for developing schoolchildren's computational thinking, which is extremely important for successful study and work in a modern digital environment.

The didactic value of introducing al-Farabi's tasks into the content of teaching trigonometry is enhanced by the choice of adequate pedagogical technologies, such as interdisciplinary projects with elements of microlearning.

The implementation of such a method of teaching trigonometry in the school algebra course, supplemented by a system of tasks from the trigonometric heritage of al-Farabi, based on the application of interdisciplinary projects with elements of micro-learning and the use of modern information and communication technologies, will equip schoolchildren with the necessary knowledge in the field of trigonometry and ensure the effective development of their computational thinking.

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