International Journal of Innovative Research and Scientific Studies, 8(5) 2025, pages: 603-614

IJIRSS
International Journal of Innovative Research & Scientific Studies

ISSN: 2617-6548

URL: www.ijirss.com

On new ridge estimators of the Conway-maxwell Poisson model in the case of highly correlated predictor variables: Application to plywood quality data

Mohamed R. Abonazel[1*], Ehab Ebrahim Mohamed Ebrahim[2], Ashrakat Adel Saber[1], Abeer R. Azazy[3]

[1]*Department of Applied Statistics and Econometrics, Faculty of Graduate Studies for Statistical Research, Cairo University, Giza, Egypt.*
[2]*Department of Economics, College of Business, Imam Mohammad Ibn Saud Islamic University (IMSIU), Riyadh, Saudi Arabia.*
[3]*Al-Alsun Higher Institute for Tourism, Hotels and Computer, Cairo, Egypt.*

Corresponding author: Mohamed R. Abonazel (*Email: mabonazel@cu.edu.eg*)

Abstract

This paper introduces a new method to handle multicollinearity in regression analysis, specifically for the Conway-Maxwell Poisson (COMP) model, which is used for count data. Multicollinearity, where predictor variables are highly correlated, often causes unstable results in such models. Our approach improves ridge parameter estimation, balancing accuracy and interpretability. Through simulations, we tested the method using mean squared error (MSE) as a main measure of the efficiency of estimation. The results show that our method performs better than traditional approaches, especially when multicollinearity is high or data is over- or underdispersed. It works particularly well with small-to-moderate datasets and complex data structures. This result was confirmed by our real-world application to plywood quality data.

Keywords: Biased estimator, Conway–Maxwell poisson model, Environmental sustainability, Multicollinearity, Ridge estimator.

DOI: 10.53894/ijirss.v8i5.8775
Funding: This research is supported by the Deanship of Scientific Research at Imam Mohammad Ibn Saud Islamic University (IMSIU), Saudi Arabia (Grant number: IMSIU-DDRSP2502).
History: Received: 28 May 2025 / **Revised:** 4 July 2025 / **Accepted:** 8 July 2025 / **Published:** 22 July 2025

Competing Interests: The authors declare that they have no competing interests.
Authors' Contributions: All authors contributed equally to the conception and design of the study. All authors have read and agreed to the published version of the manuscript.
Transparency: The authors confirm that the manuscript is an honest, accurate, and transparent account of the study; that no vital features of the study have been omitted; and that any discrepancies from the study as planned have been explained. This study followed all ethical practices during writing.
Publisher: Innovative Research Publishing

1. Introduction

Multicollinearity occurs when many explanatory variables in a regression model are moderately or strongly correlated with each other. While multicollinearity is present, the variance of the regression coefficients increases. Consequently, the standard error of the regression coefficients also grows as the variance increases, which may lead to significant changes in the results with minor variations in the data. Hoerl and Kennard [1] proposed the ridge regression estimator as an alternative to the ordinary least squares (OLS) estimate. The ridge method is based on adding biasing constants k to the diagonal of the matrix to shrink all regression coefficients, thereby decreasing the variance.

Count data regression models are applied to data that exhibits over-dispersion and under-dispersion. Also, when the response variable takes the form of counts or non-negative integer values. The Poisson regression model, Conway-Maxwell Poisson regression model are a few examples of these models. The Poisson regression model is the most commonly used; however, it cannot handle data suffering from over-dispersion or under-dispersion because, according to this model, the response variable's mean and variance are assumed to be equal. The Conway–Maxwell–Poisson (COM-Poisson) regression model is the most flexible among count data models, capable of handling data with over-dispersion and under-dispersion. Therefore, this model can accommodate the presence of multicollinearity and over-dispersion because, when these problems exist, the standard error of the maximum likelihood increases, resulting in the maximum likelihood estimator producing inefficient estimates.

There are researchers who have combined this estimator with count data regression to address the problem of multicollinearity. For example, Månsson [2] suggested the RR for negative binomial regression model, Türkan and Özel [3] introduced the modified jackknifed RR estimator for Poisson model, Kaçıranlar and Dawoud [4] proposed some ridge parameters for Zaldivar [5] examined the effectiveness of a few RR estimators for the Poisson model, Rashad and Algamal [6] developed a new RR estimator, and Yehia [7] presented the restricted RR estimator for Poisson model. Abonazel et al. [8] developed ridge estimators for the extended Poisson-Tweedie model with multicollinearity problems. For zero-inflated count regression models, Akram et al. [9] and Al-Taweel and Algamal [10] developed some estimators for the zero-inflated negative binomial model. Algamal et al. [11] developed new estimators for the zero-inflated bell regression Model. For the COMP model, Sami et al. [12] suggested ridge regression estimators for the COMP model.

Akram et al. [9] suggested a new Liu estimator for the COMP model. Algamal et al. [13] proposed the modified jackknife ridge estimator for the COMP model. Tanış and Asar [14] developed a Liu-type estimator for the COMP model. Ashraf et al. [15] proposed new ridge parameter estimators for the zero-inflated Conway-Maxwell-Poisson ridge regression model. Recently, Dawoud [16], Alreshidi et al. [17], Özkale and Mammadova [18] and Hawa et al. [19] proposed hybrid and modified estimators for the COM Model with the multicollinearity problem.

This paper is organized as follows: In Section 2, the ML estimators for the Poisson model and the COMP model are presented. Section 3 displays the theoretical comparisons between the ridge estimator and the ML estimator. In Section 4, we suggest some new methods for estimating the ridge parameter of the ridge estimator. In Section 5, we present the simulation study to evaluate the performance of the proposed ridge estimators. In Section 6, we present the results of a real-data application to assess the performance of several ridge estimators. The conclusion is given in Section 7.

2. Methodology

2.1. Poisson Regression Model and Maximum Likelihood Estimator

Poisson regression is a type of generalized linear model (GLM) in which a response variable is represented by discrete numerical values. When modeling count data, the Poisson regression model is commonly utilized. The connection between the response variable and one or more regressors is modeled using PRM. The response variable might be a count variable or a set of non-negative integers. In PRM, the regression coefficients are estimated using the maximum likelihood estimator (MLE). The dependent variable (y) in Poisson regression is an observed count that follows the Poisson distribution. As a result, the response variable consists of non-negative integers (0, 1, 2, 3, …) are the possible values of y. The probability density function looks like this:

$$P(Y_i = y_i; \mu_i) = \frac{\mu_i^{y_i} e^{-\mu_i}}{y_i!} \quad ; y_i = 0,1,2, \dots, \; \mu_i > 0 \qquad (1)$$

With mean and variance, $E(y_i) = V(y_i) = \mu_i$ (called equi-dispersion)

The Poisson regression model can be expressed as follows, similar to any other nonlinear regression model, where the response variable is a function of a vector of explanatory variables with a vector of unknown coefficients β:

$$y_i = E(y_i|x_i) + \varepsilon_i \qquad (2)$$

It is assumed that the mean response for the case, which will be represented by simplicity, is always a function of the collection of explanatory variables:

$$E(y_i|x_i) = \mu_i = \exp(x_i^T \beta) \qquad (3)$$

The response mean is used in the model's writing. According to Marcondes Filho and Sant'Anna [20], we consider the existence of a function, g, linking the response mean to a linear predictor in a way that

$$g(\mu_i) = \beta_0 + \beta_1 x_1 + \dots + \beta_p x_p = x_i^T \beta, \qquad (4)$$

Where g (.) is a monotone differentiable link function. The log link function is a popular type of this link function, such that $g(\mu_i) = \ln(\mu_i) = \exp(x_i^T \beta)$, $x_i = (x_{i0}, \dots, x_{ip})$ is ith row of *X*, which is an *n×(P+1)* matrix of P explanatory variables, and $\beta = (\beta_0, \beta_1, \dots, \beta_p)^T$ is a *(P+1) ×1* vector of regression coefficients.

For the Poisson regression model, this log link is typically used because it ensures that all fitted values for the response variable are positive.

We may estimate the Poisson regression coefficients using maximum likelihood approaches as follows:

- Step one: The likelihood function is given by:

$$L = \prod_{i=1}^{n} \frac{\mu_i^{y_i} e^{-\mu_i}}{y_i!} \qquad (5)$$

- Step two: the log likelihood function as follows:

$$L(\beta) = \sum_{i=1}^{n} y_i ln(\mu_i) - \mu_i - ln(y_i!)$$
$$= \sum_{i=1}^{n} y_i x_i^T \beta - exp(x_i^T \beta) - ln(y_i!) \quad (6)$$

- Step three: the first partial derivative with regard to β is given as follows:

$$\frac{\partial L(\beta)}{\partial \beta} = \sum_{i=1}^{n} (y_i - \mu_i) x_i \quad (7)$$
$$= \sum_{i=1}^{n} (y_i - exp(x_i^T \beta)) x_i \quad (8)$$

Because equation (8) is nonlinear in β, we may use the weighted least squares (IWLS) algorithm to find a suitable solution. The MLE of can then be derived by

$$\hat{\beta}_{MLP} = (X^T \hat{D} X)^{-1} X^T \hat{D} \hat{u} \quad (9)$$

Where u is a column vector with its ith element equal to $ln(\mu_i) + \frac{y_i - \hat{\mu}_i}{\hat{\mu}_i}$. The second derivative of the log-likelihood function is calculated to get the Hessian matrix:

$$H = -\sum_{i=1}^{n} (exp(x_i^T \beta)) x_i x_i^T \quad (10)$$

The variance-covariance matrix based on the negative inverse of the Hessian can be obtained using the maximum likelihood method as follows:

$$\Sigma = [\sum_{i=1}^{n} (exp(x_i^T \beta)) x_i x_i^T]^{-1}$$
$$\therefore \text{cov}(\hat{\beta}_{ML.P}) = (X^T \hat{D} X)^{-1} \quad (11)$$

So, the MSE of $\hat{\beta}_{ML.P}$ is given as follows:

$$MSE(\hat{\beta}_{ML.P}) = \text{tr}[(X^T \hat{D} X)^{-1}] = \sum_{j=1}^{p} \frac{1}{\delta_j} \quad (12)$$

Where δ_j is the j^{th} eigenvalue of the $(X^T \hat{D} X)$ matrix

2.2. COM Poisson Distribution and Maximum Likelihood (ML) Estimation

The COM Poisson distribution was introduced by Conway and Maxwell [21]. It is a two-parameter generalization of the Poisson distribution. When a dispersion parameter is included, it can more easily handle dispersion in count data, regardless of whether it is overdispersed or underdispersed. The probability mass function of this distribution is

$$P(Y = y;\ \vartheta, \phi) = \frac{\vartheta^Y}{Z(\vartheta, \phi)(Y!)^{\phi}} \quad , y = 0,1, \dots, \infty;\ \upsilon > 0. \quad (13)$$

The normalizing constant and infinite series are denoted by $Z(\vartheta, \phi) = \sum_{n=0}^{\infty} \frac{\vartheta^n}{(n!)^{\phi}}$, where ϑ denotes the mean parameter of the distribution, and ϕ is the dispersion parameter of the distribution. According to Sellers and Premeaux [22] the mean and variance of the COMP distribution are:

$$E(Y) \approx \vartheta^{\frac{1}{\phi}} + \frac{1}{2\phi} - \frac{1}{2}, \quad \text{var}(Y) \approx \frac{1}{\phi} \vartheta^{\frac{1}{\phi}} \quad (14)$$

The COMP distribution is the generalization of several well–known discrete distributions: the Poisson distribution is approximated by the COMP distribution if $\phi = 1$, the geometric distribution is approximated if $\phi = 0$ and $\vartheta < 1$, and the Bernoulli distribution with probability $\left(\frac{\vartheta}{1+\vartheta}\right)$ is approximated by the COMP distribution if $\phi \to \infty$. Guikema and Goffelt [23] suggested re–parameterizing the COMP distribution for the COMP regression model in order to provide a clear centering parameter, which is as follows:

$$P(Y = y) = \frac{1}{S(\mu, \phi)} \left(\frac{\mu^y}{y!}\right)^{\phi}, \quad y = 0,1, \dots, \infty;\ \upsilon > 0 \quad (15)$$

Where $\mu = \vartheta^{\frac{1}{\phi}}$ and $S(\mu, \phi) = \sum_{n=0}^{\infty} \left(\frac{\mu^n}{n!}\right)^{\phi}$. Assume that $\eta_i = \log(\mu_i) = x_i^T \beta$, when β the regression coefficient vector with the intercept included, the COMP regression model's log-likelihood function is as follows, Sami et al. [12]

$$L(y_i, \beta, \phi) = \phi \sum_{i=1}^{n} y_i \eta_i - \sum_{i=1}^{n} \phi \log(y_i!) - \sum_{i=1}^{n} \log[S(\eta_i, \phi)]. \quad (16)$$

For estimating β and ϕ, we differentiate Equation 16 for β and ϕ as follows

$$\frac{\partial L}{\partial \beta_j} = \sum_{i=1}^{n} (y_i \phi - \frac{\partial}{\partial \eta_i} \log[S(\eta_i, \upsilon)]) x_{ij} \quad (17)$$
$$\frac{\partial L}{\partial \phi} = \sum_{i=1}^{n} (-\log(y_i!) - \frac{\partial}{\partial \phi} \log[S(\eta_i, \phi)]) \quad (18)$$

The iterative reweighted least squares (IRLS) method is applied to solve Equations 17 and 18. Upon setting υ, the β estimator of COMP maximum likelihood (COMPML) is

$$\hat{\beta}_{ML} = (A)^{-1} X^T \hat{S} c \quad (19)$$

where $A = X^T \hat{S} X, c = \log(\hat{\mu}) + \frac{(y - \hat{\mu})}{\hat{\mu}^2}$, and $\hat{S} = \text{diag}(v_i);\ v_i = \frac{\tau_i}{\phi} + \frac{\phi^2 - 1}{24\phi^3} \tau_i^{-1} + \frac{\phi^2 - 1}{12\phi^4} \tau_i^{-2} + \frac{\phi^2 - 1}{6\phi^5} \tau_i^{-3};\ \tau_i = \frac{\hat{\mu}_i}{\phi}$ [12].

The MSE of $\hat{\beta}_{ML}$ is

$$\text{MSE}(\hat{\beta}_{ML}) = E(\hat{\beta}_{ML} - \beta)^T (\hat{\beta}_{ML} - \beta) = \hat{\phi}\ tr(Q \Delta^{-1} Q^T) = \hat{\phi} \sum_{j=1}^{p} \frac{1}{\delta_j}, \quad (20)$$

where $tr(\cdot)$ is the trace of the matrix, $\Delta = \text{diag}(\delta_1, \delta_2, \dots, \delta_p) = QAQ^T$, Q represents the orthogonal matrix whose columns are the eigenvectors of A, δ_j is the j^{th} eigenvalue of the A matrix, and $\hat{\phi}$ is the ML estimate of ϕ.

2.3. COM-Poisson Ridge (COMPRR) Estimator

To address the issue of multicollinearity, the RR technique for GLM was introduced by Segerstedt [24] and was based on the research of Hoerl and Kennard [1]. When there is a correlation between the explanatory variables in the COMP regression model, the MSE of MLE increases significantly and yields unproductive results. Sami et al. [12] presented the RR estimator for the COMP regression model and named it the COMPRR estimator to address the issue of multicollinearity.

The COMPRR estimator is given as follows:

$$\hat{\beta}_k = (X^T\hat{S}X + kI_P)^{-1}(X^T\hat{S}X)\hat{\beta}_{ML} \quad (21)$$

Where $k > 0$ is the ridge parameter, I_P is the identity matrix of order (p×p).

- The bias and the covariance of the COMPRR estimator is given as

$$\text{Bias}(\hat{\beta}_k) = -kQ\Delta_k^{-1}\beta$$

$$\text{COV}(\hat{\beta}_k) = \hat{\phi}Q\Delta_k^{-1}\Delta\Delta_k^{-1}Q^T$$

$$\text{MMSE}(\hat{\beta}_k) = \text{COV}(\hat{\beta}_k) + \text{Bias}(\hat{\beta}_k)\text{Bias}(\hat{\beta}_k)^T$$

$$= \hat{\phi}Q\Delta_k^{-1}\Delta\Delta_k^{-1}Q^T + (-kQ\Delta_k^{-1}\beta)(-kQ\Delta_k^{-1}\beta)^T \quad (22)$$

Where $\Delta_k = \text{diag}(\delta_1 + k, \delta_2 + k, \dots, \delta_p + k)$. Thus, the MSE of the COMPRR estimator is obtained from Equation 22 using the tr(.) operator is as follows:

$$\text{MSE}(\hat{\beta}_k) = \text{tr}\left(\text{MMSE}(\hat{\beta}_k)\right) = \hat{\phi}\sum_{j=1}^{p}\frac{\delta_j}{(\delta_j+k)^2} + k^2\sum_{j=1}^{p}\frac{\alpha_j^2}{(\delta_j+k)^2} \quad (23)$$

Where $\alpha = Q^T\beta$.

3. Comparison between COMPML and COMPRR Estimators

Theorm 1. Since $k > 0$, then the COMPRR estimator is better than the COMPML estimator if $\hat{\phi}[k[2\delta_j + k] > k\alpha_j^2$ $\forall j = 1, \dots, p$.

Proof.

The difference among the MMSE functions of the COMPML and COMPRR estimators:

$$\text{MMSE}(\hat{\beta}_{ML}) - \text{MMSE}(\hat{\beta}_k) = \hat{\phi}\Delta^{-1} - \left(\hat{\phi}Q\Delta_k^{-1}\Delta\Delta_k^{-1}Q^T + (-kQ\Delta_k^{-1}\beta)(-kQ\Delta_k^{-1}\beta)^T\right)$$

So, we can rewrite the previous equation as

$$\text{MSE}(\hat{\beta}_{ML}) - \text{MSE}(\hat{\beta}_k) = \sum_{j=1}^{p}\left(\frac{\hat{\phi}}{\delta_j} - \frac{\hat{\phi}\delta_j + k^2\alpha_j^2}{(\delta_j+k)^2}\right)$$

$\text{MSE}(\hat{\beta}_{ML}) - \text{MSE}(\hat{\beta}_k) > 0$ if $\hat{\phi}(\delta_j + k)^2 - \hat{\phi}\delta_j{}^2 - k^2\alpha_j^2 > 0 \rightarrow 2\hat{\phi}\delta_j k + \hat{\phi}k^2 - k^2\alpha_j^2 > 0 \rightarrow \hat{\phi}[k[2\delta_j + k] > k\alpha_j^2$. Then $\text{MSE}(\hat{\beta}_{ML}) > \text{MSE}(\hat{\beta}_k)$ if $\hat{\phi}[k[2\delta_j + k] > k\alpha_j^2$ for $k > 0$. The proof is completed.

4. Proposed Ridge Estimators

Based on previous studies on ridge estimation in different regression models, we can use the following estimators for the *k* parameter in the COMPRR estimator:

$$\hat{k}_1 = \frac{p\hat{\phi}}{\sum_{j=1}^{p}\hat{\alpha}_j^2}, \quad (24)$$

where $\hat{\alpha}_j$ is an element of the estimated vector $\hat{\alpha} = (\hat{\alpha}_1, \dots, \hat{\alpha}_p)^T$.

The following estimators can be used for the k parameter of the COMPRR estimator in the previous Equation 24 that follows [1].

$$\hat{k}_2 = \frac{\hat{\phi}}{\max_j(\hat{\alpha}_j^2)}, \quad (25)$$

$$\hat{k}_3 = \left(\prod_{j=1}^{p}\frac{\hat{\phi}}{(1+\hat{k}_2)\hat{\alpha}_j^2}\right)^{1/p}, \quad (26)$$

$$\hat{k}_4 = \min_j\left(\frac{1}{\hat{m}_j}\right), \quad (27)$$

where $\hat{m}_j = \sqrt{\frac{\max_j(\hat{\phi},\delta_j)}{\hat{\phi}(n-p)+\hat{\alpha}_j^2\max_j(\delta_j)}}$; $j = 1, \dots, p$.

$$\hat{k}_5 = \underset{j}{\text{median}}\left(\frac{1}{\hat{m}_j}\right), \quad (28)$$

The following estimators can be used for the k parameter of the COMPRR estimator in the previous Equations 27 and 28 that follow [12].

$$\hat{k}_6 = \left(\frac{1}{p}\sum_{j=1}^{p}\frac{\hat{\phi}}{\hat{\alpha}_j^2}\right)^{1/p}, \quad (29)$$

$$\hat{k}_7 = \max_j \left(\frac{\hat{\Phi}}{\sqrt{1/\hat{\alpha}_j^2}} \right). \quad (30)$$

We believe that our proposed estimators in Equations 25, 26, 29 and 30 will provide more efficient estimates of the COMP model.

5. Simulation Study

5.1. Simulation Design

The performance of suggested estimators and existing estimators is compared under different circumstances using the Monte Carlo simulation. The response variable (y_i) follows a COMP (μ_i, ϕ) distribution with μ_i is generated as follows:

$$\mu_i = exp\left(\beta_1 x_{i1} + \cdots + \beta_p x_{ip}\right),\ i = 1, \ldots, n, \quad (31)$$

Where $\sum_{j=1}^{p} \beta_j^2 = 1$; $\beta_1 = \cdots = \beta_p$, as in Alreshidi et al. [17] and Abonazel [25], and Abonazel [26].

Following Kibria [27], Shamany et al. [28], Abonazel and Dawoud [29] and Abonazel et al. [30] the correlated explanatory variables are produced as follows:

$$x_{ij} = \tau_{ij}\sqrt{1-\rho^2} + \rho\tau_{ip}\ ,\ \ i = 1, \ldots, n\ ; j = 1, \ldots, p \quad (32)$$

Where τ_{ij} are the independent standard normal pseudo–random numbers. We take different values of ρ, which are equivalent to 0.85, 0.90, 0.95, and 0.99. Additionally, we examine various values for n, p, and υ In this case, p denotes the number of explanatory variables that are assumed to be 2, 4, and 6, and n represents the sample size that is considered to be 30, 50, 75, 100, 200, and 300. υ indicates the dispersion parameter that is assumed to be 0.85, 1, 1.25. For the different combinations of n, p, ρ, υ the output data are repeated $L = 1000$ times. To evaluate the performance of the estimators, we use the MSE criteria as follows:

$$MSE(\hat{\beta}) = \frac{1}{L}\sum_{l=1}^{L}(\hat{\beta}_l - \beta)^T(\hat{\beta}_l - \beta), \quad (33)$$

where $(\hat{\beta}_l - \beta)$ is the difference between the estimated and true parameter vectors at the lth replication?

5.2. Simulation Results

In this section, we tested the performance of the ridge estimator using the proposed ridge parameters in the Conway-Maxwell Poisson regression model. The mean of MSE for each combination of n, p,ϕ and ρ is shown in Tables 1-9, the minimum value of the simulated MSE in each row is highlighted in bold. The following conclusions from the simulation results can be summed up as follows: sample size has an impact on the values of MSE, and the MSE decreases as the sample size increases. For example, $k_7 = 0.2349$ is a smaller value than the others when$\rho = 0.85, p = 2, \phi = 0.85\ and\ n = 30$, but when the sample size increases from 30 to 100 with the same dispersion parameter, correlation level and explanatory variables, the MSE decreases as$k_7 = 0.2334$. The sample size rise to 300, the MSE reduced $k_7 = 0.1383$. Also, the explanatory variables have an impact on the value of MSE. When the explanatory variables rise, the MSE increases. For example,$\rho = 0.85, p = 2, \phi = 0.85\ and\ n = 30$. The MSE for $k_7 = 0.2349$ is a smaller value than the others, but when explanatory variables increase from 2 to 4 with the same dispersion parameter, correlation level and explanatory variables, the MSE value rises as $k_7 = 0.4409$. Also, the explanatory variables rise from 4 to 6, and the MSE value rises to $k_7 = 0.9025$ but this value is not smaller than the others in this case, but $k_3 = 0.6914$. The dispersion parameter has a direct impact on the MSE; as the dispersion parameter grows, the MSE decreases as well. For example, at $\rho = 0.85, p = 4, \phi = 0.85\ and\ n = 30$, $k_7 = 0.4409$ this value is better than others in this case, but when the dispersion parameter grows up to 1 with other factors constant, k_7 decrease from 0.4409 to 0.3140, but still the smallest value, the dispersion parameter rises again from 1 to 1.25 with the factors constant, k_7 reduce from 0.3140 to 0.2683.

From tables 1 – 2, when $\phi = 0.85\ and\ 1$,$p = 2$ it is shown that the smallest MSE is k_5, k_6 and $k_7$but from Table 3, when the dispersion parameter rises to 1.25 with the same number of explanatory variables that equal 2, it is shown that the smallest MSE is k_1, k_6 andk_7. From the previous, that shown that k_5 is affected by the dispersion parameter when it rises to 1.25 and becomesk_1 is better than k_5. From tables 4 - 5, when $\phi = 0.85\ and\ 1$, $p = 4$ it is shown that the outperformed value of MSE is k_3, k_6 and k_7, From the earlier part, it demonstrated the explanatory variables' effect on the values of MSE, especially affecting k_5 because it is shown that it is not the smallest value and has become k_3 is better than k_5. From Table 6 when the dispersion parameter grows up to 1.25 with the number of explanatory variables constant, it is shown that the outperformed value of MSE is k_1, k_6 and k_7, From the previous, that shown that k_3 is affected by the dispersion parameter when it rises to 1.25 and becomes k_1 is better than k_5. Also, from tables 7 – 9 with the same values of dispersion parameter, but the number of explanatory variables increases to 6, it is shown that the best values of MSE are k_3 and k_7. From the previous, that shown that k_1, k_5, k_6 are affected by the number of explanatory variables when it rises to 6. From the above, it became clear that k_7 is the best in all cases when the number of independent variables and the dispersion parameter increases, at all levels of correlation, and when the sample size increases.

The results show that k_1, k_3, k_5, k_6, k_7 perform better than the other COMPRR parameters when evaluating the performance of the proposed parameters because they attain the lowest MSE.

Table 1.
MSE values of different estimators when ϕ=0.85 and p=2.

n	ρ	COMPML	k_1	k_2	k_3	k_4	k_5	k_6	k_7
30	**0.85**	1.1020	0.5557	0.6408	0.6227	0.3021	0.2527	0.4174	**0.2349**
	0.90	1.4226	0.6930	0.8116	0.7560	0.3078	0.2451	0.4749	**0.2213**
	0.95	2.7228	1.1241	1.4307	1.0267	0.2226	0.1519	0.4642	**0.1316**
	0.99	13.1134	4.6126	6.5810	2.0426	0.1034	0.0584	0.2857	**0.0497**
50	**0.85**	0.8003	0.4341	0.4894	0.4960	0.2908	0.2583	0.3598	**0.2529**
	0.90	1.2123	0.5977	0.6937	0.6600	0.2763	0.2303	0.4270	**0.2231**
	0.95	1.9952	0.8414	1.0359	0.8616	0.2450	0.1814	0.4544	**0.1616**
	0.99	10.7043	3.8734	5.4250	1.9050	0.1049	0.0560	0.2884	**0.0477**
75	**0.85**	0.5917	0.3360	0.3752	0.3870	0.2683	0.2445	0.2951	**0.2394**
	0.90	0.7040	0.4049	0.4535	0.4549	0.2888	0.2568	0.3389	**0.2480**
	0.95	1.0937	0.5188	0.5986	0.5910	0.2660	0.2258	0.3842	**0.2168**
	0.99	10.5573	3.9160	5.4506	2.0003	0.1152	0.0590	0.3220	**0.0499**
100	**0.85**	0.3837	0.2661	0.2909	0.2874	0.2475	0.2332	0.2422	0.2334
	0.90	0.7850	0.4336	0.4871	0.4915	0.2880	0.2531	0.3557	**0.2452**
	0.95	0.8572	0.4562	0.5159	0.5180	0.2780	0.2418	0.3652	**0.2343**
	0.99	5.6068	2.0842	2.8387	1.4304	0.1430	0.0831	0.3859	**0.0705**
200	**0.85**	0.1897	0.1558	0.1669	0.1554	0.1606	0.1541	0.1411	0.1526
	0.90	0.2656	0.2009	0.2180	0.2091	0.2008	0.1909	0.1838	0.1901
	0.95	0.5659	0.3507	0.3862	0.3941	0.2819	0.2603	0.3108	0.2610
	0.99	2.6538	1.0726	1.3608	1.0123	0.2122	0.1445	0.4594	**0.1267**
300	**0.85**	0.1618	0.1392	0.1473	0.1375	0.1449	0.1402	0.1275	0.1383
	0.90	0.1822	0.1489	0.1599	0.1505	0.1558	0.1493	0.1373	0.1467
	0.95	0.3766	0.2564	0.2813	0.2800	0.2391	0.2247	0.2350	**0.2239**
	0.99	1.4994	0.7001	0.8352	0.7462	0.2620	0.2028	0.4396	**0.1836**

Table 2.
MSE values of different estimators when $\phi = 1$ and $p = 2$.

n	ρ	COMPML	k_1	k_2	k_3	k_4	k_5	k_6	k_7
30	**0.85**	1.0885	0.4331	0.5354	0.4965	0.2392	0.1877	0.3054	**0.1733**
	0.90	1.3523	0.5200	0.6554	0.5688	0.2412	0.1796	0.3283	**0.1604**
	0.95	2.8315	0.9800	1.3388	0.7690	0.1761	0.1040	0.3019	**0.0826**
	0.99	13.7322	4.2662	6.3775	1.3608	0.0674	0.0336	0.1522	**0.0322**
50	**0.85**	0.7810	0.3307	0.3953	0.4072	0.2275	0.1948	0.2741	**0.1940**
	0.90	1.1624	0.4577	0.5670	0.5132	0.2050	0.1604	0.3045	**0.1580**
	0.95	2.0547	0.7204	0.9602	0.6700	0.1889	0.1201	0.3096	**0.1026**
	0.99	10.5397	3.2181	4.7805	1.1658	0.0631	0.0218	0.1435	**0.0172**
75	**0.85**	0.5646	0.2449	0.2865	0.3176	0.2093	**0.1874**	0.2236	0.1893
	0.90	0.6751	0.3060	0.3592	0.3742	0.2251	0.1939	0.2569	**0.1915**
	0.95	1.1186	0.4187	0.5228	0.4786	0.2039	0.1592	0.2783	**0.1520**
	0.99	10.4474	3.3616	4.9242	1.2950	0.0712	0.0201	0.1622	**0.0139**
100	**0.85**	0.3487	0.1931	0.2177	0.2401	0.1936	**0.1831**	0.1913	0.1925
	0.90	0.7785	0.3446	0.4072	0.4135	0.2318	0.1948	0.2729	**0.1922**
	0.95	0.8185	0.3430	0.4122	0.4103	0.2091	0.1723	0.2630	**0.1699**
	0.99	5.9351	1.9224	2.7759	1.0289	0.0933	0.0354	0.2163	**0.0255**
200	**0.85**	0.1477	**0.0995**	0.1117	0.1159	0.1124	0.1082	0.1021	0.1110
	0.90	0.2279	**0.1372**	0.1549	0.1661	0.1497	0.1432	0.1383	0.1493
	0.95	0.5442	0.2588	0.2983	0.3259	0.2164	**0.1961**	0.2345	0.2041
	0.99	2.6550	0.8651	1.2001	0.7109	0.1453	0.0794	0.2705	**0.0641**
300	**0.85**	0.1235	0.0903	0.1002	0.0991	0.1027	0.0993	**0.0889**	0.1003
	0.90	0.1426	0.0955	0.1077	0.1092	0.1103	0.1060	**0.0950**	0.1075
	0.95	0.3313	0.1757	0.1997	0.2205	0.1795	**0.1693**	0.1716	0.1779
	0.99	1.4814	0.5636	0.7231	0.5710	0.1950	0.1327	0.2917	**0.1184**

Table 3.
MSE values of different estimators when $\phi = 1.25$ and $p = 2$.

n	ρ	COMPML	k_1	k_2	k_3	k_4	k_5	k_6	k_7
30	**0.85**	1.2500	0.4473	0.5814	0.4468	0.2828	0.2238	0.2909	**0.2106**
	0.90	1.4902	0.4954	0.6620	0.4751	0.2783	0.2140	0.2906	**0.1942**
	0.95	3.1387	0.8955	1.3036	0.5481	0.2128	0.1541	0.2541	**0.1370**
	0.99	13.5383	3.2811	5.2435	0.6735	0.1406	**0.1270**	0.1648	0.1334
50	**0.85**	0.8880	0.3365	0.4179	0.3820	0.2666	**0.2302**	0.2672	0.2341
	0.90	1.2314	0.4327	0.5598	0.4359	0.2476	0.2060	0.2840	**0.2048**
	0.95	2.1958	0.6630	0.9371	0.4975	0.2251	0.1602	0.2496	**0.1473**
	0.99	10.7155	2.8334	4.4003	0.6676	0.1267	0.0959	0.1408	**0.0951**
75	**0.85**	0.6506	0.2461	0.2959	0.3180	0.2410	**0.2171**	0.2302	0.2245
	0.90	0.7723	0.3082	0.3766	0.3652	0.2594	**0.2245**	0.2575	0.2264
	0.95	1.2589	0.4005	0.5331	0.4050	0.2327	0.1866	0.2479	**0.1826**
	0.99	10.4848	2.7197	4.2698	0.6626	0.1221	0.0844	0.1351	**0.0824**
100	**0.85**	0.4300	**0.2180**	0.2432	0.2870	0.2384	0.2285	0.2340	0.2478
	0.90	0.8552	0.3272	0.4059	0.3729	0.2541	**0.2158**	0.2522	0.2170
	0.95	0.9357	0.3306	0.4199	0.3674	0.2339	**0.1959**	0.2412	0.1971
	0.99	5.8975	1.4614	2.2895	0.5371	0.1301	0.0919	0.1544	**0.0869**
200	**0.85**	0.2222	**0.1507**	0.1612	0.1883	0.1766	0.1742	0.1710	0.1841
	0.90	0.2981	**0.1731**	0.1885	0.2276	0.2038	0.1993	0.1957	0.2156
	0.95	0.6462	0.2774	0.3305	0.3482	0.2602	**0.2347**	0.2546	0.2496
	0.99	2.7317	0.7625	1.1288	0.4819	0.1803	0.1209	0.2031	**0.1104**
300	**0.85**	0.1887	**0.1371**	0.1462	0.1641	0.1610	0.1594	0.1510	0.1655
	0.90	0.2106	**0.1383**	0.1492	0.1736	0.1675	0.1651	0.1554	0.1733
	0.95	0.3998	**0.1919**	0.2164	0.2599	0.2221	0.2132	0.2063	0.2332
	0.99	1.6148	0.5085	0.7056	0.4380	0.2223	0.1620	0.2326	**0.1528**

Table 4.
MSE values of different estimators when $\phi = 0.85$ and $p = 4$.

n	ρ	COMPML	k_1	k_2	k_3	k_4	k_5	k_6	k_7
30	**0.85**	4.4443	1.6854	2.5970	1.2626	1.5019	0.7784	0.7589	**0.4409**
	0.90	4.4691	1.5533	2.4927	1.0582	1.5091	0.7338	0.6589	**0.4286**
	0.95	9.1558	3.0299	5.1101	1.7923	1.4382	0.5048	0.7034	**0.2230**
	0.99	31.5666	9.5575	17.6034	3.6984	0.4913	0.1399	0.4300	**0.0563**
50	**0.85**	1.5259	0.6955	0.9879	0.5696	1.0510	0.7747	**0.5201**	0.5966
	0.90	2.6038	1.0928	1.5895	0.8648	1.4247	0.8759	0.6587	**0.6020**
	0.95	4.8414	1.8284	2.8202	1.2910	1.5213	0.7345	0.7262	**0.4061**
	0.99	24.3604	8.0233	13.9164	3.6871	0.8168	0.1980	0.5873	**0.0747**
75	**0.85**	0.8079	0.4481	0.6095	**0.3550**	0.6631	0.5840	0.3896	0.5010
	0.90	1.8021	0.8111	1.1541	0.6612	1.1586	0.8311	**0.5774**	0.6341
	0.95	2.8045	1.1496	1.6970	0.8793	1.4648	0.8640	0.6510	**0.5663**
	0.99	13.7834	4.4613	7.6595	2.4237	0.9769	0.3030	0.6071	**0.1142**
100	**0.85**	0.8563	0.4798	0.6467	**0.3801**	0.7043	0.6167	0.4117	0.5284
	0.90	1.3367	0.6603	0.9186	0.5461	0.9778	0.7683	**0.5245**	0.6135
	0.95	1.6177	0.7415	1.0535	0.5859	1.0859	0.7953	**0.5298**	0.6088
	0.99	10.2263	3.4606	5.8419	2.0373	1.2986	0.4257	0.6771	**0.1648**
200	**0.85**	0.4053	0.2877	0.3561	**0.2175**	0.3769	0.3626	0.2687	0.3285
	0.90	0.6321	0.3989	0.5173	**0.3147**	0.5568	0.5144	0.3622	0.4529
	0.95	1.1403	0.5779	0.8009	0.4681	0.8754	0.7105	**0.4664**	0.5843
	0.99	5.8867	2.1664	3.4004	1.5015	1.5115	0.6629	0.7518	**0.3348**
300	**0.85**	0.3061	0.2338	0.2786	**0.1716**	0.2898	0.2835	0.2193	0.2625
	0.90	0.4226	0.3006	0.3705	**0.2311**	0.3914	0.3755	0.2816	0.3403
	0.95	0.7507	0.4454	0.5898	**0.3660**	0.6428	0.5795	0.4044	0.5023
	0.99	3.7600	1.4300	2.1693	1.0921	1.4880	0.7702	0.6826	**0.4648**

Table 5.
MSE values of different estimators when $\phi = 1$ and $p = 4$.

n	ρ	COMPML	k_1	k_2	k_3	k_4	k_5	k_6	k_7
30	**0.85**	4.6701	1.4760	2.4968	1.0365	1.2183	0.6350	0.5903	**0.3140**
	0.90	4.7528	1.3619	2.4114	0.8599	1.2293	0.6128	0.5164	**0.3213**
	0.95	9.8274	2.7859	5.0548	1.4874	1.0717	0.3732	0.5158	**0.1369**
	0.99	34.7199	8.9563	17.7629	2.9006	0.2936	0.0739	0.2683	**0.0225**
50	**0.85**	1.6428	0.5899	0.9146	0.4946	0.9832	0.7024	**0.4354**	0.5236
	0.90	2.7829	0.9516	1.5172	0.7436	1.2409	0.7642	0.5487	**0.4828**
	0.95	5.1447	1.6087	2.7432	1.0754	1.1690	0.5911	0.5718	**0.2794**
	0.99	25.4876	7.1635	13.4270	2.7909	0.5217	0.1198	0.3772	**0.0293**
75	**0.85**	0.8403	0.3599	0.5337	**0.3187**	0.6271	0.5425	0.3371	0.4763
	0.90	1.8622	0.6619	1.0302	0.5587	1.0211	0.7304	**0.4750**	0.5400
	0.95	3.0585	1.0178	1.6515	0.7607	1.2691	0.7441	0.5379	**0.4437**
	0.99	14.5868	4.0451	7.4902	1.8707	0.6695	0.1998	0.4002	**0.0542**
100	**0.85**	0.8557	0.3741	0.5499	**0.3184**	0.6473	0.5583	0.3389	0.4876
	0.90	1.4182	0.5572	0.8386	0.4844	0.9147	0.7084	**0.4498**	0.5602
	0.95	1.7331	0.6298	0.9770	0.5095	0.9939	0.7163	**0.4434**	0.5294
	0.99	10.8110	3.1013	5.6898	1.6058	0.9285	0.3050	0.4706	**0.0895**
200	**0.85**	0.3937	0.2244	0.3093	**0.1839**	0.3517	0.3343	0.2266	0.3076
	0.90	0.6333	0.3165	0.4494	**0.2733**	0.5242	0.4770	0.3079	0.4294
	0.95	1.1737	0.4616	0.6947	0.4016	0.8058	0.6442	**0.3904**	0.5321
	0.99	6.2351	1.9167	3.2873	1.2558	1.1419	0.5076	0.5783	**0.2219**
300	**0.85**	0.2896	0.1807	0.2407	**0.1407**	0.2649	0.2574	0.1827	0.2407
	0.90	0.4139	0.2362	0.3232	**0.1998**	0.3672	0.3479	0.2414	0.3203
	0.95	0.7724	0.3560	0.5136	**0.3247**	0.6138	0.5416	0.3474	0.4810
	0.99	4.0343	1.2603	2.1126	0.9068	1.2105	0.6261	0.5359	**0.3293**

Table 6.
MSE values of different estimators when $\phi = 1.25$ and $p = 4$.

n	ρ	COMPML	k_1	k_2	k_3	k_4	k_5	k_6	k_7
30	**0.85**	4.9549	1.2561	2.3782	0.7722	0.9656	0.5503	0.4670	**0.2683**
	0.90	5.1077	1.1973	2.3391	0.6777	0.9469	0.5401	0.4306	**0.2800**
	0.95	10.5960	2.3818	4.8132	1.0396	0.7779	0.3169	0.3821	**0.1394**
	0.99	37.8163	7.8740	17.0847	1.9570	0.2521	0.1229	0.2236	**0.0918**
50	**0.85**	1.8200	0.5051	0.8599	0.4460	0.8751	0.6571	**0.4011**	0.4741
	0.90	3.0139	0.7953	1.4394	0.5855	1.0043	0.6678	0.4541	**0.3911**
	0.95	5.5091	1.3230	2.5381	0.7838	0.8351	0.4667	0.4343	**0.2262**
	0.99	27.0311	5.9289	12.2750	1.8574	0.3578	0.1252	0.2607	**0.0748**
75	**0.85**	0.9739	**0.3259**	0.5002	0.3435	0.6292	0.5565	0.3514	0.5064
	0.90	2.0272	0.5629	0.9669	0.4895	0.8909	0.6780	**0.4333**	0.4927
	0.95	3.3821	0.9264	1.6626	0.6417	1.0748	0.6746	0.4673	**0.3784**
	0.99	15.6977	3.4453	7.0893	1.3361	0.4533	0.1852	0.3018	**0.0881**
100	**0.85**	0.9652	**0.3480**	0.5233	0.3582	0.6591	0.5814	0.3646	0.5309
	0.90	1.5394	0.4895	0.7890	0.4627	0.8518	0.6907	**0.4361**	0.5481
	0.95	1.9258	0.5633	0.9577	0.4658	0.9086	0.6905	**0.4198**	0.4977
	0.99	11.5971	2.7419	5.5191	1.1236	0.6787	0.2634	0.3442	**0.1049**
200	**0.85**	0.4640	**0.2181**	0.3078	0.2331	0.3962	0.3779	0.2611	0.3652
	0.90	0.7360	**0.2947**	0.4311	0.3148	0.5599	0.5141	0.3352	0.4833
	0.95	1.2895	0.3925	0.6296	0.3830	0.7553	0.6216	**0.3710**	0.5234
	0.99	6.7107	1.6387	3.1791	0.8844	0.8459	0.4216	0.4236	**0.1841**
300	**0.85**	0.3611	**0.1903**	0.2607	0.1986	0.3189	0.3103	0.2272	0.3037
	0.90	0.4911	**0.2266**	0.3203	0.2415	0.4125	0.3917	0.2692	0.3772
	0.95	0.8613	**0.3044**	0.4595	0.3305	0.6126	0.5458	0.3406	0.5005
	0.99	4.3508	1.0467	1.9834	0.6853	0.8974	0.5114	0.4182	**0.2588**

Table 7.
MSE values of different estimators when $\phi = 0.85$ and $p = 6$.

n	ρ	COMPML	k_1	k_2	k_3	k_4	k_5	k_6	k_7
30	**0.85**	3.0063	1.1234	1.9184	**0.6914**	2.1313	1.4670	0.8556	0.9025
	0.90	4.9830	1.7482	3.0346	1.1015	2.9007	1.5345	1.0271	**0.8244**
	0.95	10.5805	3.4982	6.4953	1.8040	3.7328	1.2249	1.0583	**0.4448**
	0.99	62.5657	18.4402	37.8865	6.1226	2.2271	0.2364	0.6564	**0.0631**
50	**0.85**	1.7274	0.7986	1.2449	**0.5433**	1.4677	1.2216	0.7599	0.9122
	0.90	2.8117	1.1228	1.8257	**0.7545**	2.1336	1.4828	0.8911	0.9835
	0.95	6.0510	2.0823	3.6451	1.2808	3.1981	1.4899	1.0543	**0.7409**
	0.99	32.3803	9.9146	19.5210	4.0601	2.9150	0.4284	0.8095	**0.1078**
75	**0.85**	1.4625	0.7008	1.0807	**0.4764**	1.2711	1.0994	0.6901	0.8483
	0.90	1.7492	0.8047	1.2577	**0.5510**	1.4953	1.2453	0.7734	0.9288
	0.95	3.5364	1.3379	2.2336	**0.8869**	2.4397	1.5503	0.9585	0.9366
	0.99	18.7715	6.1142	11.5077	2.7676	3.7129	0.7301	0.9292	**0.2017**
100	**0.85**	1.0081	0.5446	0.8126	**0.3564**	0.9232	0.8563	0.5760	0.7043
	0.90	1.1007	0.5695	0.8661	**0.3841**	0.9984	0.9098	0.6036	0.7297
	0.95	2.8770	1.1323	1.8612	**0.7678**	2.1482	1.4772	0.8969	0.9512
	0.99	15.1410	5.0688	9.2917	2.6003	3.7678	0.9258	1.0571	**0.2921**
200	**0.85**	0.4805	0.3257	0.4340	**0.1997**	0.4635	0.4542	0.3563	0.4073
	0.90	0.6615	0.4123	0.5759	**0.2621**	0.6286	0.6068	0.4486	0.5288
	0.95	1.4408	0.7031	1.0778	**0.4890**	1.2598	1.1035	0.7114	0.8644
	0.99	6.7163	2.4138	4.1643	1.3628	3.4574	1.4978	1.0565	**0.6879**
300	**0.85**	0.3518	0.2607	0.3278	**0.1564**	0.3431	0.3395	0.2803	0.3136
	0.90	0.4457	0.3079	0.4056	**0.1878**	0.4312	0.4234	0.3360	0.3824
	0.95	0.8178	0.4797	0.6885	**0.3188**	0.7656	0.7252	0.5185	0.6184
	0.99	4.1140	1.5209	2.5766	0.9525	2.6993	1.5723	0.9685	**0.8719**

Table 8.
MSE values of different estimators when $\phi = 1$ and $p = 6$.

n	ρ	COMPML	k_1	k_2	k_3	k_4	k_5	k_6	k_7
30	**0.85**	3.4033	1.0154	1.9404	**0.6138**	2.0526	1.3305	0.7404	0.7660
	0.90	5.4444	1.5769	3.0488	0.9114	2.6614	1.3271	0.8243	**0.6340**
	0.95	11.8777	3.3391	6.8173	1.4976	3.1386	0.9636	0.7996	**0.2859**
	0.99	71.8876	18.4547	40.7879	5.4868	1.3016	0.1312	0.4409	**0.0198**
50	**0.85**	1.8875	0.6843	1.1747	**0.4914**	1.4754	1.1814	0.6856	0.8625
	0.90	3.1028	0.9841	1.7785	**0.6643**	2.0581	1.3492	0.7711	0.8371
	0.95	6.8419	1.9490	3.8238	1.0654	2.8679	1.2515	0.8386	**0.5281**
	0.99	35.5422	9.2235	19.9561	3.1942	1.9811	0.2751	0.5334	**0.0453**
75	**0.85**	1.5634	0.5863	0.9943	**0.4305**	1.2683	1.0639	0.6214	0.8159
	0.90	1.9213	0.6896	1.1956	**0.4991**	1.5079	1.2101	0.6929	0.8705
	0.95	3.8325	1.1591	2.1716	**0.7445**	2.2494	1.3660	0.7984	0.7582
	0.99	20.9890	5.7910	11.9319	2.3795	2.5485	0.5018	0.6708	**0.1029**
100	**0.85**	1.0925	0.4565	0.7587	**0.3403**	0.9513	0.8613	0.5390	0.7122
	0.90	1.2070	0.4794	0.8127	**0.3538**	1.0361	0.9157	0.5545	0.7348
	0.95	3.1746	0.9973	1.8227	**0.6614**	2.0704	1.3462	0.7595	0.8103
	0.99	16.8374	4.7963	9.5599	2.2682	2.7591	0.6514	0.7927	**0.1673**
200	**0.85**	0.4993	0.2641	0.4101	**0.1726**	0.4702	0.4569	0.3284	0.4088
	0.90	0.7033	0.3390	0.5430	**0.2469**	0.6469	0.6166	0.4233	0.5391
	0.95	1.5787	0.5974	1.0081	**0.4571**	1.2817	1.0826	0.6506	0.8444
	0.99	7.5467	2.2806	4.3373	1.2098	3.0077	1.2282	0.8634	**0.4732**
300	**0.85**	0.3576	0.2131	0.3109	**0.1338**	0.3429	0.3377	0.2611	0.3108
	0.90	0.4609	0.2505	0.3837	**0.1646**	0.4361	0.4250	0.3125	0.3833
	0.95	0.8801	0.4010	0.6489	**0.3020**	0.7916	0.7368	0.4879	0.6317
	0.99	4.6385	1.4080	2.6473	0.8539	2.5258	1.3754	0.8222	**0.6809**

Table 9.
MSE values of different estimators when $\phi = 1.25$ and $p = 6$.

n	ρ	COMPML	k_1	k_2	k_3	k_4	k_5	k_6	k_7
30	**0.85**	3.9082	0.8720	1.9244	**0.5261**	1.7776	1.1592	0.6308	0.6150
	0.90	6.1759	1.3879	3.0814	0.7423	2.1891	1.1189	0.6704	**0.4728**
	0.95	13.1663	2.8553	6.6706	1.1136	2.1174	0.7060	0.5790	**0.2075**
	0.99	83.3779	16.5587	41.7990	3.8061	0.6917	0.1186	0.2842	**0.0631**
50	**0.85**	2.1357	0.6116	1.1504	**0.4722**	1.4751	1.1601	0.6445	0.8317
	0.90	3.5629	0.9013	1.8379	**0.5921**	1.9214	1.2608	0.6863	0.7154
	0.95	7.6898	1.7593	3.8808	0.8594	2.2919	1.0290	0.6704	**0.3982**
	0.99	39.6810	8.5491	20.2823	2.4983	1.2305	0.2209	0.3893	**0.0732**
75	**0.85**	1.7594	0.5276	0.9608	**0.4415**	1.2879	1.0696	0.6096	0.8226
	0.90	2.2009	0.6189	1.1747	**0.4759**	1.5129	1.1838	0.6415	0.8346
	0.95	4.2417	1.0139	2.1416	0.6275	1.9782	1.2087	0.6750	**0.6220**
	0.99	23.6035	5.3286	12.1443	1.8847	1.6043	0.3723	0.4802	**0.1024**
100	**0.85**	1.2633	0.4100	0.7247	**0.3695**	1.0131	0.9041	0.5432	0.7594
	0.90	1.4157	0.4314	0.7814	**0.3706**	1.1057	0.9572	0.5433	0.7707
	0.95	3.5470	0.8612	1.7908	**0.5674**	1.8604	1.2045	0.6524	0.6665
	0.99	18.9142	4.3388	9.7651	1.6635	1.8969	0.4869	0.5429	**0.1344**
200	**0.85**	0.6027	0.2549	0.4206	**0.2317**	0.5488	0.5304	0.3702	0.4901
	0.90	0.8267	0.3112	0.5249	**0.2933**	0.7244	0.6839	0.4492	0.6179
	0.95	1.7881	0.5128	0.9585	**0.4272**	1.2821	1.0646	0.6023	0.8133
	0.99	8.4640	2.0584	4.4077	0.9787	2.3255	0.9905	0.6779	**0.3458**
300	**0.85**	0.4462	0.2195	0.3441	**0.1934**	0.4190	0.4118	0.3120	0.3896
	0.90	0.5625	0.2452	0.4012	**0.2202**	0.5165	0.5012	0.3538	0.4667
	0.95	1.0315	0.3711	0.6296	**0.3464**	0.8740	0.8025	0.5079	0.7081
	0.99	5.2347	1.2731	2.7128	0.7007	2.1698	1.1809	0.6724	**0.5211**

6. Real-life Application: Plywood Quality Data

In this section, we use a dataset from Marcondes Filho and Sant'Anna [20] to evaluate the effect of four variables over the number of defects found in produced plywood. This data is also used by Algamal et al. [13] and showed that the data is suitable for the COM Poisson model.

As in Marcondes Filho and Sant'Anna [20], we are considering the number of defects per laminated plastic plywood area as the response variable (y) and the following input variables: volumetric shrinkage (x_1), assembly time (x_2), wood density (x_3), and drying temperature (x_4). See Marcondes Filho and Sant'Anna [20] for more details about this data.

In fact, this data was chosen because it focuses on the number of defects in laminated plastic plywood, directly reflecting the efficiency of production processes. By analyzing factors such as volumetric shrinkage, assembly time, wood density, and drying temperature, the research aims to identify optimal manufacturing conditions that minimize waste and improve product quality. Improving quality and reducing defects contribute to lower material and energy consumption, which aligns with broader sustainability goals in industrial operations. Although the variables used in our study are primarily technical, reducing defects in production leads to less raw material waste and energy usage. This indirectly supports environmental sustainability by promoting more responsible resource use and minimizing the environmental footprint of manufacturing processes. Optimizing process parameters can help manufacturers adopt more sustainable practices, such as lowering drying temperatures or reducing assembly time, which may result in reduced emissions and lower overall energy demand.

To investigate the existence of multicollinearity in this data, we calculate correlation coefficients between the explanatory variables, the variance inflation factors (VIFs), and the condition number (CN). The correlation coefficients are presented in Figure 1. The VIFs of the four independent variables (x_1, x_2, x_3, x_4) are 33.579316 31.306277, 1.455364, 1.873734, respectively. While the CN is 1041.749. The correlation, VIF, and CN values confirm the presence of severe multicollinearity.

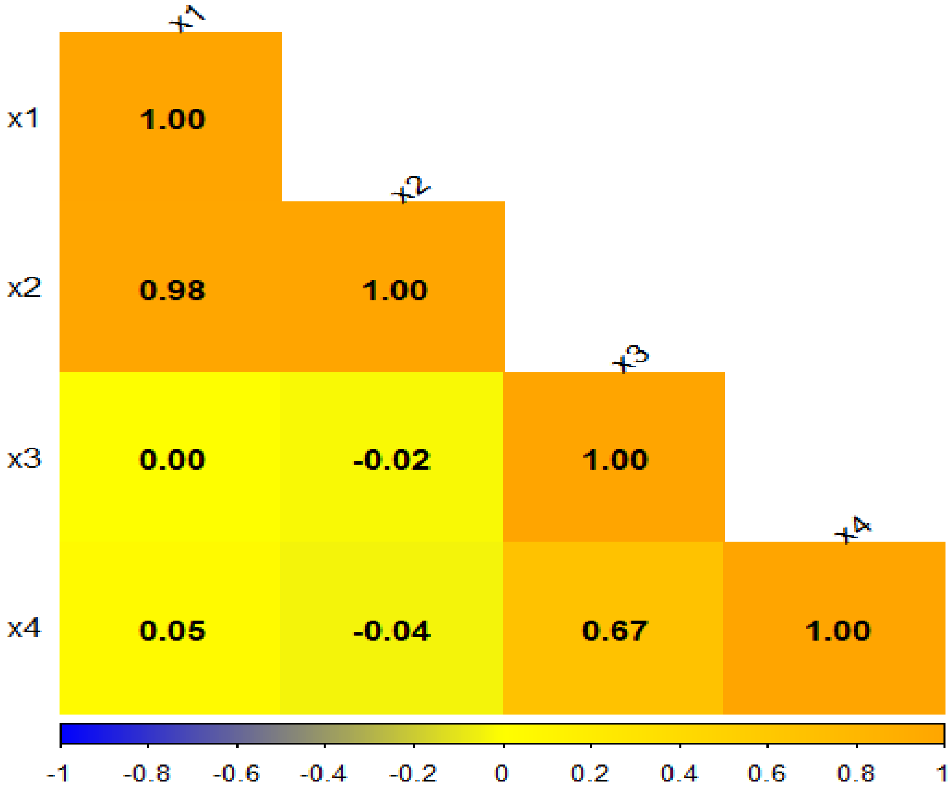

Figure 1.
Heatmap correlation matrix of the independent variables of the plywood data.

Table 8.
Estimated coefficients and MSEs for several ridge estimators of the plywood data.

Variable	COMPML	COMPRR Estimators						
		k_1	k_2	k_3	k_4	k_5	k_6	k_7
x_1	0.9594	0.9587	0.9592	0.9592	0.9594	0.9594	0.9548	0.9591
x_2	-0.7003	-0.6997	-0.7002	-0.7002	-0.7003	-0.7003	-0.6969	-0.7001
x_3	-0.0799	-0.0791	-0.0797	-0.0797	-0.0799	-0.0799	-0.0753	-0.0796
x_4	0.0149	0.0149	0.0149	0.0149	0.0149	0.0149	0.0149	0.0149
MSE	0.02792	0.00523	0.00533	0.00533	0.00536	0.00536	**0.00467**	0.00531
Estimated k	--------	1.0268	0.2574	0.2574	0.0103	0.0366	6.0097	0.4327

Table 8 shows the estimates of the regression parameters and the estimated MSE values for the different estimators. From Table 8 we see the following:

1. The COMPML estimator has the worst performance among all ridge estimators (from k_1 to k_7).
2. The COMPRR estimator with k_6, which has the lowest MSE value, is the best-performing estimator.

7. Conclusion

In this paper, we have presented new methods for ridge parameter estimation when multicollinearity exists. Through a simulation study, the effectiveness of the suggested estimators was investigated for various combinations of correlation, number of explanatory variables, sample size, and the value of the dispersion parameter. The estimators k_1, k_3, k_5, k_6, k_7 were chosen because they give the lowest MSE values. In summary, the estimators' MSE demonstrated superior performance when the number of explanatory variables, dispersion parameters, rose. Additionally, it was noted that the average performance of k_1, k_3, k_5, k_6, k_7 was superior across all correlation levels, sample size, dispersion parameter, and number of explanatory variables. This result was confirmed by applying real data on plywood quality. Future work could expand a new estimation method to more advanced count regression models and larger datasets.

References

[1] A. E. Hoerl and R. W. Kennard, "Ridge regression: Biased estimation for nonorthogonal problems," *Technometrics*, vol. 12, no. 1, pp. 55-67, 1970.

[2] K. Månsson, "On ridge estimators for the negative binomial regression model," *Economic Modelling*, vol. 29, no. 2, pp. 178-184, 2012.

[3] S. Türkan and G. Özel, "A new modified Jackknifed estimator for the poisson regression model," *Journal of Applied Statistics*, vol. 43, no. 10, pp. 1892-1905, 2016.

[4] S. Kaçıranlar and I. Dawoud, "On the performance of the poisson and the negative binomial ridge predictors," *Communications in Statistics-Simulation and Computation*, vol. 47, no. 6, pp. 1751-1770, 2018.

[5] C. Zaldivar, "On the performance of some poisson ridge regression estimators," Master's Thesis, 2019. https://doi.org/10.25148/etd.fidc006538

[6] N. K. Rashad and Z. Y. Algamal, "A new ridge estimator for the poisson regression model," *Iranian Journal of Science and Technology, Transactions A: Science*, vol. 43, no. 6, pp. 2921-2928, 2019.

[7] E. G. Yehia, "On the restricted poisson ridge regression estimator," *Science Journal of Applied Mathematics and Statistics*, vol. 9, no. 4, pp. 106-112, 2021.

[8] M. R. Abonazel, A. R. R. Alzahrani, A. A. Saber, I. Dawoud, E. Tageldin, and A. R. Azazy, "Developing ridge estimators for the extended poisson-tweedie regression model: Method, simulation, and application," *Scientific African*, vol. 23, p. e02006, 2024.

[9] M. N. Akram, M. Amin, F. Sami, A. B. Mastor, O. M. Egeh, and A. H. Muse, "A new conway maxwell–poisson liuregression estimator—method and application," *Journal of Mathematics*, vol. 2022, no. 1, p. 3323955, 2022.

[10] Y. Al-Taweel and Z. Algamal, "Some almost unbiased ridge regression estimators for the zero-inflated negative binomial regression model," *Periodicals of Engineering and Natural Sciences*, vol. 8, no. 1, pp. 248-255, 2020.

[11] Z. Y. Algamal, A. F. Lukman, M. R. Abonazel, and F. A. Awwad, "Performance of the ridge and liu estimators in the zero-inflated bell regression model," *Journal of Mathematics*, vol. 2022, no. 1, p. 9503460, 2022. https://doi.org/10.1155/2022/9503460

[12] F. Sami, M. Amin, and M. M. Butt, "On the ridge estimation of the conway-maxwell poisson regression model with multicollinearity: Methods and applications," *Concurrency and Computation: Practice and Experience*, vol. 34, no. 1, p. e6477, 2022.

[13] Z. Y. Algamal, M. R. Abonazel, F. A. Awwad, and E. T. Eldin, "Modified jackknife ridge estimator for the conway-maxwell-poisson model," *Scientific African*, vol. 19, p. e01543, 2023.

[14] C. Tanış and Y. Asar, "Liu-type estimator in Conway–Maxwell–Poisson regression model: Theory, simulation and application," *Statistics*, vol. 58, no. 1, pp. 65-86, 2024.

[15] B. Ashraf, M. Amin, and M. N. Akram, "New ridge parameter estimators for the zero-inflated Conway Maxwell Poisson ridge regression model," *Journal of Statistical Computation and Simulation*, vol. 94, no. 8, pp. 1814-1840, 2024.

[16] I. Dawoud, "New biased estimators for the conway–maxwell-poisson model," *Journal of Statistical Computation and Simulation*, vol. 95, no. 1, pp. 117-136, 2025.

[17] N. A. Alreshidi, M. A. Alrasheedi, A. F. Lukman, H. Alrweili, and R. A. Farghali, "Modified kibria–lukman estimator for the conway–maxwell–poisson regression model: Simulation and application," *Mathematics*, vol. 13, no. 5, p. 794, 2025. https://doi.org/10.3390/math13050794

[18] M. R. Özkale and U. Mammadova, "Performance analysis of shrinkage estimators in conway-maxwell-poisson regression model," *Communications in Statistics - Theory and Methods*, pp. 1-29, 2025. https://doi.org/10.1080/03610926.2025.2484449

[19] N. S. Hawa, M. Y. Mustafa, B. M. G. Kibria, and Z. Y. Algamal, "Bootstrap liu-type estimator for conway-maxwell-poisson regression model," *Communications in Statistics - Simulation and Computation*, pp. 1-12, 2025. https://doi.org/10.1080/03610918.2025.2462680

[20] D. Marcondes Filho and A. M. O. Sant'Anna, "Principal component regression-based control charts for monitoring count data," *The International Journal of Advanced Manufacturing Technology*, vol. 85, no. 5, pp. 1565-1574, 2016.

[21] R. W. Conway and W. L. Maxwell, "A queuing model with state dependent service rates," *Journal of Industrial Engineering*, vol. 12, no. 2, pp. 132–136, 1962.

[22] K. F. Sellers and B. Premeaux, "Conway–maxwell–poisson regression models for dispersed count data," *Wiley Interdisciplinary Reviews: Computational Statistics*, vol. 13, no. 6, p. e1533, 2021.

[23] S. D. Guikema and J. P. Goffelt, "A flexible count data regression model for risk analysis," *Risk Analysis: An International Journal*, vol. 28, no. 1, pp. 213-223, 2008.

[24] B. Segerstedt, "On ordinary ridge regression in generalized linear models," *Communications in Statistics-Theory and Methods*, vol. 21, no. 8, pp. 2227-2246, 1992.

[25] M. R. Abonazel, "New modified two-parameter liu estimator for the conway–maxwell poisson regression model," *Journal of Statistical Computation and Simulation*, vol. 93, no. 12, pp. 1976-1996, 2023.

[26] M. R. Abonazel, "A new biased estimation class to combat the multicollinearity in regression models: Modified two--parameter liu estimator," *Computational Journal of Mathematical and Statistical Sciences*, vol. 4, no. 1, pp. 316-347, 2025.

[27] B. G. Kibria, "Performance of some new ridge regression estimators," *Communications in Statistics-Simulation and Computation*, vol. 32, no. 2, pp. 419-435, 2003.

[28] R. Shamany, N. N. Alobaidi, and Z. Y. Algamal, "A new two-parameter estimator for the inverse gaussian regression model with application in chemometrics," *Electronic Journal of Applied Statistical Analysis*, vol. 12, no. 2, pp. 453-464, 2019.

[29] M. R. Abonazel and I. Dawoud, "Developing robust ridge estimators for poisson regression model," *Concurrency and Computation: Practice and Experience*, vol. 34, no. 15, p. e6979, 2022.

[30] M. R. Abonazel, A. A. Saber, and F. A. Awwad, "Kibria–lukman estimator for the conway–maxwell poisson regression model: Simulation and applications," *Scientific African*, vol. 19, p. e01553, 2023.