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An unbiased ridge estimator for the poisson regression model: Method, simulation, and application to plywood quality

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Abstract

The Poisson model is a commonly used model for analyzing count data in regression. The Poisson maximum likelihood (PML) estimator is used to estimate the regression parameters of the model. Such the PML is affected by multicollinearity problems, then biased estimators have been developed to address this problem, such as the Poisson ridge (PR) estimator. Even though the PR estimator reduces the effect of multicollinearity problems in the model, it increases the bias in estimation. So, recently, some estimators have been proposed to reduce this bias, such as the Jackknifed Poisson ridge (JPR) and modified Jackknifed Poisson ridge (MJPR) estimators. These estimators have two advantages: firstly, they reduce the effect of multicollinearity, and secondly, they decrease the bias and consequently improve their performance. To remove the effect of bias from the estimator, we introduce an unbiased Poisson ridge (UPR) estimator by analogy with the unbiased ridge estimator. We derive the properties of the UPR estimator. We compare the proposed estimator (UPR) with other existing estimators (PML, PR, JPR, and MJPR) theoretically using the mean square error matrix as a measure of goodness of fit. Furthermore, a simulation study and real-life application have been provided to support the theoretical findings.

Keywords: Biased estimators, Environmental sustainability, Jackknifed Poisson ridge, Mean square error, Modified Jackknifed Poisson ridge, Multicollinearity, Poisson maximum likelihood.

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1. Introduction

In a multiple linear regression model, the ordinary least squares (OLS) estimator is the best linear unbiased estimator when all assumptions of the model are satisfied. However, in real life, the explanatory variables often suffer from correlation, which is called multicollinearity. For this reason, researchers have developed some methods to address this problem. One popular method is to use biased estimation techniques, such as ridge regression, which is a well-known approach used for this purpose.

In case we have account data, the Poisson regression is the model used to model the data by using the maximum likelihood estimation method (MLE). The MLE is often used to estimate the coefficients [1]. However, unfortunately, the problem of multicollinearity affects the variance of MLE, and the OLS estimator becomes unstable [2]. Therefore, some methods have been identified to reduce the effect of multicollinearity on the estimation of parameters in the Poisson regression model.

Månsson and Shukur [2] suggested a Poisson ridge regression estimator and developed some existing ridge estimators to the Poisson regression model. Also, Månsson et al. [3] introduced the generalization of the Liu estimator for the Poisson regression model and suggested some procedures for choosing the parameter d. Asar and Genç [4] proposed a two-parameter estimator for the Poisson regression model and suggested some methods to select its parameters (k, d). Türkan and Özel [5] proposed the jackknifed Poisson ridge (JPR) estimator and the modified Jackknifed Poisson ridge (MJPR) estimator as a solution to the problem of multicollinearity in the Poisson model. Recently, Qasim et al. [6] derived the properties of JPR and MJPR estimators and proposed a new ridge estimator for the ridge parameter for them. Recently, Alheety et al. [7] proposed a new biased estimator to improve the performance of biased estimators in Poisson regression. Also, Alkhateeb and Algamal [8] proposed a jackknifed Liu-type estimator in a Poisson regression model.

In this paper, we utilize the philosophy of developing a new estimator that is unbiased, a procedure originally introduced by Crouse et al. [9]. We refer to the proposed estimator as the unbiased Poisson ridge (UPR) estimator.

The paper is organized as follows: In Section 2, statistical methodology is presented, while in Section 3, a comparison study between the proposed UPR estimator and other existing estimators is conducted through a Monte Carlo simulation study. Section 4 contains a real-life application. Finally, the conclusions with some remarks are provided in Section 5.

2. Statistical Methodology

2.1. Maximum Likelihood Estimation and the Biased Estimators

Suppose y_j for $j = 1, \dots, n$ is the dependent variable and follows a Poisson distribution with parameter μ_j with the following probability mass function:

$$f(y_j) = \frac{\exp(\mu_j)\mu_j{}^{y_j}}{y_j!}; \quad y_j = 0,1,2, \dots \tag{1}$$

For the Poisson regression model, μ_j can be written as $\mu_j = \exp(x_j\beta)$ as the mean response function for the Poisson regression model, where x_j is the jth row of X which is a $n \times (p + 1)$ matrix with p explanatory variables and β is a $(p + 1) \times 1$ vector of regression coefficients. The traditional MLE is used to estimate β. The log likelihood of this model corresponds to:

$$L(\beta; y) = \sum_{j=1}^{n}\{y_j(x_j\beta) - ex\,p(x_j\beta) - \log(\prod_{j=1}^{n} y_j\,!)\} \tag{2}$$

Solving $L(\beta; y)$ with respect to β results in:

$$\frac{\partial L}{\partial \beta} = \sum_{j=1}^{n}(y_j - \exp\,(x_j\beta))\,x_j = 0 \tag{3}$$

Now, we use the iteratively re-weighted least squares (IRLS) algorithm to get the Poisson maximum likelihood (PML) estimator, which can be written as follows:

$$\hat{\beta}_{\text{PML}} = \left(X'\widehat{W}X\right)^{-1}X'\widehat{W}Z = S^{-1}X'\widehat{W}Z, \tag{4}$$

where $S = (X'\widehat{W}X)$, $\widehat{W} = diag(\hat{\mu}_j)$ and Z is the column vector with $z_j = \log(\hat{\mu}_j) + \frac{y_j - \hat{\mu}_j}{\hat{\mu}_j}$. The $\hat{\beta}_{\text{PML}}$ is an asymptotically unbiased estimator of β. Then the variance-covariance matrix of the PML estimator is

$$Var(\hat{\beta}_{\text{PML}}) = \left(X'\widehat{W}X\right)^{-1} = S^{-1}. \tag{5}$$

In general case, the mean squared error matrix (MSEM) of an estimator $\tilde{\beta}$ of β can be written as:

$$MSEM(\tilde{\beta}) = E[(\tilde{\beta} - \beta)(\tilde{\beta} - \beta)'] = Var(\tilde{\beta}) + Bias(\tilde{\beta})Bias(\tilde{\beta})'. \tag{6}$$

Then the scalar mean squared error (MSE) is denoted as

$$MSE(\tilde{\beta}) = trac\{MSEM(\tilde{\beta})\}. \tag{7}$$

Suppose that P and Λ are the eigenvector and eigenvalues of the S matrix, where $\Lambda = diag\{\lambda_i\}$; such that $\lambda_1 > \lambda_2 > \dots > \lambda_{p+1}$ and $\lambda_i > 0$ for all $i = 1,2, \dots, (p + 1)$. Hence, the MSE of the PML estimator is defined as

$$MSE(\hat{\beta}_{PML}) = trac\,\{S^{-1}\} = trac\,\{(P'\Lambda P)^{-1}\} = \sum_{i=1}^{p+1}\left(\frac{1}{\lambda_i}\right). \tag{8}$$

If the explanatory variables are suffering for high correlation, the matrix S is ill-conditioned and the MLE becomes unstable with high variance. To solve this problem, Månsson and Shukur [2] introduced the Poisson ridge (PR) estimator as follows:

$$\hat{\beta}_{PR} = (S + kI)^{-1}S\hat{\beta}_{PML} = \mathrm{S}_{\mathrm{k}}^{-1}S\hat{\beta}_{PML}\,, \quad k > 0 \tag{9}$$

where $S_k = (S + kI)$, the MSEM and MSE of the PR estimator are

$$MSEM(\hat{\beta}_{PR}) = Var(\hat{\beta}_{PR}) + Bias(\hat{\beta}_{PR})Bias(\hat{\beta}_{PR})' = \mathrm{S}_{\mathrm{k}}^{-1}\mathrm{S}\,\mathrm{S}_{\mathrm{k}}^{-1} + (-\mathrm{k}\mathrm{S}_{\mathrm{k}}^{-1}\beta)(-\mathrm{k}\mathrm{S}_{\mathrm{k}}^{-1}\beta)' \tag{10}$$

$$MSE(\hat{\beta}_{PR}) = \sum_{i=1}^{p+1} \frac{\lambda_i}{(\lambda_i+k)^2} + \sum_{i=1}^{p+1} \frac{k^2\alpha_i^2}{(\lambda_i+k)^2}, \quad (11)$$

where α_i is defined as the ith element of $P'\beta$. The PR estimator overcomes the problem of multicollinearity, but it has some amount of bias. For this reason, Türkan and Özel [5] proposed the Jackknifed Poisson ridge (JPR) estimator and the modified Jackknifed Poisson ridge (MJPR) estimator. These estimators will reduce the bias and the effect of multicollinearity simultaneously.

The JPR estimator is given as follows:

$$\hat{\beta}_{JPR} = (I - (k(S+kI)^{-1})^2)\hat{\beta}_{PML} = (I - (k\mathrm{S}_\mathrm{k}^{-1})^2)\hat{\beta}_{PML}, \qquad k > 0. \quad (12)$$

And the MSEM and MSE of the JPR estimator are

$$MSEM(\hat{\beta}_{JPR}) = Var(\hat{\beta}_{JPR}) + Bias(\hat{\beta}_{JPR})Bias(\hat{\beta}_{JPR})' = (\mathrm{I} - \mathrm{k}^2\mathrm{S}_\mathrm{k}^{-2})\mathrm{S}^{-1}(\mathrm{I} - \mathrm{k}^2\mathrm{S}_\mathrm{k}^{-2}) + (-\mathrm{k}^2\mathrm{S}_\mathrm{k}^{-2}\beta)(-\mathrm{k}^2\mathrm{S}_\mathrm{k}^{-2}\beta)' \quad (13)$$

$$MSE(\hat{\beta}_{JPR}) = \sum_{i=1}^{p+1}\left\{\frac{1}{\lambda_i}\left(1 - \frac{k^2}{(\lambda_i+k)^2}\right)^2\right\} + \sum_{i=1}^{p+1}\left(\frac{k^4\alpha_i^2}{(\lambda_i+k)^4}\right). \quad (14)$$

While the MJPR estimator will be as follows:

$$\hat{\beta}_{MJPR} = (I - (k\mathrm{S}_\mathrm{k}^{-1})^2)(I - k\mathrm{S}_\mathrm{k}^{-1})\hat{\beta}_{PML}, \qquad k > 0. \quad (15)$$

And the MSEM and MSE of the MJPR estimator are

$$MSEM(\hat{\beta}_{MJPR}) = Var(\hat{\beta}_{MJPR}) + Bias(\hat{\beta}_{MJPR})Bias(\hat{\beta}_{MJPR})'$$
$$= (\mathrm{I} - \mathrm{k}^2\mathrm{S}_\mathrm{k}^{-2})(\mathrm{I} - \mathrm{kS}_\mathrm{k}^{-1})\mathrm{S}^{-1}(\mathrm{I} - \mathrm{kS}_\mathrm{k}^{-1})(\mathrm{I} - \mathrm{k}^2\mathrm{S}_\mathrm{k}^{-2}) + (\mathrm{k}S_\mathrm{k}^{-2}(I + \mathrm{kS}_\mathrm{k}^{-1} - \mathrm{k}^2\mathrm{S}_\mathrm{k}^{-2})\beta)\,(\mathrm{k}S_\mathrm{k}^{-2}(I + \mathrm{kS}_\mathrm{k}^{-1} - \mathrm{k}^2\mathrm{S}_\mathrm{k}^{-2})\beta)' \quad (16)$$

$$MSE(\hat{\beta}_{MJPR}) = \sum_{i=1}^{p+1}\left\{\frac{1}{\lambda_i}\left(\frac{(\lambda_i+k)^2-k^2}{(\lambda_i+k)^2}\right)^2\left(\frac{\lambda_i}{(\lambda_i+k)^2}\right)^2\right\} + \sum_{i=1}^{p+1}\left\{\frac{k^2\alpha_i^2}{(\lambda_i+k)^4}\left(1 + \frac{k}{\lambda_i+k} - \frac{k^2}{(\lambda_i+k)^2}\right)^2\right\}. \quad (17)$$

Qasim et al. [6] showed that the MJPR estimator with special ridge estimator k has a smaller MSE than PML, PR, and JPR estimators.

2.2. The Proposed Estimator

In order to remove the effect of the bias in the ridge regression estimator for linear regression model, Crouse et al. [9] presented the unbiased ridge estimator (URE) as a linear combination of prior information with the OLS estimator as follows.:

$$\hat{\beta}(k,J) = (X'X + kI)^{-1}(X'Y + kJ), \quad (18)$$

where J is a random vector with $J \sim N(\beta, \frac{\sigma^2}{k}I)$, where σ^2 is the variance of the error term in the linear regression model. Özkale and Kaçiranlar [10] compared the URE with the OLS estimator for the linear regression model and showed that the URE is better than the OLS estimator in the MSEM sense. With the method used by Crouse et al. [9], we suggest an unbiased ridge estimator for the Poisson regression model as follows:

$$\hat{\beta}_{UPR} = (S + kI)^{-1}(X'\widehat{W}Z + \mathrm{kJ}) = \mathrm{S}_\mathrm{k}^{-1}(X'\widehat{W}Z + \mathrm{kJ}), \quad (19)$$

where J~ $(\beta, \frac{1}{k}I)$. For (19), it is easy to see that $\hat{\beta}_{UPR}$ is an unbiased estimator of β and we call this estimator as unbiased Poisson ridge (UPR) estimator. And the MSEM and MSE of the UPR estimator are

$$MSEM(\hat{\beta}_{UPR}) = \mathrm{S}_\mathrm{k}^{-1}, \quad (20)$$

$$MSE(\hat{\beta}_{UPR}) = \sum_{\mathrm{i}=1}^{\mathrm{p}+1}\left(\frac{1}{\lambda_\mathrm{i}+\mathrm{k}}\right). \quad (21)$$

In the following section, we will compare the new estimator with the MLE estimator, the PR estimator, the JPR estimator, and the MJPR estimator in the MSEM. Firstly, we must use the MSE as a criterion for goodness of fit, as it contains all relevant information regarding the estimators, such as variance and bias.

To evaluate the performance of the proposed estimator in comparison to other estimators presented in this study, it is necessary to provide the following lemma, which will be useful for this purpose.

Lemma 1: (See Farebrother [11]): *Let* M *be a positive definite (p.d.) matrix and* a *be a vector, then* $\mathrm{M} - aa' \geq 0$ *if and only if* $a'\mathrm{M}^{-1}a \leq 1$.

2.3. The Comparison between the MLE and UPR Estimators

Theorem 1: The UPR estimator is always better than the PML estimator in the MSEM sense.

Proof:

Let $\Delta_1 = MSEM(\hat{\beta}_{\mathrm{PML}}) - MSEM(\hat{\beta}_{\mathrm{UPR}})$

$$= S^{-1} - S_k^{-1} = P\{\Lambda^{-1} - (\Lambda + kI)^{-1}\}P'$$
$$= P\left(diag\left\{\frac{1}{\lambda_i} - \frac{1}{\lambda_i+k}\right\}_{i=1}^{p+1}\right)P'.$$

Since $\lambda_i + k > \lambda_i$, then $\frac{1}{\lambda_i} > \frac{1}{\lambda_i+k}$, for all i. Then Δ_1 is p.d. and the proof is completed.

2.4. The comparison between the PR and UPR estimators

Theorem 2: If $k > (1/\alpha_i^2)$, the UPR estimator will be better than the PR estimator in the MSEM sense.

Proof:
Let $\Delta_2 = MSEM(\hat{\beta}_{PR}) - MSEM(\hat{\beta}_{UPR})$
$= S_k^{-1} S S_k^{-1} + k^2 S_k^{-1}\beta\beta' S_k^{-1} - S_k^{-1}$
$= (S_k^{-1} S - I) S_k^{-1} + k^2 S_k^{-1}\beta\beta' S_k^{-1}$
$= P\left(diag\left\{\frac{\lambda_i}{(\lambda_i + k)^2} - \frac{1}{(\lambda_i+k)} + \frac{k^2\alpha_i^2}{(\lambda_i+k)^2}\right\}_{i=1}^{p+1}\right)P' = P\left(diag\left\{\frac{k(k\alpha_i^2 - 1)}{(\lambda_i+k)^2}\right\}_{i=1}^{p+1}\right)P'.$
Then Δ_2 will be p.d. when $k > (1/\alpha_i^2)$ and the proof is completed.

2.5. The Comparison Between the JPR and UPR Estimators

Theorem 3: Under the Poisson regression model, if $\lambda_i(\lambda_i + 2k)^2/(\lambda_i + k)^3 < 1$, the UPR estimator is better than the JPR estimator if $B_1' D_1^{-1} B_1 > 1$.
Proof:
Let $\Delta_3 = MSEM(\hat{\beta}_{UPR}) - MSEM(\hat{\beta}_{JPR})$
$= s_k^{-1} - (I - k^2 S_k^{-2}) S^{-1} (I - k^2 S_k^{-2}) - k^4 S_k^{-2}\beta\beta' S_k^{-2}$
$= D_1 - B_1 B_1',$
where $D_1 = s_k^{-1} - (I - k^2 S_k^{-2}) S^{-1} (I - k^2 S_k^{-2})$ and $B_1 = k^2 S_k^{-2}\beta$.
So, we are looking for the condition that makes D_1 p. d.

$$D_1 = s_k^{-1} - (I - k^2 S_k^{-2}) S^{-1} (I - k^2 S_k^{-2}) = P\left(diag\left\{\frac{1}{\lambda_i+k} - \frac{\left(1-\left(\frac{k}{\lambda_i+k}\right)^2\right)^2}{\lambda_i}\right\}_{i=1}^{p+1}\right)P'$$

Let $g_i = \frac{1}{\lambda_i+k} - \frac{\left(1-\left(\frac{k}{\lambda_i+k}\right)^2\right)^2}{\lambda_i} = \frac{\lambda_i - (\lambda_i+k)\left(1-\left(\frac{k}{\lambda_i+k}\right)^2\right)^2}{\lambda_i(\lambda_i+k)} = \frac{\lambda_i - {\lambda_i}^2(\lambda_i+2k)^2/(\lambda_i+k)^3}{\lambda_i(\lambda_i+k)}$, then $g_i > 0$, if $\lambda_i(\lambda_i + 2k)^2/(\lambda_i + k)^3 < 1$. Now by applying Lemma 1, the proof is completed.

2.6. The comparison between the MJPR and UPR estimators

Theorem 4: Under the Poisson regression model, when $\frac{{\lambda_i}^3(\lambda_i+k)^2}{(\lambda_i+k)^5} < 1$, the UPR estimator is better than the MJPR estimator if $B_2' D_2^{-1} B_2 > 1$.
Proof:
Let us write the properties of MJPR and UPR in the canonical form. So,

$$\Delta_4 = MSEM(\hat{\beta}_{UPR}) - MSEM(\hat{\beta}_{MJPR})$$
$$= D_2 - B_2 B_2',$$

where $D_2 = S_k^{-1} - (I - k^2 S_k^{-2})(I - k S_k^{-1}) S^{-1} (I - k S_k^{-1})(I - k^2 S_k^{-2})$,
and $B_2 = (k S_k^{-2}(I + k S_k^{-1} - k^2 S_k^{-2})\beta)$.
Now we are searching for the case when D_2 will be p.d. Therefore,
$D_2 = P\left(diag\left\{\frac{1}{\lambda_i+k} - \frac{\left(\lambda_i((\lambda_i+k)^2 - k^2)^2\right)}{(\lambda_i+k)^2}\right\}_{i=1}^{p+1}\right)P'$. So, after some simplifications, $D_2 > 0$ if $\frac{{\lambda_i}^3(\lambda_i+k)^2}{(\lambda_i+k)^5} < 1$. Now by applying Lemma 1, the proof is completed.

2.7. Selection of the Biasing Parameters

In this section, we can suggest the biasing parameters for PR, JPR, MJPR, and UPR estimators as follows:

- The suggested estimator of *k* for the PR estimator is (Qasim et al. [6])

$$\hat{k}_{PR} = \min\left(\frac{1}{\hat{\alpha}_i^2}\right)_{i=1}^{p+1}. \quad (22)$$

Where $\hat{\alpha}_i$ is defined as the *i*th element of $P'\hat{\beta}_{PML}$.

- The suggested estimator of *k* for JPR and MJPR estimators is (Qasim et al. [6])

$$\hat{k}_{JPR} = \min\left\{\frac{\left(1 + \sqrt{\left(1 + \lambda_i\hat{\alpha}_i^2\right)}\right)}{\hat{\alpha}_i^2}\right\}_{i=1}^{p+1}. \quad (23)$$

- The suggested $\hat{k}$ for the UPR estimator is

$$\hat{k}_1 = median\left\{\frac{\left(1+\sqrt{\left(1+\lambda_i\hat{\alpha}_i^2\right)}\right)}{\hat{\alpha}_i^2}\right\}_{i=1}^{p+1}, \tag{24}$$

$$\hat{k}_2 = mean\left\{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}\right\}_{i=1}^{p+1}, \tag{25}$$

$$\hat{k}_3 = median\left\{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}\right\}_{i=1}^{p+1}, \tag{26}$$

$$\hat{k}_4 = \max\left\{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}\right\}_{i=1}^{p+1}, \tag{27}$$

where $\hat{\sigma}^2 = \frac{1}{n-p-1}\sum_{j=1}^{n}\left(y_j - \hat{\mu}_j\right)^2$.

3. Monte-Carlo Simulation Study

We conducted a simulation study to examine the performance of the PML, PRR, JPR, MJPR, and UPR estimators. The program for the simulation study is written in the R programming language.

3.1. Simulation Design

The simulated data is carried with the following settings:

1. The response variable (y_j) is generated from the Poisson distribution with a mean equal to $\mu_j = \exp\left(x_j\beta\right); j = 1, \dots, n$.
2. The values of sample sizes (*n*) were chosen to be 30, 50, 75, 100, 200 and 300.
3. The number of the explanatory variables (p) is chosen to be 3, 6 and 9. Where $\sum_{i=2}^{p+1}\beta_i^2 = 1$ and $\beta_2 = \cdots = \beta_{p+1}$ as in [12-17]. While the intercept is defined as β_1= 0 as in [18, 19].
4. The explanatory variables are generated as in [16, 20-23]: $x_{ji} = \omega_{ji}\sqrt{1-\rho^2} + \rho\omega_{j(p+1)}; j = 1, \dots, n; i = 2, \dots, (p+1)$, where ω_{ji} are the independent standard uniform pseudorandom numbers and ρ is defined as the correlation between the explanatory variables: ρ = 0.80, 0.85, 0.90, 0.95, and 0.99.
5. We used the simulated MSE (SMSE) and simulated total absolute bias (STAB) criteria for verification, which are computed as

$$SMSE\left(\hat{\beta}\right) = \frac{1}{1000}\sum_{l=1}^{1000}\left(\hat{\beta}_l - \beta\right)'\left(\hat{\beta}_l - \beta\right); \tag{28}$$

$$STAB\left(\hat{\beta}\right) = \frac{1}{1000}\sum_{l=1}^{1000}\left(\sum_{j=1}^{p+1}\left|\hat{\beta}_j - \beta_j\right|\right)_l, \tag{29}$$

where $\hat{\beta}_l$ is the estimated value vector at the l^{th} experiment of the simulation and β is the true parameter vector. The experiment is replicated 1000 times.

3.2. Simulation Results

The simulation results are summarized in Tables 1–6. The best value of the averaged SMSE and STAB is highlighted in bold. We can summarize the main simulation results from Tables 1–6 as follows:

1. As expected, the performance of the PML estimator is the lowest among all the considered estimators and should not be recommended for parameter estimation in the Poisson regression model with multicollinearity problems.
2. If the multicollinearity degree (ρ) and the explanatory variables numbers (p) are increasing, then the SMSE and STAB values for all estimators (PML, PRR, JPR, MJPR and UPR) increase.
3. The SMSE and STAB values of all estimators decrease as the sample size (*n*) increases, assuming other factors remain fixed.
4. For the four biased estimators, we can note that the MJPR and UPR estimators are better than PRR and JPR estimators in all simulation cases.
5. Finally, the UPR estimator, especially UPR with $\hat{k}_1$, is the best performing among all the other estimators mentioned in most cases, especially when the model contains highly multicollinearity (ρ > 0.90).

Table 1.
STAB values of different estimators when $p = 3$.

ρ	n	PML	PR	JPR	MJPR	UPR (1)	UPR (2)	UPR (3)	UPR (4)
0.80	30	1.759	1.318	1.299	0.935	0.728	0.774	0.743	0.801
	50	1.149	0.936	0.884	0.690	0.652	0.727	0.665	0.776
	75	0.948	0.787	0.708	0.553	0.539	0.652	0.556	0.723
	100	0.828	0.726	0.658	0.561	0.537	0.654	0.552	0.723
	200	0.615	0.562	0.519	0.478	0.475	0.618	0.471	0.714
	300	0.506	0.477	0.438	0.411	0.437	0.598	0.428	0.701
0.85	30	1.827	1.321	1.291	0.861	0.650	0.725	0.691	0.753
	50	1.346	1.046	0.989	0.699	0.604	0.696	0.633	0.748
	75	1.130	0.905	0.836	0.626	0.553	0.649	0.588	0.706
	100	0.964	0.795	0.736	0.587	0.549	0.662	0.559	0.732
	200	0.707	0.628	0.554	0.453	0.453	0.600	0.466	0.688
	300	0.550	0.510	0.454	0.404	0.429	0.594	0.433	0.694
0.90	30	2.308	1.573	1.565	0.942	0.670	0.716	0.681	0.745
	50	1.564	1.122	1.069	0.697	0.565	0.649	0.602	0.701
	75	1.488	1.079	1.023	0.676	0.537	0.641	0.575	0.694
	100	1.142	0.902	0.821	0.573	0.524	0.632	0.550	0.695
	200	0.803	0.687	0.591	0.450	0.446	0.571	0.466	0.655
	300	0.670	0.601	0.517	0.418	0.420	0.565	0.445	0.657
0.95	30	3.364	2.172	2.179	1.215	0.619	0.666	0.644	0.691
	50	2.494	1.632	1.618	0.913	0.531	0.602	0.569	0.643
	75	1.891	1.277	1.245	0.727	0.497	0.598	0.539	0.638
	100	1.819	1.235	1.194	0.694	0.465	0.564	0.517	0.597
	200	1.189	0.878	0.794	0.511	0.415	0.537	0.461	0.606
	300	0.974	0.775	0.669	0.436	0.390	0.524	0.433	0.601
0.99	30	7.553	4.719	4.735	2.343	0.576	0.587	0.584	0.599
	50	5.541	3.471	3.497	1.817	0.521	0.565	0.541	0.573
	75	3.980	2.445	2.450	1.205	0.426	0.496	0.470	0.516
	100	4.172	2.586	2.595	1.317	0.410	0.488	0.465	0.495
	200	2.525	1.612	1.595	0.822	0.324	0.440	0.424	0.437
	300	2.235	1.425	1.400	0.716	0.297	0.438	0.400	0.434

Table 2.
SMSE values of different estimators when $p = 3$.

ρ	n	PML	PR	JPR	MJPR	UPR (1)	UPR (2)	UPR (3)	UPR (4)
0.80	30	1.260	0.760	0.747	0.350	0.188	0.208	0.204	0.217
	50	0.533	0.360	0.329	0.180	0.154	0.185	0.162	0.205
	75	0.373	0.259	0.220	0.120	0.108	0.154	0.118	0.183
	100	0.275	0.211	0.177	0.115	0.105	0.152	0.114	0.180
	200	0.153	0.128	0.109	0.083	0.083	0.137	0.084	0.173
	300	0.104	0.092	0.077	0.062	0.072	0.130	0.072	0.170
0.85	30	1.396	0.802	0.788	0.320	0.154	0.186	0.181	0.196
	50	0.750	0.469	0.438	0.198	0.133	0.172	0.150	0.192
	75	0.536	0.350	0.314	0.155	0.112	0.153	0.131	0.173
	100	0.379	0.260	0.228	0.129	0.110	0.156	0.118	0.184
	200	0.206	0.162	0.130	0.077	0.076	0.131	0.084	0.165
	300	0.126	0.108	0.086	0.060	0.069	0.129	0.073	0.167
0.90	30	2.283	1.215	1.210	0.424	0.164	0.183	0.176	0.193
	50	1.047	0.576	0.549	0.212	0.118	0.154	0.140	0.173
	75	0.938	0.529	0.504	0.201	0.105	0.148	0.126	0.166
	100	0.558	0.356	0.317	0.140	0.101	0.145	0.116	0.169
	200	0.275	0.201	0.158	0.081	0.074	0.121	0.084	0.151
	300	0.189	0.151	0.117	0.066	0.065	0.118	0.076	0.152
0.95	30	5.143	2.666	2.686	0.910	0.143	0.164	0.165	0.169
	50	2.781	1.437	1.440	0.472	0.107	0.138	0.132	0.152
	75	1.574	0.825	0.815	0.275	0.092	0.134	0.115	0.146
	100	1.461	0.768	0.755	0.253	0.080	0.121	0.108	0.129
	200	0.636	0.364	0.330	0.122	0.063	0.108	0.086	0.131
	300	0.414	0.266	0.221	0.086	0.056	0.105	0.076	0.131
0.99	30	27.648	13.764	13.827	4.082	0.131	0.134	0.146	0.136
	50	15.163	7.531	7.622	2.360	0.103	0.120	0.119	0.121
	75	7.634	3.674	3.691	1.050	0.070	0.096	0.094	0.101
	100	8.456	4.211	4.250	1.296	0.064	0.094	0.092	0.093
	200	3.118	1.583	1.587	0.480	0.041	0.081	0.082	0.076
	300	2.436	1.246	1.244	0.386	0.034	0.083	0.074	0.077

Table 3.
STAB values of different estimators when $p = 6$.

$\boldsymbol{\rho}$	n	PML	PR	JPR	MJPR	UPR (1)	UPR (2)	UPR (3)	UPR (4)
0.80	30	2.671	2.076	2.002	1.305	0.738	**0.691**	0.703	0.696
	50	1.906	1.598	1.446	0.948	**0.627**	0.669	0.633	0.697
	75	1.558	1.370	1.207	0.831	**0.591**	0.663	0.603	0.693
	100	1.399	1.234	1.107	0.807	**0.586**	0.673	0.604	0.698
	200	0.876	0.836	0.744	0.602	**0.514**	0.663	0.560	0.693
	300	0.717	0.696	0.637	0.547	**0.487**	0.661	0.541	0.692
0.85	30	2.850	2.196	2.115	1.276	0.693	**0.668**	0.688	0.687
	50	2.346	1.858	1.729	1.067	0.634	0.651	**0.632**	0.681
	75	1.764	1.496	1.322	0.854	**0.575**	0.644	0.579	0.680
	100	1.530	1.331	1.177	0.805	**0.558**	0.657	0.576	0.688
	200	1.047	0.977	0.843	0.628	**0.501**	0.648	0.537	0.686
	300	0.871	0.828	0.723	0.560	**0.466**	0.643	0.515	0.684
0.90	30	3.600	2.670	2.647	1.593	0.754	**0.675**	0.721	0.689
	50	2.716	2.062	1.974	1.190	0.631	0.642	**0.628**	0.675
	75	2.293	1.805	1.686	1.059	**0.569**	0.634	0.576	0.678
	100	1.840	1.537	1.377	0.875	**0.546**	0.633	0.554	0.677
	200	1.319	1.176	0.995	0.662	**0.458**	0.607	0.483	0.666
	300	1.060	0.981	0.834	0.588	**0.441**	0.613	0.476	0.673
0.95	30	6.336	4.453	4.510	2.614	0.791	**0.625**	0.779	0.636
	50	4.167	2.977	2.980	1.764	0.650	**0.603**	0.647	0.643
	75	3.168	2.303	2.262	1.361	0.600	0.612	**0.582**	0.661
	100	2.893	2.153	2.086	1.244	0.539	0.573	**0.537**	0.629
	200	1.835	1.508	1.331	0.811	**0.478**	0.590	0.485	0.655
	300	1.441	1.259	1.064	0.661	**0.417**	0.576	0.430	0.650
0.99	30	12.299	8.684	8.821	5.094	1.368	**0.599**	0.910	0.611
	50	9.388	6.506	6.628	3.878	0.952	**0.536**	0.723	0.567
	75	7.750	5.423	5.503	3.175	0.926	**0.532**	0.710	0.548
	100	6.346	4.470	4.531	2.566	0.739	**0.505**	0.613	0.538
	200	4.352	3.124	3.139	1.795	0.572	**0.477**	0.535	0.540
	300	3.513	2.520	2.505	1.437	0.486	**0.467**	0.486	0.547

Table 4.
SMSE values of different estimators when $p = 6$.

ρ	n	PML	PR	JPR	MJPR	UPR (1)	UPR (2)	UPR (3)	UPR (4)
0.80	30	1.625	1.020	0.986	0.430	0.144	0.137	0.135	0.141
	50	0.830	0.588	0.509	0.219	0.101	0.123	0.104	0.135
	75	0.559	0.432	0.353	0.166	0.090	0.123	0.097	0.135
	100	0.453	0.351	0.291	0.150	0.089	0.127	0.098	0.137
	200	0.176	0.159	0.127	0.080	0.069	0.125	0.088	0.137
	300	0.117	0.110	0.092	0.065	0.062	0.124	0.081	0.136
0.85	30	1.909	1.178	1.141	0.437	0.126	0.125	0.126	0.134
	50	1.262	0.821	0.754	0.295	0.104	0.118	0.106	0.132
	75	0.727	0.526	0.441	0.187	0.085	0.118	0.090	0.132
	100	0.547	0.414	0.342	0.158	0.082	0.123	0.091	0.135
	200	0.256	0.222	0.170	0.090	0.064	0.119	0.079	0.134
	300	0.175	0.158	0.122	0.069	0.056	0.118	0.073	0.133
0.90	30	3.052	1.801	1.811	0.697	0.153	0.128	0.139	0.135
	50	1.754	1.063	1.025	0.394	0.106	0.117	0.106	0.130
	75	1.284	0.814	0.760	0.310	0.083	0.113	0.087	0.129
	100	0.812	0.569	0.490	0.199	0.077	0.112	0.080	0.130
	200	0.414	0.326	0.251	0.108	0.053	0.105	0.061	0.126
	300	0.272	0.230	0.174	0.082	0.048	0.107	0.060	0.128
0.95	30	10.098	5.639	5.757	2.102	0.190	0.110	0.167	0.117
	50	4.404	2.463	2.503	0.938	0.118	0.103	0.112	0.118
	75	2.560	1.447	1.447	0.551	0.097	0.105	0.090	0.123
	100	2.124	1.241	1.221	0.460	0.078	0.093	0.078	0.114
	200	0.823	0.554	0.471	0.181	0.057	0.097	0.059	0.120
	300	0.513	0.385	0.299	0.116	0.043	0.096	0.048	0.120
0.99	30	41.415	23.189	23.820	8.914	0.721	0.103	0.311	0.108
	50	25.278	13.721	14.106	5.265	0.346	0.083	0.173	0.096
	75	16.509	9.126	9.363	3.481	0.312	0.078	0.154	0.086
	100	10.823	5.983	6.135	2.221	0.213	0.073	0.119	0.086
	200	5.074	2.873	2.930	1.065	0.107	0.065	0.081	0.085
	300	3.287	1.820	1.845	0.672	0.079	0.064	0.070	0.086

Table 5.
STAB values of different estimators when $p = 9$.

ρ	n	PML	PR	JPR	MJPR	UPR (1)	UPR (2)	UPR (3)	UPR (4)
0.80	30	3.072	2.514	2.372	1.530	0.692	0.596	0.596	0.605
	50	2.348	2.041	1.853	1.233	0.632	0.592	0.574	0.603
	75	2.026	1.809	1.607	1.078	0.597	0.599	0.562	0.606
	100	1.709	1.540	1.368	0.971	0.559	0.592	0.545	0.601
	200	1.085	1.047	0.928	0.716	0.514	0.596	0.544	0.604
	300	0.907	0.885	0.801	0.655	0.496	0.597	0.537	0.605
0.85	30	4.203	3.242	3.208	2.060	0.743	0.598	0.615	0.606
	50	2.955	2.443	2.287	1.447	0.663	0.595	0.581	0.604
	75	2.096	1.871	1.643	1.066	0.581	0.590	0.542	0.605
	100	1.822	1.642	1.445	0.993	0.559	0.589	0.541	0.601
	200	1.248	1.192	1.027	0.732	0.490	0.588	0.519	0.600
	300	1.044	1.005	0.880	0.665	0.472	0.589	0.520	0.601
0.90	30	4.673	3.569	3.555	2.195	0.749	0.586	0.621	0.600
	50	3.566	2.848	2.742	1.714	0.664	0.573	0.565	0.595
	75	2.685	2.264	2.066	1.308	0.622	0.584	0.544	0.603
	100	2.269	1.955	1.761	1.162	0.573	0.592	0.534	0.605
	200	1.584	1.470	1.257	0.842	0.487	0.580	0.491	0.602
	300	1.270	1.206	1.023	0.714	0.452	0.574	0.479	0.598
0.95	30	7.998	5.881	5.992	3.811	0.873	0.601	0.728	0.620
	50	5.334	4.006	4.040	2.524	0.759	0.562	0.617	0.593
	75	3.939	3.057	2.994	1.867	0.642	0.564	0.530	0.596
	100	3.162	2.573	2.435	1.517	0.569	0.555	0.482	0.593
	200	2.220	1.932	1.720	1.102	0.492	0.564	0.458	0.597
	300	1.860	1.657	1.438	0.933	0.438	0.550	0.420	0.593
0.99	30	18.822	13.730	14.032	9.082	1.530	0.547	0.767	0.581
	50	12.418	9.172	9.372	6.055	1.245	0.518	0.714	0.560
	75	9.974	7.365	7.532	4.859	1.037	0.500	0.614	0.564
	100	8.126	6.133	6.247	3.892	0.909	0.481	0.641	0.538
	200	5.393	4.064	4.113	2.582	0.658	0.459	0.461	0.546
	300	4.467	3.443	3.424	2.152	0.561	0.454	0.409	0.538

Table 6.
SMSE values of different estimators when $p = 9$.

ρ	n	PML	PR	JPR	MJPR	UPR (1)	UPR (2)	UPR (3)	UPR (4)
0.80	30	1.535	1.035	0.964	0.414	0.099	0.097	0.090	0.100
	50	0.893	0.676	0.584	0.267	0.081	0.094	0.081	0.099
	75	0.677	0.534	0.443	0.204	0.073	0.095	0.079	0.099
	100	0.495	0.393	0.322	0.161	0.065	0.094	0.076	0.099
	200	0.190	0.176	0.138	0.082	0.057	0.095	0.077	0.099
	300	0.132	0.126	0.102	0.067	0.052	0.094	0.074	0.099
0.85	30	2.952	1.832	1.834	0.789	0.110	0.096	0.090	0.100
	50	1.400	0.969	0.889	0.372	0.087	0.094	0.081	0.099
	75	0.713	0.561	0.456	0.198	0.069	0.093	0.075	0.098
	100	0.549	0.439	0.356	0.169	0.065	0.094	0.077	0.099
	200	0.253	0.229	0.173	0.087	0.051	0.094	0.071	0.099
	300	0.182	0.167	0.129	0.072	0.048	0.093	0.070	0.098
0.90	30	3.582	2.194	2.211	0.893	0.112	0.093	0.089	0.099
	50	2.141	1.384	1.335	0.544	0.086	0.088	0.073	0.096
	75	1.195	0.847	0.746	0.309	0.075	0.090	0.071	0.097
	100	0.878	0.637	0.549	0.244	0.066	0.093	0.072	0.098
	200	0.423	0.358	0.275	0.124	0.046	0.089	0.059	0.097
	300	0.268	0.238	0.176	0.085	0.040	0.088	0.058	0.097
0.95	30	12.342	7.145	7.351	3.020	0.157	0.091	0.113	0.099
	50	4.955	2.949	3.019	1.246	0.114	0.081	0.080	0.094
	75	2.748	1.706	1.692	0.702	0.082	0.083	0.063	0.094
	100	1.771	1.179	1.112	0.456	0.063	0.082	0.053	0.094
	200	0.885	0.652	0.557	0.239	0.047	0.084	0.050	0.095
	300	0.626	0.480	0.390	0.167	0.035	0.080	0.041	0.094
0.99	30	79.267	44.617	45.798	18.954	0.559	0.080	0.142	0.092
	50	31.380	17.941	18.519	7.849	0.364	0.069	0.110	0.084
	75	20.513	11.705	12.112	5.169	0.251	0.064	0.078	0.084
	100	12.551	7.456	7.701	3.163	0.191	0.058	0.085	0.076
	200	5.675	3.350	3.442	1.426	0.094	0.055	0.045	0.079
	300	3.886	2.360	2.389	1.002	0.068	0.054	0.035	0.076

4. Empirical Application: Plywood Quality Data

In this section, we use a dataset from Marcondes Filho and Sant'Anna [24] for evaluating the effect of four variables on the number of defects found in produced plywood. As in Marcondes Filho and Sant'Anna [24] we are considering the number of defects per laminated plastic plywood area as the response variable (y) and the following input variables: volumetric shrinkage (x_1), assembly time (x_2), wood density (x_3), and drying temperature (x_4). This data is also used by Algamal et al. [15].

To investigate the existence of multicollinearity, we calculate correlation coefficients between the explanatory variables, the variance inflation factors (VIFs), and the condition number (CN). The correlation coefficients are $\rho_{X1,X2} = 0.98, \rho_{X1,X3} = -0.02, \rho_{X1,X4} = 0.05, \rho_{X2,X3} = -0.05, \rho_{X2,X4} = -0.04, \rho_{X3,X4} = 0.93$. The VIFs of the explanatory variables are 223.20, .19560, 48.48, 56.08, and the CN is 85460.49. The correlation coefficients, VIFs, and CN values confirm the presence of severe multicollinearity.

Table 7 shows the estimates of the regression parameters and the estimated MSE values for the different estimators. From Table 7, we see the following:

1. The PML estimator has the worst performance among all biased estimators (PR, JPR, MJPR, and UPR).
2. UPR(1)−UPR(4) estimators are better than the PR, JPR, and MJPR estimators.
3. The UPR(3) estimator, which has the lowest MSE value, is the best-performing estimator.

Table 7.
Estimated coefficients and MSEs for the wood defects data.

Variable	PML	PR	JPR	MJPR	UPR(1)	UPR(2)	UPR(3)	UPR(4)
Intercept	-20.373	-20.706	-21.301	-21.230	-20.293	-20.248	-20.243	-20.325
x1	-0.620	-0.584	-0.539	-0.449	-0.562	-0.555	-0.554	-0.568
x2	1.506	1.470	1.425	1.330	1.445	1.436	1.435	1.451
x3	-10.890	-9.531	-7.432	-6.008	-10.038	-10.004	-10.000	-10.067
x4	0.089	0.087	0.085	0.082	0.087	0.087	0.087	0.087
MSE	8.433	8.350	8.377	8.112	8.065	8.051	8.050	8.085

5. Conclusion

The presence of multicollinearity problems in the Poisson regression model renders the Poisson maximum likelihood (PML) estimator inefficient and unstable. To address this issue, researchers have proposed several biased Poisson regression estimators, such as the Poisson ridge (PR) estimator, the Jackknifed Poisson ridge (JPR) estimator, and the modified Jackknifed Poisson ridge (MJPR) estimator. In this paper, we proposed an unbiased Poisson ridge (UPR) estimator and demonstrated, both theoretically and through a simulation study, that the proposed (UPR) estimator is more efficient than the existing estimators (PML, PR, JPR, and MJPR). Furthermore, this result was validated by a real-life application. In future work, we can develop a robust version of the UPR estimator as an extension of Dawoud and Abonazel [14], Awwad et al. [17] and Dawoud et al. [20].

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